Q.P Code: 21376

FLUID MECHANICS SOLUTION

SEM 4 (CBCGS-MAY 2018)

BRANCH-MECHANICAL ENGINEERING

Q 1) A) Explain briefly boundary layer formation and define boundary layer thickness. (05)

Solution -

Boundary layer formation :

When a real fluid (viscous fluid) flows past a stationary solid boundary, a layer of fluid which comes in contact with the boundary surface, adheres to it (on account of viscosity) and condition of no slip occurs (The no-slip condition implies that the velocity of fluid at a solid boundary must be same as that of boundary itself). Thus the layer of fluid which cannot slip away from the boundary surface undergoes retardation; this retarded layer further causes retardation for the adjacent layers of the fluid, thereby developing a small region in the immediate vicinity of the boundary surface in which the velocity of the flowing fluid increases rapidly from zero at the boundary surface and approaches the velocity of main stream. The layer adjacent to the boundary is known as boundary layer. Boundary layer is formed whenever there is relative motion between the boundary and the fluid.

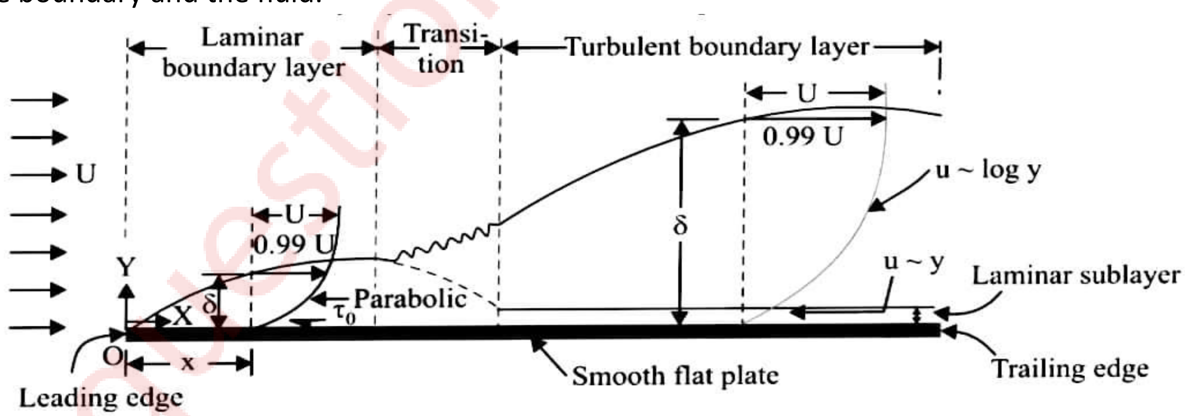


FIG : BOUNDARY LAYER ON A FLAT PLATE

Boundary layer thickness :

Thickness of the boundary layer is arbitrarily defined as that distance from the boundary in which the velocity reaches 99 per cent of the velocity of the free stream ($u = 0.99U$). It is denoted by the symbol δ.

Q 1) B) With neat sketch explain working and construction of pitot tube. (05)

Solution –

Construction:

It consists of a glass tube in the form of a 90° bend of short length open at both its ends. It is placed in the flow with its bent leg directed upstream so that a stagnation point is created immediately in front of the opening. The kinetic energy at this point gets converted into pressure energy causing the liquid to rise in the vertical limb, to a height equal to the stagnation pressure.

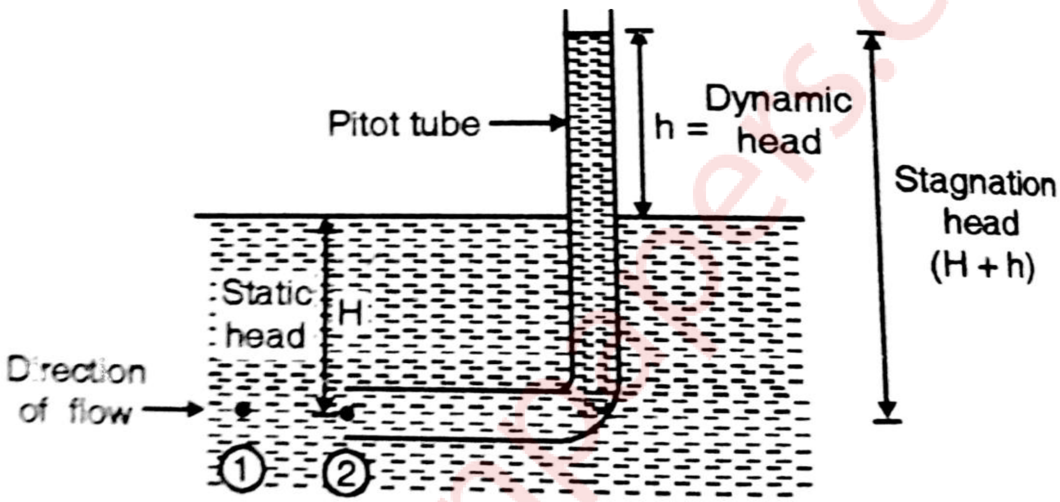


FIG : PITOT TUBE

Working :

Pitot tube is one of the most accurate devices for velocity measurement. It works on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to conversion of kinetic energy into pressure.

