FLUID MECHANICS SOLUTION

SEM 4 (CBCGS-DEC 2019)

BRANCH-MECHANICAL ENGINEERING

Q 1) a) What is the effect of temperature on viscosity of water and that of the air. (04)

Solution:

1) For Water:

Viscous forces in a fluid are due to cohesive forces and molecular momentum transfer. Therefore, if the temperature of liquid increased, its cohesion and hence viscosity will decrease.

2) For Air :

In gases momentum exchange is dominant. Therefore, if the temperature of gases is increases, its momentum exchange will increase and hence viscosity will increase.

Q 1) b) The weight of block of dimensions $1.2\mathrm{m} \times 1\mathrm{m} \times 1.8\mathrm{m}$ in water is 2542.8 N. Find its weight in air. (04)

Solution:

Given:

Volume of body $= 1.2 \times 1 \times 1.8 = 2.16\ \mathrm{m}^3$

Weight of body in water 2542.8 N n

Volume of the water displaced = Volume of the body $= 2.16\ \mathrm{m}^3$

Weight of the water displaced $= 1000 \times 9.81 \times 2.16 = 21189.6$ N

For the equilibrium of the body

Weight of body in air $-$ Weight of the water displaced = Weight in water

$$W_{air} = 21189.6 + 2542.8 = 23732.4 \text{ N}$$

Q 1) c) If the velocity field is given by $\mathrm{u} = \mathrm{x}^2 - \mathrm{y}^2$ and $\mathrm{v} = -(2\mathrm{xy})$, Check whether i) flow is possible or not ii) rotational or irrotational. (04)

Solution:

Given: $\mathrm{u} = \mathrm{x}^2 - \mathrm{y}^2$

$$\frac{\partial u}{\partial x} = 2x$$

$$\frac{\partial u}{\partial y} = -2y$$

Also $\mathrm{v} = -(2\mathrm{xy})$

$$\frac{\partial v}{\partial x} = -2y$$

$$\frac{\partial v}{\partial y} = -2x$$

i) For a two-dimensional flow, continuity equation is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Substituting the value of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2x - 2x = 0$$

It is a possible case of fluid flow.

ii) Rotation, ω_z is given by $\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = -2y + 2y = 0$

Rotation is zero, which means it is case of irrotational flow.

Q 1) d) The velocity profile within boundary layer for steady, two-dimensional, incompressible, laminar flow over a flat is given by $\frac{\mathbf{u}}{\mathbf{U}_\infty} = \mathbf{A} + \mathbf{B}\left(\frac{\mathbf{y}}{\boldsymbol{\delta}}\right)$ Using suitable boundary condition, evaluate the constants A and B. **(04)**

Solution:

$$\frac{\mathrm{u}}{\mathrm{U}_\infty} = \mathrm{A} + \mathrm{B}\left(\frac{\mathrm{y}}{\delta}\right)$$

A) Boundary condition at wall surface

i) When $\mathrm{y} = 0,\ \mathrm{u} = 0 \quad \therefore \mathrm{a} = 0$

ii) When $\mathrm{y} = 0,\ \frac{\mathrm{d}^2\mathrm{u}}{\mathrm{dy}^2} = 0$

$$\frac{1}{\mathrm{U}}\frac{\mathrm{du}}{\mathrm{dy}} = 0 + \mathrm{b}$$

$$\frac{1}{U}\frac{d^2u}{dy^2} = 0$$

$\therefore 0 = b$

ii) Boundary condition at outer edge

i) $y = \delta \quad u = U_0$

$1 = a + b$

ii) $y = \delta \quad \frac{du}{dy} = 0$

$0 = a + 3b$

$A = \frac{3}{2}$

$B = -\frac{1}{2}$

Q 1) e) Explain losses of energy in the flow through pipe. (04)

Solution:

MAJOR ENERGY LOSSES :

These losses which are due to friction are calculated by :

1. Darcy-Weisbach formula

The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach formula which is given by:

$$h_f = \frac{4fLV^2}{D \times 2g}$$

where, hf = Loss of head due to friction,

f = Co-efficient of friction, (a function of Reynolds number, Re)

$h = \frac{0.0791}{(Re)^{1/4}}$ 0.0791 for varying from 4000 to 10^6

$= \frac{16}{Re}$ for Re < 2000 (laminar/viscous flow)

L = Length of the pipe,

V = Mean velocity of flow, and

D = Diameter of the pipe.

2. Chezy's formula.

Mean velocity, $V = C\sqrt{m\,i}$

2. MINOR ENERGY LOSSES

Whereas the major loss of energy or head is due to friction, the minor loss of energy (or head) includes the following cases :

i) Loss of head due to sudden enlargement

ii) Loss of head due to sudden contraction

iii) Loss of head due to an obstruction in the pipe

iv) Loss of head at the entrance to a pipe

v) Loss of head at the exit of a pipe

vi) Loss of head due to bend in the pipe.

vii) Loss of head in various pipe fittings.