Q 1) C) The following represents the velocity components. Calculate the unknown velocity component so that they satisfy the continuity equation. (05)

$$\boldsymbol{u = 2x^2\ ;\ \ v = 2xyz}$$

Solution:

$$u = 2x^2\ ;\ \ v = 2xyz$$

Differentiating

$$\frac{\partial u}{\partial x} = 4x \qquad \frac{\partial v}{\partial y} = 2xz$$

Using continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$4x + 2xz + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial w}{\partial z} = -4x - 2xz \quad \text{...(i)}$$

Integrating Equation (i), We get,

$$w = -4xz - xz^2$$

$$= -xz\,(4 + z)$$

Q 1) D) The absolute viscosity of liquid having specific gravity of 0.87 is 0.073 poise. Find kinematics viscosity in m/s and in Stokes. (05)

Solution:

Given:

Specific gravity s $= 0.87$

Absolute viscosity μ $= 0.0073$ Ns/m^2

i) Kinematic viscosity v

weight density $\rho = s \times \rho_w = 0.87 \times 1000$

$= 870\ Kg/m^3$

Kinematic viscosity V $= \frac{\mu}{\rho} = \frac{0.0073}{870}$

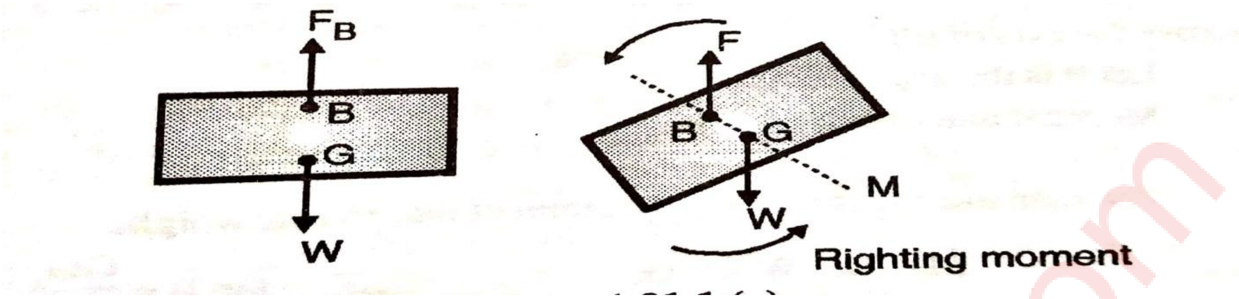
$= 8.39 \times 10^{-6} m^2$/sec

$= 0.0839$ strokes

Q 1) E) Explain stability of floating bodies. (05)

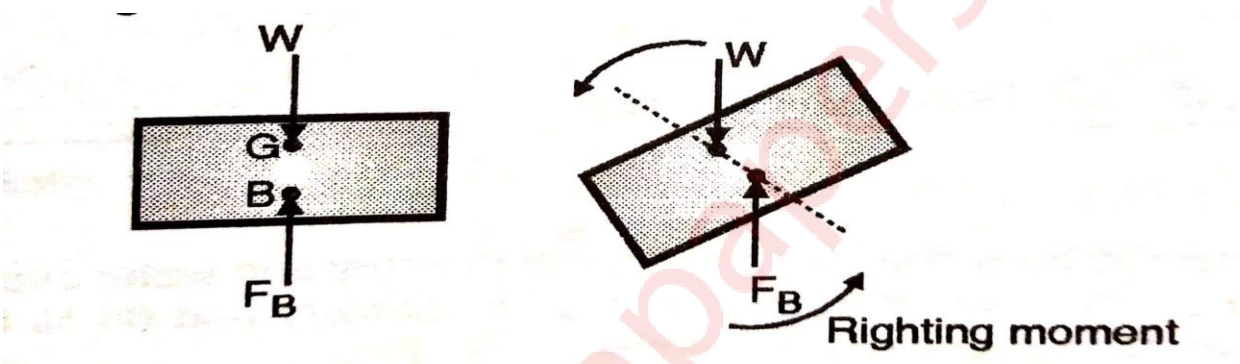
Solution:

Conditions of stable equilibrium:
Stable equilibrium:
When centre of buoyancy is lies above the centre of gravity, submerged body is stable.

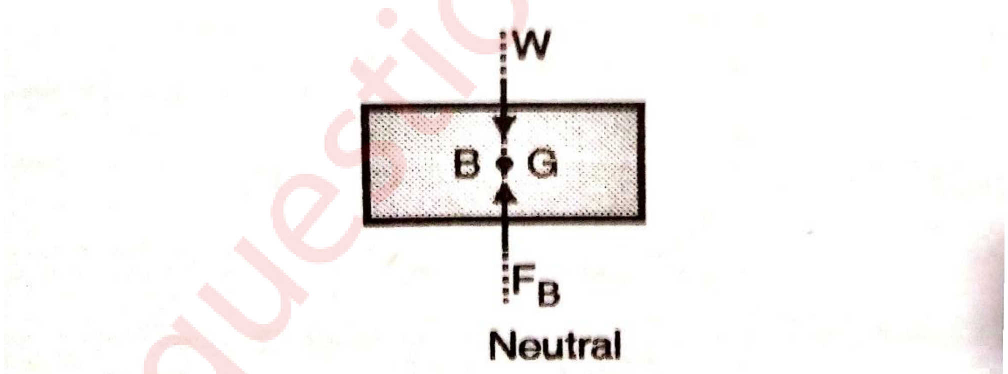


Unstable equilibrium:

When centre of buoyancy is lies below centre of gravity, submerged body is in unstable equilibrium.