Q 1) f) State the bernoulli's Theorem. List out the assumptions and limitations of Bernoulli's equation. (04)

Solution:

Bernoulli's theorem states as follows:

"In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line."

Assumptions:

It may be mentioned that the following assumptions are made in the derivation of Bernoulli's equation:

1. The liquid is ideal and incompressible.

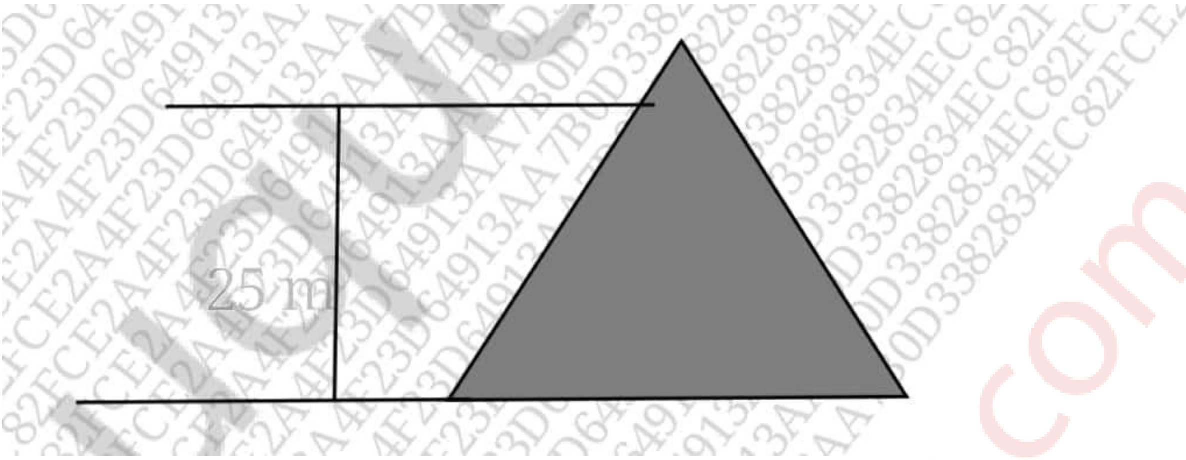
2. The flow is steady and continuous.

3. The flow is along the stream line, i.e., it is one-dimensional.

4. The velocity is uniform over the section and is equal to the mean velocity.

5. The only forces acting on the fluid are the gravity forces and the pressure forces.

Q 2) a) The water in a 25 m deep reservoir is kept inside by a 150 m wide wall whose cross section is an equilateral triangle as shown in fig.1. Determine the total force acting on the inner surface of the wall and its line of action. (06)



Solution:

Given :

Since triangle is equilateral

$\mathrm{b} = 150\ \mathrm{m} \quad \mathrm{h} = 150\ \mathrm{m}$

Distance of centroid from free surface,

$\bar{x} = 25 \times \frac{1}{3} \times 150 = 1250\ \mathrm{m}$

$A = \frac{1}{2} \times 150 \times 150 = 11250\ \mathrm{m}^2$

Moment of inertia $I_G = \frac{\mathrm{bh}^3}{36} = \frac{150 \times 150^3}{36} = 14062.5 \times 10^3 \mathrm{m}^4$

Total pressure force,

$$P = \gamma . A . \bar{x} = 9.81 \times 11250 \times 1250 = 137.95 \times 10^6\ KN$$

Q 2) b) The stream function in a two-dimensional, incompressible flow field is given by $\boldsymbol{\Psi = x^3 - 3xy^2}$. Find velocity at a point (1, 2) and the velocity potential function. (10)

Solution:

Given: $\Psi = \mathrm{x}^3 - 3\mathrm{xy}^2$

The velocity components u and v in terms of Ψ are

$$\mathrm{u} = -\frac{\partial \psi}{\partial \mathrm{y}} = 6xy$$

$$\mathrm{v} = -\frac{\partial \psi}{\partial \mathrm{x}} = 3x^2 - 3y^2$$

At the point P (1,2), we get $u = 6 \times 1 \times 2 = 12 \text{ units/sec}$

$$v = (3 \times 1) - (3 \times 2^2) = -9 \text{ units/sec}$$

Resultant velocity at $P = \sqrt{u^2 + v^2} = \sqrt{12^2 + (-9)^2} = 15 \text{ units/sec}$

Velocity potential function ϕ

$$\frac{\partial \phi}{\partial y} = -v = -3x^2 + 3y^2 \quad \text{...(i)}$$

$$\frac{\partial \phi}{\partial x} = -u = -6xy \quad \text{...(ii)}$$

Now integrating Equation (i),

$$\phi = -x^3 + y^3 \quad \text{...(iii)}$$

Differentiating w.r.t. x,

$$\frac{\partial \phi}{\partial x} = -3x^2 + f'(x)$$

From equation (ii)