Neutral equilibrium:

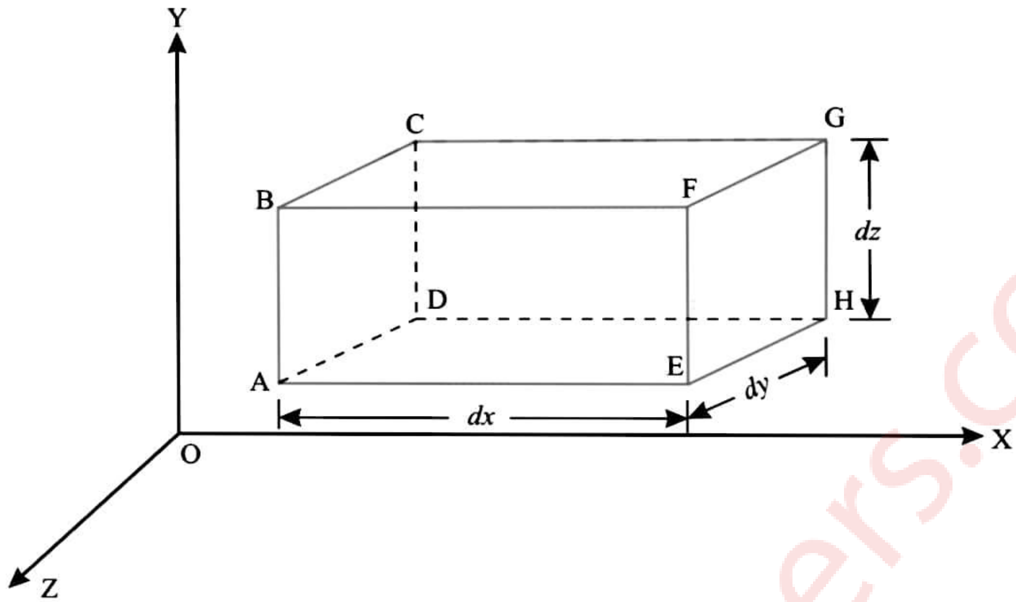


When centre of buoyancy and centre of gravity is coincide, body is in neutral equilibrium.

Q 2) A) Derive the differential form of the general mass conservation equation in Cartesian coordinate for a fluid. (10)

Solution:

Continuity equation in Cartesian coordinate:



Consider a fluid element (control volume) – parallelopiped with sides dx, dy and dz as shown in Fig.

Let, ρ = Mass density of the fluid at a particular instant;

u, v, w = Components of velocity of flow entering the three faces of the parallelopiped.

Rate of mass of fluid entering the face ABCD (i.e. fluid influx).

= ρ × velocity in X-direction × area of ABCD

= $\rho\, u dy\, dz$ ...(i)

Rate of mass of fluid leaving the face EFGH (i.e. fluid efflux).

= ρ u dy dz $+ \frac{\partial}{\partial x}(\rho u\ dy\ dz)dx$ ...(ii)

The gain in mass per unit time due to flow in the X-direction is given by the difference between

the fluid influx and fluid efflux.

∴ Mass accumulated per unit time due to flow in X-direction

= $\rho u\ dy\ dz - [\rho u + \frac{\partial}{\partial x}(\rho u)\ dx]dy\ dz$

= $-\frac{\partial}{\partial x}(\rho u)dx\ dy\ dz$ ...(iii)

Similarly, the gain in fluid mass per unit time in the parallelopiped due to flow in Y and Z-directions

$= -\dfrac{\partial}{\partial y}(\rho v) dx\, dy\, dz$ (in Y-direction) ...(iv)

$= -\dfrac{\partial}{\partial z}(\rho w) dx\, dy\, dz$ (in Z-direction) ...(v)

The total (or net) gain in fluid mass per unit for fluid along three co-ordinate axes

$= -\left[\dfrac{\partial}{\partial x}(\rho u) + \dfrac{\partial}{\partial y}(\rho v) + (\rho w)\dfrac{\partial}{\partial z}\right] dx\, dy\, dz$ ...(vi)

Rate of change of mass of the parallelopiped (control volume)

$= \dfrac{\partial}{\partial t}(\rho\, dx\, dy\, dz)$ ...(vii)

Equations (vi) and (vii), we get:

$$-\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + (\rho w)\frac{\partial}{\partial z}\right] dx\, dy\, dz = \frac{\partial}{\partial t}(\rho\, dx\, dy\, dz)$$

Simplification and rearrangement of terms would reduce the above expression to:

$\dfrac{\partial}{\partial x}(\rho u) + \dfrac{\partial}{\partial y}(\rho v) + \dfrac{\partial}{\partial z}(\rho w) + \dfrac{\partial}{\partial t} = 0$ ...(a)

This eqn. (a) is the general equation of continuity in three-dimensions and is applicable to any type of flow and for any fluid whether compressible or incompressible.

For steady flow $\left(\frac{\partial \rho}{\partial t} = 0\right)$ incompressible fluids (ρ = constant) the equation reduces to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial y} = 0$$

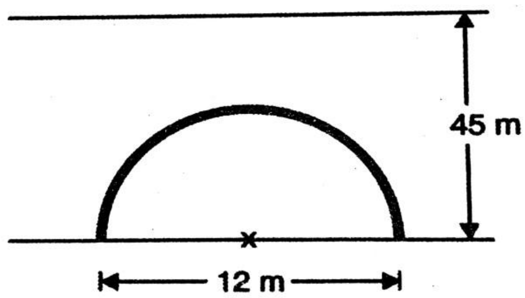
For two dimensional flow, above eqn. reduces to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

For one dimensional flow, say in X-direction, eqn. takes the form:

$$\frac{\partial u}{\partial x} = 0$$

Q 2) B) A semicircular 12 m diameter tunnel is to be built under a 45 m deep, 240 m long lake. Determine the magnitude and direction of total hydrostatic force acting on the roof of the tunnel. **(10)**

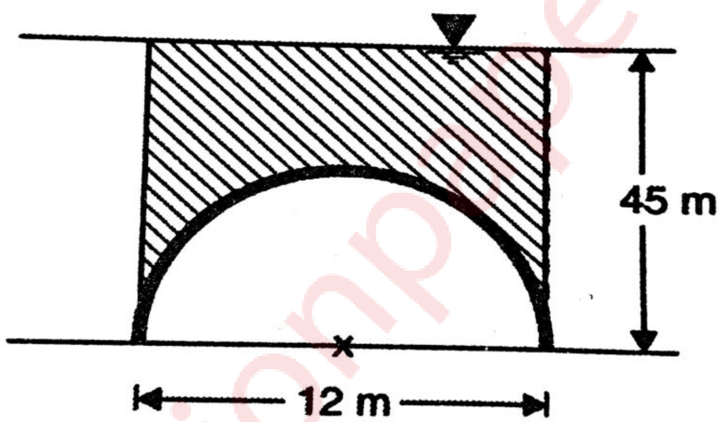


Solution:

Given :

Semi-circular diameter d = 12 m depth of water in lake 45 m

To find : hydrostatic forces



Total hydrostatic force acting on the roof of the tunnel

= Weight of liquid over the tunnel

= Weight – density × volume

= γ_w × L(Area of rectangle – Area of semi circle)

= $9.81 \times 240 \left[45 \times 12 - \frac{\pi \times 6^2}{2}\right]$

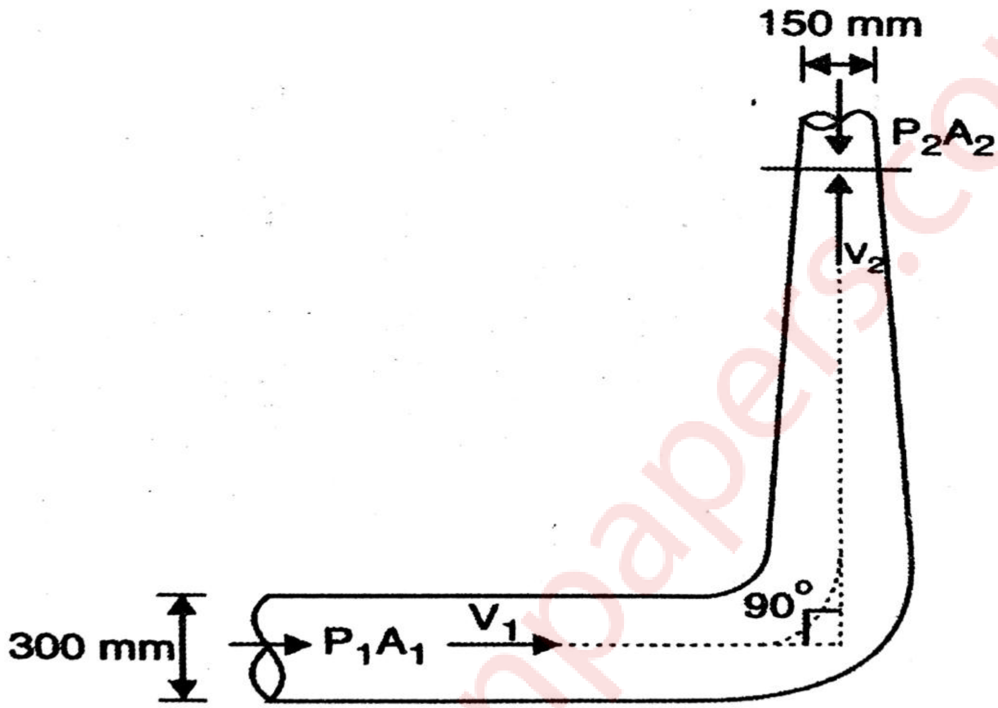
= 2354.5(540-56.55)

= 1138234.68 kN

Q 3) A) A 90° reducing bend has a diameter 300 mm at inlet and 150 mm at exit carries 0.6 m^3/s oil of specific gravity 0.85 with a pressure of 120 kN/ m^2 at inlet to the bend. The

volume of bend is 0.15 m^3. Find the magnitude and direction of the force on the bend. Neglect the frictional losses and assume both inlet and outlet sections to be at same horizontal level. **(10)**

Solution:



Discharge through bend $Q = 0.6\ m^3/s$

$$A_1 = \frac{\pi}{4}(0.3)^2 = 0.0707\ m^2$$

$$A_2 = \frac{\pi}{4}(0.15)^2 = 0.0177\ m^2$$

Specific gravity $s = 0.85$

Mass density of oil $\rho = s \times \rho_{water} = 0.85 \times 1000 = 850\ kg/\ m^3$