$$\frac{\partial \phi}{\partial x} = -6xy$$

$$-6xy = f'(x)$$

Integrating $-3\frac{y^2}{2} + C = f(x)$

Put value of $f(x) = -3\frac{y^2}{2}$ in equation (iii),

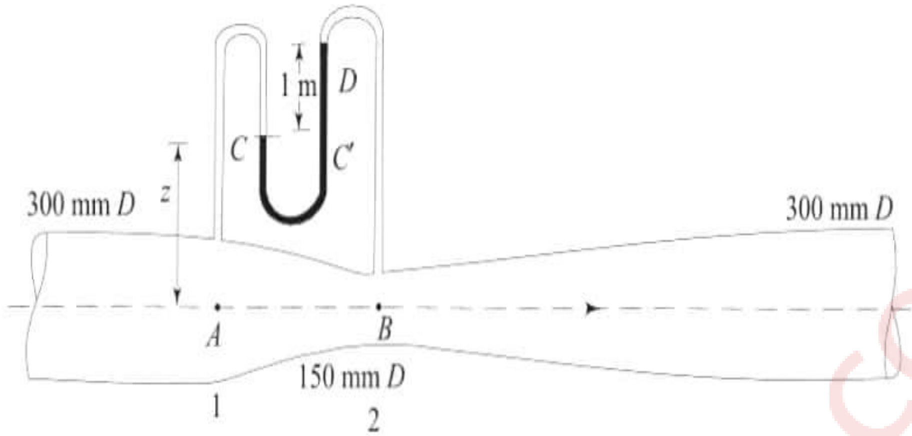
$$\phi = \frac{x^4}{4} - 3\frac{y^2}{2} + C$$

Q 2) c) Water flows through $300 \text{ mm} \times 150 \text{ mm}$ venturimeter at a rate of 0.065 m^3/s and the differential gauge is deflected 1.2 m. Specific gravity of the manometric fluid is 1.6. Determine the coefficient of discharge of the venturimeter. (04)

Solution:

Applying Bernoulli's equation between A and B, and considering the fluid to be inviscid, we get

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + 0 = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + 0 \quad \text{...(i)}$$



Again from continuity equation,

$$V_A^2 = (A_B/A_A)^2 V_B^2 \qquad \text{...(ii)}$$

Solving for V_B from eqn. (i) with the help of eqn. (ii) we have

$$V_B = \sqrt{\frac{2(P_A - P_B)/\rho}{1-(A_B/A_A)^2}}$$

The actual rate of discharge Q can be written as

$$Q = C_D A_B V_B$$

$$= C_D A_B \sqrt{\frac{2(P_A - P_B)/\rho}{1-(A_B/A_A)^2}} \qquad \text{...(iii)}$$

Where C_D is the coefficient of discharge.

From the principle of hydrostatics applied to the differential gauge, we gat

$$(P_A/\rho g - z) = P_B/\rho g - (z + 1.2) + 1.6 \times 1.2$$

$$\frac{P_A - P_B}{\rho g} = 0.72 \text{ m}$$

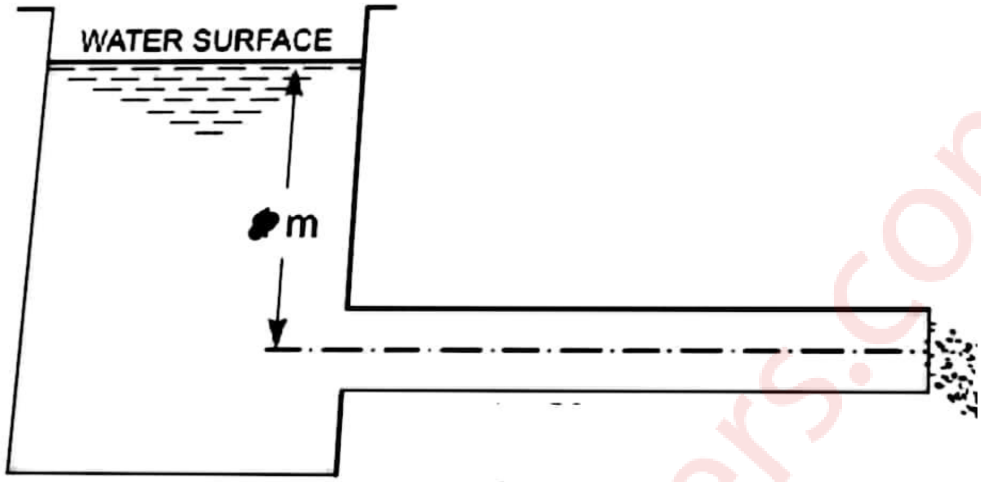
Hence, from eqn (iii), we can write

$$0.065 = C_D \frac{\pi}{4} (0.15)^2 \sqrt{2 \times 9.81\,(0.72)/(1-0.25)}$$

$$C_D = 0.847$$

Q 3) a) Water is flowing through a horizontal pipe of 15 cm diameter and of length 30 m. While one end of the pipe is connected to a tank, the other end is open to the atmosphere. If the height of water in the tank is 5 m above the centre of pipe, determine the rate of water through the pipe. Take $f = 0.03$ (10)

Solution:



Deia of pipep, $\mathrm{d} = 15 \text{ icm} = 0.15 \text{ m}$

Length of pipe, $\mathrm{L} = 30 \text{ m}$

Height of water, $\mathrm{H} = 5 \text{ m}$

Co-efficeint of friction, $\mathrm{f} = 0.03$

Let the velocity of water in pipe $= \mathrm{v\ m/s}$

Applying bernoulli's equation at the top of the water surface in the tank and at the outlet of pipe, we have

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_1 = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + \text{all losses}$$

Considering datum line passing through the centre of pipe

$$0 + 0 + 5 = 0 + \frac{V_2^2}{2g} + 0 + (h_i + h_f)$$

$$5 = \frac{V_2^2}{2g} + h_i + h_f$$

But the velocity in pipe $= \mathrm{V}$ $\quad \therefore \mathrm{V} = \mathrm{V_2}$

Loss of head at entrance of a pipe $\therefore \quad h_i = 0.5\ \frac{V^2}{2g}$

$$h_f = \frac{4.f.L.v^2}{2gd}$$

Substituting the values we have

$$5 = \frac{V^2}{2g} + 0.5\frac{V^2}{2g} + \frac{4.f.L.v^2}{2gd}$$

$$= \frac{V^2}{2g}\left[1 + 0.5 + \frac{4\times0.03\times30}{0.15}\right] = \frac{V^2}{2g}[1 + 0.5 + 24]$$

$$= \frac{V^2}{2g} \times 25.5$$

$$V = \sqrt{\frac{5\times2\times9.81}{25.5}} = 1.96 \text{ m/s}$$

$$Q = A \times v = \frac{\pi}{4} \times (0.15)^2 \times 1.96 = 0.0346 \text{ m}^3/s$$

$$= 34.63 \text{ litres/s}$$

Q 3) b) A 45° reducing bend in a horizontal plane has an inlet diameter OD 300 mm and outlet diameter of 150 mm. The pressure at outlet is 20 kPa gauge and rate of flow of water through bend is 0.09 0.09 m^3/s. Neglecting friction. Determine the magnitude and direction of force required to keep the bend in position. Neglect the weight of the water in the weight of the water in bend. (10)

Solution: Applying Bernoulli's equation between A and B, and considering the fluid to be inviscid, we get

$$\frac{P_1}{\Upsilon} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\Upsilon} + \frac{V_2^2}{2g} + Z_2$$

Discharge through bend $Q = 0.09 \text{ m}^3/\text{s}$

$$A_1 = \frac{\pi}{4}(0.3)^2 = 0.07068 \text{ m}^2$$

$$A_2 = \frac{\pi}{4}(0.15)^2 = 0.01767 \text{ m}^2$$

Pressure at inlet $P_2 = 215.8 \text{ kN/ m}^2$

Velocity at inlet and outlet

$$V_1 = \frac{Q}{A_1} = \frac{0.09}{0.07068} = 1.27 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.09}{0.01767} = 5.09 \text{ m/s}$$

Using bernoulli's equation

$$\frac{P_1}{\Upsilon} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\Upsilon} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1}{1000 \times 9.81} + \frac{1.27^2}{2 \times 9.81} + Z_1 = \frac{20}{1000 \times 9.81} + \frac{5.09^2}{2 \times 9.81} + Z_2$$

Where, $Z_1 = Z_2$

$$\frac{P_1}{1000 \times 9.81} + 0.0822 = 0.002 + 1.32$$

$P_1 = 1.24 \times 9810 = 12.16 \times 10^3 \ N/\text{m}^2$

By momentum principle

Net force in x-direction = mass flow rate × change in velocity in x-direction

$$P_1A_1 - P_2A_2\cos\theta + F_x = \rho Q(V_2\cos\theta - V_1)$$

$$12.16 \times 10^3 \times 0.07068 - 20 \times 0.01767\cos 45° + F_x = 1000 \times 0.09(5.09\cos 45° - 1.27)$$

$$859.28 + F_x = 126.35$$

$$F_x = -732.93\text{N}$$

$$= 732.93\text{N} \ (\leftarrow)$$

Q 4) a) Derive an expression for the area velocity relationship for a compressible fluid flow in the form $\frac{dA}{A} = -\frac{dV}{V}(1 - M^2)$. Explain properly, with the help of diagrams, what are the important conclusions derived from the above relationship. (10)

Solution:

For an incompressible flow the continuity equation may be expressed as :

AV = Constant, which when differentiated gives,

$$\text{AdC} + \text{VdA} = 0$$

or, $$\frac{dA}{A} = -\frac{dV}{V} \quad \text{...(A)}$$

But in case of compressible flow, the continuity equation is given by:

ρAV = Constant, which can be differentiated to give

$$\rho d(AV) + AVd\rho = 0$$

or, $\rho(AdV + VdA) + AVd\rho = 0$

or, $\rho AdV + \rho VdA + AVd\rho = 0$

Dividing both sides by ρAV, we get:

$$\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0 \quad \text{...(B)}$$

$$\text{or, } \frac{dA}{A} = -\frac{dV}{V} - \frac{d\rho}{\rho} \quad \text{...(C)}$$