Weight density $\Upsilon = \rho g = 850 \times 9.81 = 8338.5 kg/\ m^3$

$= 8.34 kN/\ m^3$

Pressure at inlet $\rho_1 = 120\ kN/\ m^2$

Volume of bend $V = 0.15\ m^3$

Both inlet and outlet at same level,

$$\text{Weight of bent } w = \Upsilon \times v = (\rho q)\ v = 850 \times 0.15$$

$= 127.5$ N

Velocity at inlet and outlet

$$V_1 = \frac{Q}{A_1} = \frac{0.6}{0.0707} = 8.49\ m^3/s$$

$$V_2 = \frac{Q}{A_2} = \frac{0.6}{0.0177} = 3.39\ m^3/s$$

Using bernoulli's equation

$$\frac{P_1}{\Upsilon} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\Upsilon} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{120}{8.34} + \frac{8.49^2}{2 \times 9.81} + Z_1 = \frac{P_2}{8.34} + \frac{3.39^2}{2 \times 9.81} + Z_2$$

$$13.59 + 3.67 = \frac{P_2}{8.34} + 0.586$$

$$P_2 = 16.674 \times 8.34 = 139\ kN/\ m^2$$

By momentum principle

Net force in x-direction = mass flow rate × change in velocity in x-direction

$$P_1A_1 + F_x = \rho Q(0 - V_1)$$

$$120 \times 0.0707 + F_x = \left(\frac{850}{1000}\right) \times 0.6(-8.49)$$

$$8.485 + F_x = -4.33$$

$$F_x = -(8.485 + 8.33) = -12.8\text{kN}$$

$$= 12.8\text{kN} (\leftarrow)$$

Net force in y-direction $= \rho Q\ (V_2)$

$$-P_2A_2 + F_y = \left(\frac{850}{1000}\right) \times 0.6(3.39)$$

$$-139 \times 0.0177 + F_y = 1.729$$

$$-2.46 + F_y = 1.729$$

$$\therefore \quad F_y = 4.189\ \text{kN}$$

$$\therefore \quad F = \sqrt{\left(F_x^2 + F_y^2\right)}$$

$$= \sqrt{(12.8^2 + 4.189^2)}$$

$$= 13.47 \text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{F_x}{F_y}\right) = \tan^{-1}\left(\frac{4.189}{12.8}\right)$$

$$= 18.12°$$

Q 3) B) Consider a two dimensionless viscous incompressible flow of a Newtonian fluid between two parallel plates, separately by a distance 'b'. One of the plates is stationary and the other is moving with uniform velocity u. There is no pressure gradient in the flow. Obtain the general equation from the general Navier-Stokes equation. (10)

Solution:

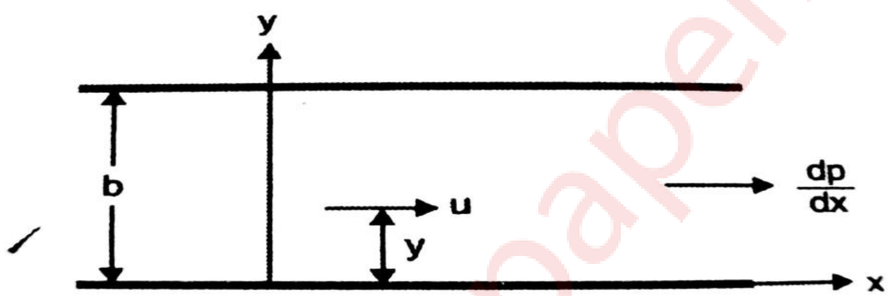


FIG : LAMINAR FLOW BETWEEN TWO STRAIGHT PARALLEL PLATES

Flow in x-direction

$$u = f_1(x, y, t), v = 0, w = 0$$

$$p = f_2(x, y, t)$$

$\partial u/\partial z = 0$ Represent flow is two-dimensional flow. ...(i)

For this flow, continuity equation is reduced to

$$\frac{\partial u}{\partial x} = 0$$

$$\therefore u = f_1(y, t) \quad \text{...(ii)}$$

The body forces are assuming to be zero.

$$\therefore x = 0$$

Using Navier-Stokes equation,

$$\frac{Du}{Dt} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$= X - \frac{1}{\rho}\frac{\partial p}{\partial x} + v\,\overline{V}^2\ u$$

$$\overline{V}^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

From the above condition,

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial x^2} \qquad \text{...(iii)}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$p = f_2\ (x, t) \qquad \text{...(iv)}$$

Since u is not function of x from equation (ii) and p is not a function of y, it can be concluded from equation (iii) and (iv), that $\frac{\partial p}{\partial x}$ must be zero or function of time.

Assuming steady flow, $\frac{du}{dt} = 0$

Integrating equation (iii) w.r.t. y, we get,

$$\frac{\partial u}{\partial y} = -\frac{1}{\mu}\frac{dp}{dx}.y + A \qquad \text{...(v)}$$

Again integrating equation v, we get equation velocity distribution.

$$u = -\frac{1}{2\mu}\frac{dp}{dx}.y^2 + Ay + B \qquad \text{...(vi)}$$

where A and B are constant

When one plate is stationary and other is moving, flow in such boundary condition is known as simple coquette flow.

$$\frac{dp}{dx} = 0 \qquad \text{for this type of flow}$$

Velocity of moving boundary is U.