The Euler's equation for compressible fluid is given by:

$$\frac{dp}{\rho} + VdV + gdz = 0$$

Neglecting the z terms the above equation reduces to:

$$\frac{dp}{\rho} + VdV = 0$$

This equation can also be expressed as:

$$\frac{dp}{\rho} \times \frac{dp}{d\rho} + VdV = 0$$

or, $\frac{dp}{d\rho} \times \frac{dp}{\rho} + VdV = 0$

But, $\frac{dp}{d\rho} = c^2$

$$c^2 \times \frac{dp}{\rho} + VdV = 0$$

$$c^2 \frac{dp}{\rho} = -VdV \text{ or } \frac{dp}{\rho} = -\frac{VdV}{c^2}$$

Substituting the value of $\frac{dp}{\rho}$ in eqn. (A), we get:

$$\frac{dV}{V} + \frac{dA}{A} - \frac{VdV}{c^2} = 0$$

$$\frac{dA}{A} - \frac{VdV}{c^2} = \frac{VdV}{c^2} - \frac{dV}{V} = \frac{dV}{V}\left(\frac{V^2}{c^2} - 1\right)$$

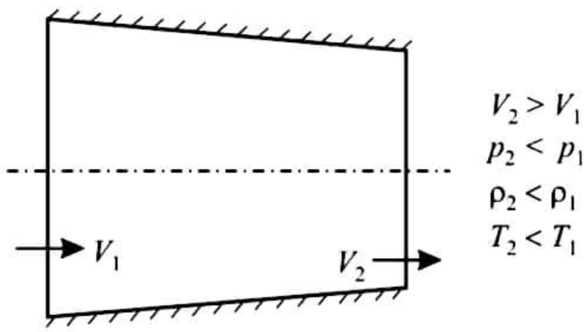
$$\frac{dA}{A} = \frac{dV}{V}(M^2 - 1)$$

From above eqn. it is possible to formulate the following conclusions of practical significance:

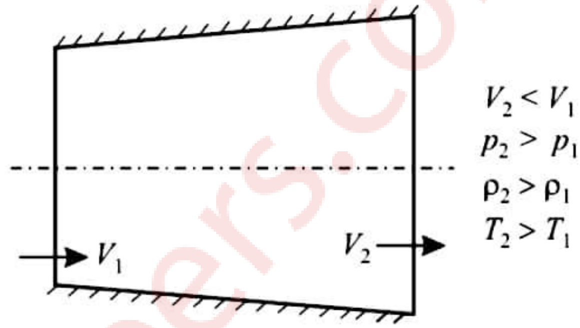
(i) For subsonic flow (M < 1) :

$\frac{dV}{V} > 0$; $\frac{dA}{A} < 0$; $dp < 0$ (convergent nozzle)

$\frac{dV}{V} < 0$; $\frac{dA}{A} > 0$; $dp > 0$ (divergent diffuser)



(*a*) Convergent nozzle.

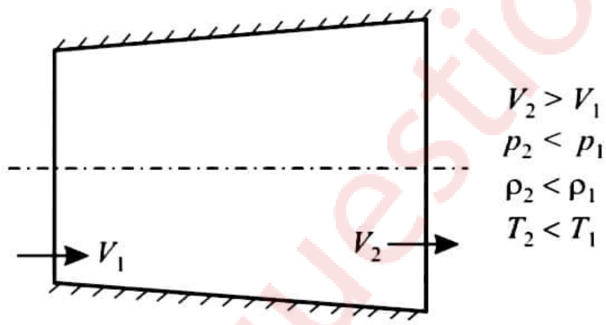


(*b*) Divergent diffuser

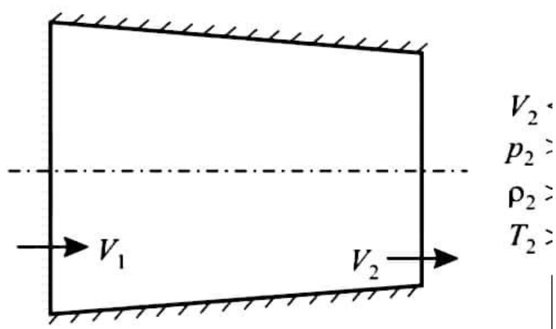
(ii) For supersonic flow (M > 1) :

$\frac{dV}{V} > 0$; $\frac{dA}{A} > 0$; $dp < 0$ (divergent nozzle)

$\frac{dV}{V} < 0$; $\frac{dA}{A} < 0$; $dp > 0$ (convergent diffuser)



(*a*) Divergent nozzle



(*a*) Convergent diffuser.

(iii) For sonic flow (M = 1) :

$\frac{dA}{A} = 0$ (straight flow passage since dA must be zero)

dp = (zero/zero) i.e. indeterminate, but when evaluated, the change of pressure dp = 0, since dA = 0 and the flow is frictionless.