(i) $y = 0, u = 0$ (ii) $y = b, u = U$

For this type of flow, $\frac{dp}{dx}$ does not exist

From boundary conition,

From (i) $\therefore\ B = 0$

From (ii) $\therefore\ u = U$

This equation of velocity distribution is

$$u = \frac{y}{b} U$$

This equation is the general equation from Navier Stokes equation.

Q 4) A) Using the laminar boundary layer velocity distribution:

$$\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2 - \left(\frac{y}{\delta}\right)^4$$

(i) Check if boundary layer separation occurs.

(ii) Determine boundary layer thickness (in terms of Re) **(10)**

Solution:

Velocity profile,

$$\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2 - \left(\frac{y}{\delta}\right)^4$$

(i) Boundary layer thickness (δ)

Shear stress by using Von-Karman equation

$$\tau_0 = \rho\, U^2 \frac{\partial \theta}{\partial x} \qquad \text{...(i)}$$

But momentum thickness,

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left[\left(2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2 - \left(\frac{y}{\delta}\right)^4\right)\left(1 - \frac{2y}{\delta} + \frac{2y^2}{\delta^2} + \frac{y^4}{\delta^4}\right)\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{4y^3}{\delta^3} + \frac{2y^5}{\delta^5} - \frac{2y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{4y^4}{\delta^4} - \frac{2y^6}{\delta^6} - \frac{y^4}{\delta^4} + \frac{2y^5}{\delta^5} - \frac{2y^6}{\delta^6} - \frac{y^8}{\delta^8}\right] dy$$

$$= \left[\frac{2y}{2\delta} - \frac{4y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} + \frac{2y^6}{6\delta^5} - \frac{2y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{4y^5}{5\delta^4} - \frac{2y^7}{7\delta^6} - \frac{y^5}{5\delta^4} + \frac{2y^6}{6\delta^5} - \frac{2y^7}{7\delta^6} - \frac{y^9}{9\delta^8}\right]_0^\delta$$

$$= \left[\delta - \frac{4}{3}\delta + \delta + \frac{1}{3}\delta - \frac{2}{3}\delta + \delta - \frac{4}{5}\delta - \frac{2}{7}\delta - \frac{1}{5}\delta + \frac{1}{3}\delta - \frac{2}{7}\delta - \frac{1}{9}\delta\right]$$

$$\theta = -\frac{1}{63}\delta$$

Put value of θ in equation (i)

$$\tau_0 = \rho U^2 \frac{\partial}{\partial x}\left(-\frac{1}{63}\delta\right)$$

$$\tau_0 = -\frac{1}{63}\rho U^2 \frac{\partial \delta}{\partial x} \qquad \text{...(ii)}$$

Now, shear stress by using Newton's law of viscosity,

$$\tau_0 = \mu\left(\frac{du}{dx}\right)$$

But $u = 2U\frac{y}{\delta} - \frac{6Uy^2}{\delta^2} - \frac{4Uy^4}{\delta^4}$

Differentiating w.r.t. y

$$\frac{du}{dx} = \frac{2U}{\delta} - \frac{4Uy}{\delta^2} - \frac{4Uy^4}{\delta^4}$$

At $y = 0$

$$\left(\frac{du}{dx}\right)_{y=0} = \frac{2U}{\delta}$$

$$\tau_0 = \mu\left(\frac{2U}{\delta}\right) \qquad \text{...(iii)}$$

Equating the value of shear stress from equations (ii) and (iii),

$$-\frac{1}{63}\rho U^2 \frac{\partial \delta}{\partial x} = \frac{2U\mu}{\delta}$$

$$\delta.\partial\delta = \frac{-126\mu}{\rho U}\delta x$$

By integrating,

$$\int \delta.\partial\delta = \frac{-126\mu}{\rho U}\int \delta x$$

$$\frac{\delta^2}{2} = \frac{-126\mu}{\rho U}.x + c$$

Apply boundary condition

When $y = 0,\ \delta = 0,\ \therefore c = 0$

$$\frac{\delta^2}{2} = \frac{-126\,\mu}{\rho U}.x$$

$$\delta^2 = \frac{-126\,\mu}{\rho U}.\frac{x}{x}$$

$$\delta = 5.84x.\sqrt{\frac{\mu}{\rho U}} \qquad \therefore R_e = \frac{\rho U x}{\mu}$$

$$\delta = 5.84x\,\frac{x}{\sqrt{R_e}}$$

Q 4) B) Derive Euler's equation of motion in Cartesian co-ordinate. (10)

Solution:

Euler's equation in Cartesian coordinates:

Consider an infinitely small mass of fluid enclosed in an elementary parallelopiped of sides *dx, dy* and *dz* as shown in Fig. The motion of the fluid element is influenced by the following forces:

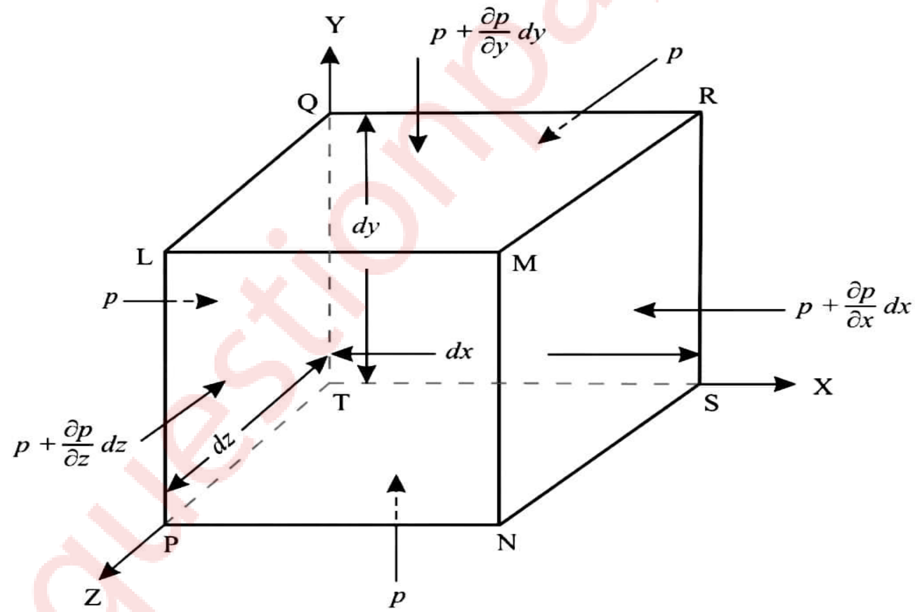


FIG : NORMAL SURFACE FORCES ON A NON—VISCOUS FLUID ELEMENTT

(i) Normal forces due to pressure:

The intensities of hydrostatic pressure acting normal to each face of the parallelepiped are shown in Fig.

The net pressure force in the X-direction

$= p.dy.dz - (p + \frac{\partial p}{\partial x}dx)\,dy\,dz$

$= -\frac{\partial p}{\partial x}\;dx.dy.dz$

(ii) Gravity or body force:

Let B be the body force per unit mass of fluid having components Bx , By and Bz in the X, Y and Z directions respectively.

Then, the body force acting on the parallelopiped in the direction of X-coordinate is
$= B_x\,.\rho.dx.dy.dz$

(iii) Inertia forces:

The inertia force acting on the fluid mass, along the X-coordinate is given by,

Mass × acceleration $= \rho.dx.dy.dz.dudt$

As per Newton's second law of motion summation of forces acting in the fluid element in any direction equals the resulting inertia forces in that direction. Thus, along X-direction:

$$B_x\,.\rho.dx.dy.dz - \frac{\partial p}{\partial x}\;dx.dy.dz = \rho.dx.dy.dz.\frac{du}{dt}$$

Dividing both sides by $\rho.dx.dy.dz.$, we have:

$$B_x - \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{du}{dt} \qquad \text{...(i)}$$

In this equation each term has dimensions of force per unit mass or acceleration. Obviously the total acceleration in a given direction is prescribed by the algebraic sum of the body force and the pressure gradient in that direction since the velocity components are functions of position and time, i.e.,$u = f(x, y, z, t)$,therefore, the total derivative of velocity u in the X-direction

can be written as:

$$= \frac{\partial u}{\partial t}\;dt + \frac{\partial u}{\partial x}\;dx + \frac{\partial u}{\partial y}\;dy + \frac{\partial u}{\partial z}\;dz$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}.\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}.\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}.\frac{\partial z}{\partial t}$$

Substituting, $\frac{\partial x}{\partial t} = u$, $\frac{\partial y}{\partial t} = v$ and $\frac{\partial z}{\partial t} = w$; we have:

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad \text{...(ii)}$$

Combining equation (i) and (ii), we get the force components as:

$$B_x - \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad \text{...(iii)}$$

Similarly, $B_y - \frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{\partial v}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$ ...(iv)

$$B_z - \frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \qquad \text{...(iv)}$$

For steady flow: $\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial w}{\partial t} = 0$

Thus, the Euler's equation for a steady three-dimensional flow can be written as:

$$B_x - \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad \text{...(vi)}$$

$$B_y - \frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{\partial v}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \qquad \text{...(vii)}$$

$$B_z - \frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \qquad \text{...(viii)}$$

In Euler's equation each term represents force per unit mass. Thus, if each equation is multiplied by the respective projections of the elementary displacement, the resulting equation would represent energy. Thus, in order to get total energy in the three-dimensional-steady-incompressible flow, the energy terms can be combined as follows:

$$B_x dx - \frac{1}{\rho}\frac{\partial p}{\partial x}dx = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad \text{...(ix)}$$

$$B_y dy - \frac{1}{\rho}\frac{\partial p}{\partial y}dy = \frac{\partial v}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \qquad \text{...(x)}$$

$$B_z dz - \frac{1}{\rho}\frac{\partial p}{\partial z}dz = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \qquad \text{...(xi)}$$

From the equation of a stream line in a three-dimensional flow, we have:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$udy = vdx;\ vdz = wdy;\ udz = wdx$$