Q 4) b) The local velocity u in a laminar, incompressible flow over a flat plate is given by

$\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^3 + \left(\frac{y}{\delta}\right)^4$ **where y is perpendicular distance from the plate, δ is the boundary layer thickness and U_∞ is free stream velocity. Obtain the expression for the displacement thickness and momentum thickness.** **(10)**

Solution:

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right)$$

$$= \int_0^\delta \left(1 - \frac{2y}{\delta} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right) dy$$

$$= \left[\delta - \frac{2y^3}{3\delta^2} + \frac{2y^4}{4\delta^3} - \frac{2y^5}{5\delta^4}\right]_0^\delta$$

$$= \delta - \frac{2\delta}{3} + \frac{2\delta}{4} - \frac{2\delta}{5}$$

$$\delta^* = \frac{29}{15}\delta$$

But momentum thickness,

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left[\left(2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2 + \left(\frac{y}{\delta}\right)^4\right)\left(1 - \frac{2y}{\delta} + \frac{2y^2}{\delta^2} - \frac{y^4}{\delta^4}\right)\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{4y^4}{\delta^3} - \frac{2y^5}{\delta^5} - \frac{2y^3}{\delta^3} + \frac{4y^4}{\delta^4} - \frac{4y^6}{\delta^6} + \frac{2y^7}{\delta^7} + \frac{y^4}{\delta^4} - \frac{2y^5}{\delta^5} + \frac{2y^6}{\delta^6} - \frac{y^8}{\delta^8}\right] dy$$

$$= \left[\frac{2y}{2\delta} - \frac{4y^3}{3\delta^2} + \frac{4y^5}{4\delta^4} - \frac{2y^6}{6\delta^5} - \frac{2y^4}{4\delta^3} + \frac{4y^5}{5\delta^4} - \frac{4y^7}{7\delta^6} + \frac{2y^8}{8\delta^7} + \frac{y^5}{5\delta^4} - \frac{2y^6}{6\delta^5} + \frac{2y^8}{8\delta^7} - \frac{y^9}{9\delta^8}\right]_0^\delta$$

$$= \left[\delta - \frac{4}{3}\delta + \frac{4}{5}\delta - \frac{1}{3}\delta - \frac{1}{2}\delta + \frac{4}{5}\delta - \frac{4}{7}\delta + \frac{2}{8}\delta + \frac{1}{5}\delta - \frac{1}{3}\delta + \frac{1}{4}\delta - \frac{1}{9}\delta\right]$$

$$\theta = \frac{37}{315}\delta$$

Q 5) a) An aeroplane is flying at 900 km/hr through still air having pressure of 80 KN/$\mathbf{m^2}$ and temperature of -8℃. Find the mach number. Also find stagnation properties on the nose of the plane. Take $\mathbf{R = 287\ J/kg^\circ K}$ and $\mathbf{k = 1.4}$. **(10)**

Solution:

Velocity of air, $V = \frac{1000}{3600} \times 900 = 250 \text{ m/s}$

Pressure, $p_1 = 80 \text{ KN/m}^2 = 80 \times 10^3 \text{ N/ m}^2$

Temperature $t_1 = -8°\text{C} = -15 + 273 = 265 \text{ K}$

R= 287 J/kgK , and assuming k = 1.4

Velocity of sound C $= \sqrt{\text{k}RT} = \sqrt{1.4 \times 287 \times 265}$

$$C_1 = 326.3 \text{ m/s}$$

Mach number

$$\text{M}_1 = \frac{V_1}{C_1} = \frac{250}{326.3} = 0.766$$

Stagnation pressure

$$\text{P}_\text{s} = \text{P}_1 \left(1 + \left(\frac{\gamma - 1}{2} M_1^2\right)\right)^{\frac{\gamma}{\gamma - 1}}$$

$$= 80 \left(1 + \left(\frac{1.4 - 1}{2} \times 0.766^2\right)\right)^{\frac{1.4}{1.4 - 1}}$$

$$= 117.96 \text{ kPa}$$

Stagnation temperature T_s :

$$\text{T}_\text{s} = \text{T}_1 \left(1 + \left(\frac{\gamma - 1}{2} M_1^2\right)\right)$$

$$= 265 \left(1 + \left(\frac{1.4 - 1}{2} \times 0.766^2\right)\right)$$

$$\text{T}_\text{s} = 296.09 - 273 = 23.09℃$$

Stagnation density ρ_s :

$$\rho_\text{s} = \frac{\text{P}_\text{S}}{\text{RT}_\text{S}} = \frac{117.96}{287 \times 296.09} = 1.38 \text{ kg/m}^3$$

Q 5) b) Derive Euler's equation of motion along streamline. **(10)**

Solution:

Consider steady flow of an ideal fluid along the stream tube. Separate out a small element of fluid of cross-sectional area dA and length ds from stream tube as a free body from the moving fluid.

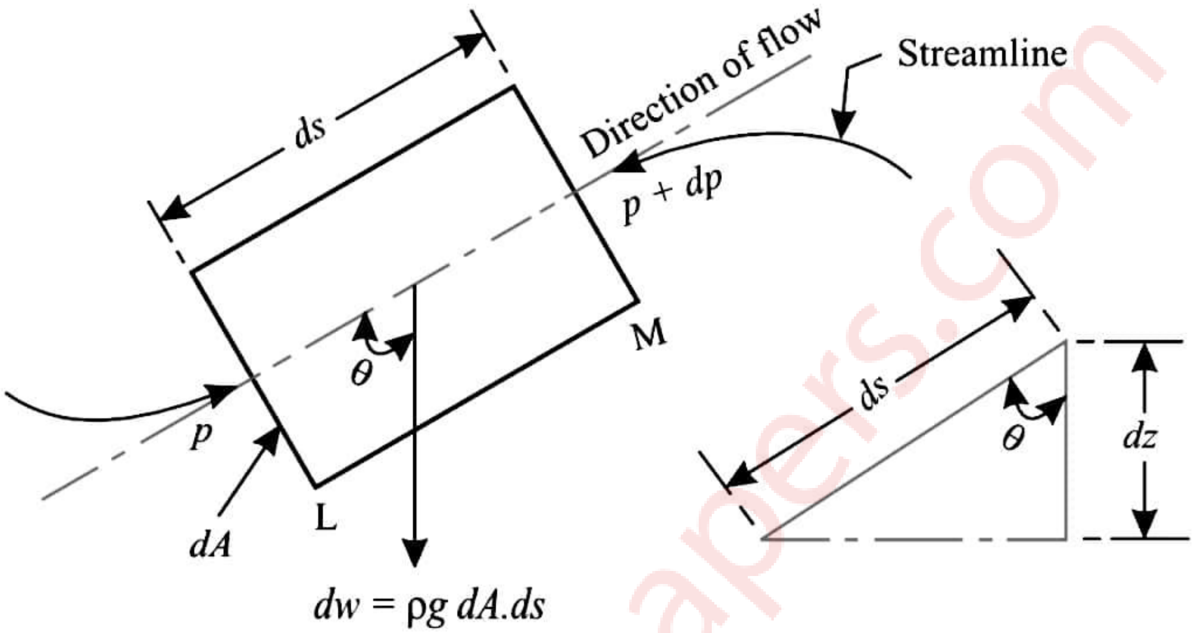


Fig. shows such a small element LM of fluid of cross-section area dA and length ds.

Let, p = Pressure on the element at L,

$p + dp$ = Pressure on the element at M, and

V = Velocity of the fluid element.

The external forces tending to accelerate the fluid element in the direction of stream line are as follows:

1. Net pressure force in the direction of flow is,

$$p.dA - (p + dp)\,dA = -\,dp\,.\,dA \qquad \text{...(i)}$$

2. Component of the weight of the fluid element in the direction of flow is

$$= -\,\rho.g.dA.ds.\cos\theta$$

$$= -\,\rho.g.dA.ds\left(\frac{dz}{ds}\right) \qquad \left(\because \cos\theta = \frac{dz}{ds}\right)$$

$$= -\,\rho.g.dA.dz \qquad \text{...(ii)}$$

mass of the fluid element $= \rho.dA.ds$ ...(iii)

The acceleration of the fluid element

$$a = \frac{dV}{dt} = \frac{dV}{dt} \times \frac{ds}{dt} = V \times \frac{dV}{ds}$$

Now according to the newton's second law of motion, Force = mass× acceleration

$$-\ dp\ .\ dA - \ \rho . g . dA . dz = \ \rho . dA . ds\ V \times \frac{dV}{ds}$$

Dividing both sides by $\rho . dA$, we get:

$$-\frac{dp}{\rho} - \ g . dz = V . dV$$

This is the required Euler's equation for motion.

Q 6) a) Explain what is meant by separation of boundary layer and describe in detail the methods to control this? (06)

Solution:

(i) When a solid body is immersed in a flowing fluid, a thin layer of fluid called boundary layer is formed, adjacent to the solid body. In this thin layer of fluid, the velocity varies from zero to free stream velocity in the direction normal to the solid body.

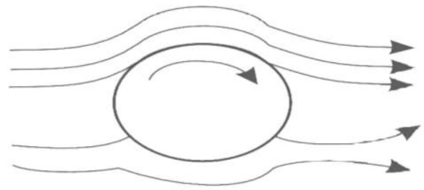
(ii) Along the length of the solid body, the thickness of the boundary layer increases. The fluid layer adjacent to the solid surface has to do work against surface friction at the expense of kinetic energy.

(iii) This loss of kinetic energy is recovered from the immediate fluid layer in contact with the layer adjacent to the solid surface through momentum exchange process. Thus the velocity of the layer goes on decreasing.

(iv) Along the length of solid body, at a certain point a stage may come when the boundary layer may not be able to keep sticking to the solid body, if it cannot provide kinetic energy to overcome the resistance offered by the solid body. In other words, the boundary layer will be separated from the surface. This phenomenon is called the boundary layer separation.

Methods to control separation:

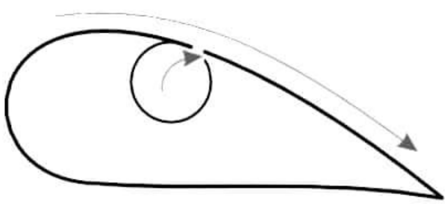
where the fluid as well as the cylinder move in the same direction, the boundary layer does not form. However on the lower side of cylinder where the fluid motion is opposite to that of cylinder separation would occur (Fig. A).



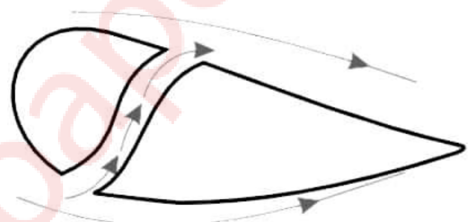
FIG(A)

2. Acceleration of fluid in the boundary layer:

This method of controlling separation consists of supplying additional energy to particles of fluid which are being retarded in the boundary layer. This may be achieved either by injecting the fluid into the region of boundary layer from the interior of the body with the help of some available device as shown in Fig. B or by diverting a portion of fluid of the main stream from the region of high pressure to the retarded region of boundary layer through a slot provided in the body (Fig. C).



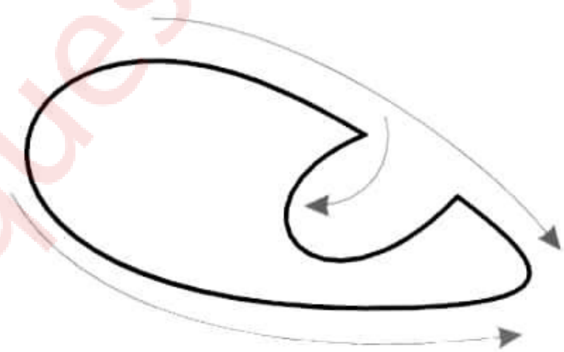
FIG(B)



FIG(C)

3. Suction of fluid from the boundary layer:

In this method, the slow moving fluid in the boundary layer is removed by suction through slots or through a porous surface as shown in the Fig. D.



FIG(D)

4. Streamlining of body shapes:

By the use of suitably shaped bodies, the point of transition of the boundary layer from laminar to turbulent can be moved downstream which results in the reduction of the skin friction drag. Further more by streamlining of body shapes, the separation may be eliminated.

Q 6) b) State Reynold's Transport theorem and explain each term in detail. (04)

Solution:

Reynold's Transport Theorem:

"The amount of an extensive property 'N' that system, possesses at a given instant, N_{system} can be determined by summing the amount associated with each fluid particle in the system."

$$\frac{\mathrm{DN_{sys}}}{\mathrm{Dt}} = \frac{\mathrm{d}}{\mathrm{dt}} \int_{\mathrm{cv}} \eta\rho \, \mathrm{dV} + \int_{\mathrm{cs}} \eta\rho \, \mathrm{V} \, . \, \widehat{\mathrm{n}} \, \mathrm{dA}$$

$\frac{\mathrm{DN_{sys}}}{\mathrm{Dt}}$ = Time rate of change of N within a system

$\frac{\mathrm{d}}{\mathrm{dt}} \int_{\mathrm{cv}} \eta\rho \, \mathrm{dV}$ = Time rate of change of N within a control volume

$\int_{\mathrm{cs}} \eta\rho \, \mathrm{V} \, . \, \widehat{\mathrm{n}} \, \mathrm{dA}$ = Net flux of N through control system

This is Reynold's Transport theore m.

Q 6) c) An oil with density 850 kg/m^3 and viscocity 0.16 Ns/m^2 flows through a 20 cm diameter pipe at a rate of 1.2 lit/sec. If the length of pipe is 500 m, find the pressure drop between the two ends of the pipe . Also calculate the shear stress at the pipe wall.
(10)

Solution:

Viscosity of the oil = $0.16 \, Ns/m^2$

Relative density = $850 \, kg/m^3$

Diameter of the pipe, D = 0.2 m

Length of the pipe, L = 500 m

i) Difference of pressure, (p1 – p2):

The difference of pressure for viscous or laminar flow is given by

$$(p_1 - p_2) = \frac{32\mu\bar{\mathrm{u}}\mathrm{L}}{\mathrm{D}^2}$$

Average velocity, $\bar{u} = \frac{Q}{Area} = \frac{0.0012}{(\pi/4)D^2} = \frac{0.0012}{(\pi/4)0.2^2} = 0.0382 \text{ m/s}$

Reynolds number, $R_e = \frac{\rho VD}{\mu} = \frac{850 \times 0.0382 \times 0.2}{0.16} = 40.59$

Since Re < 2000, therefore, the flow is laminar/viscous.

Substituting the values in eqn, we get :

$$(p_1 - p_2) = \frac{32 \times 0.16 \times 0.0382 \times 500}{(0.2)^2} = 2444.8 \text{ N/m}^2$$

ii) Shear stress at wall $\tau_0 = -\frac{\partial p}{\partial x} . \frac{R}{2} = 2444.8 \times \frac{(0.2/2)}{2}$

$$= 122.24 \text{ N/m}^2$$