Substituting these values in eqns. (ix), (x) and (xi), we get:

$$B_x dx - \frac{1}{\rho}\frac{\partial p}{\partial x}\,dx = u\frac{\partial u}{\partial x}\,dx + u\frac{\partial u}{\partial y}\,dy + u\frac{\partial u}{\partial z}\,dz \qquad \text{...(xii)}$$

$$B_y dy - \frac{1}{\rho}\frac{\partial p}{\partial x}\,dy = v\frac{\partial v}{\partial x}\,dx + v\frac{\partial v}{\partial y}\,dy + v\frac{\partial v}{\partial z}\,dz \qquad \text{...(xii)}$$

$$B_z dz - \frac{1}{\rho}\frac{\partial p}{\partial x}\,dz = w\frac{\partial w}{\partial x}\,dx + w\frac{\partial w}{\partial y}\,dy + w\frac{\partial w}{\partial z}\,dz \qquad \text{...(xiii)}$$

Acceleration terms are of form $u\frac{\partial u}{\partial x}$ which can be replaced by $\frac{1}{2}\frac{\partial(u^2)}{\partial x}$. Thus,

$$B_x dx - \frac{1}{\rho}\frac{\partial p}{\partial x}\,dx = \frac{1}{2}\left[\frac{\partial(u^2)}{\partial x}\,dx + \frac{\partial(u^2)}{\partial y}\,dy + \frac{\partial(z^2)}{\partial y}\,dz\right] = \frac{1}{2}\,\partial(u^2) \qquad \text{...(xv)}$$

$$B_y dy - \frac{1}{\rho}\frac{\partial p}{\partial x}\,dy = \frac{1}{2}\,\partial(v^2) \qquad \text{...(xvi)}$$

$$B_z dz - \frac{1}{\rho}\frac{\partial p}{\partial x}\,dz = \frac{1}{2}\,\partial(w^2) \qquad \text{...(xvii)}$$

Adding eqns. (xv), (xvi) and (xvii), we get:

$$B_x dx + B_y dy + B_z dz - \frac{1}{\rho}\left[\frac{\partial p}{\partial x}\,dx + \frac{\partial p}{\partial y}\,dy + \frac{\partial p}{\partial y}\,dz\right]$$

$$= \frac{1}{2}\left[d(u^2) + d(v^2) + d(w^2)\right]$$

$$B_x dx + B_y dy + B_z dz - \frac{1}{\rho}\,dp = \frac{1}{2}\,d(V^2) \qquad \text{...(xviii)}$$

Where, $V =$ Total velocity vector

When gravity is the only body force acting on the third element, then:

$B_x = 0, B_z = 0$ and $B_y = -g$

$B_y = -g$ since the gravitational force acts in the downword direction which is negative 'with' respect to Y, Which is positive upward. Inserting these values in (xviii), we get:

$$-g - \frac{1}{\rho}\,dp = \frac{1}{2}\,d(V^2)$$

$$-g - \frac{1}{\rho} dp = VDV$$

$$\frac{dp}{\rho} + VDV + g = 0$$

Q 5) A) Air has a velocity of 1000 km/hr at pressure of 9.81 kN/ $\mathbf{m^2}$ vacuum and a temperature of 47°C. Compute its stagnation properties (Pressure, Temperature and Density). Take atmospheric Pressure 98.1 kN/ $\mathbf{m^2}$, R= $\mathbf{287}$ J/kg °k and $\boldsymbol{\gamma} = \mathbf{1.4}$. (10)

Solution:

Velocity of air, $\mathrm{V} = 1000\ \mathrm{km/hr} = 1000 \times \frac{5}{8} = 277.78\ \mathrm{m/s}$

Pressure, $p_1 = -9.81\ \mathrm{kN/\ m^2}$ (vacuum)

Temperature $t_1 = 47°\mathrm{C} = 47 + 273 = 320\ \mathrm{K}$

Atmospheric Pressure, $p_{atm} = 98.1\ \mathrm{kN/\ m^2}$

R= 287 J/kg °k , γ =1.4

Pressure of air $p_1 = p_{atm} + p_{gauge} = 98.1 + (-9.81) = 88.29\ \mathrm{kN/\ m^2}$

Velocity of sound C = $\sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 320}$

$$C_1 = 358.6\ \mathrm{m/s}$$

Mach number

$$\mathrm{M_1} = \frac{V_1}{C_1} = \frac{277.78}{358.6} = 0.775$$

Stagnation pressure

$$\mathrm{P_s} = \mathrm{P_1}\left(1 + \left(\frac{\gamma-1}{2} M_1^2\right)\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 88.29\left(1 + \left(\frac{1.4-1}{2} \times 0.775^2\right)\right)^{\frac{1.4}{1.4-1}}$$

$$= 131.27\ kN/\ \mathrm{m^2}$$

Stagnation temperature T_s

$$T_s = T_1\left(1 + \left(\frac{\gamma - 1}{2}\right) M_1^2\right)$$

$$= 320\left(1 + \left(\frac{1.4 - 1}{2}\right) \times 0.775^2\right) = 358.4 \text{ k}$$

$$T_s = 358.4 - 273 = 85.4°C$$

Stagnation density ρ_s :

$$P_s = \frac{\rho_S}{RT_S} = \frac{131.27}{287 \times 358.4} = 1.276\ kg/m^3$$

Q 5) B) A flow has a velocity potential function is given by $\phi = x^3 - 3xy^2$. Verify whether it represents a valid flow field. If it does then determine the stream function. (10)

Solution:

Velocity potential function,

(i) Velocity components in x and y direction

$$u = -\frac{\partial \phi}{\partial x} - \frac{\partial}{\partial x}(x^3 - 3xy^2) = -(3x^2 - 3y^2)$$

$$v = -\frac{\partial \phi}{\partial x} - \frac{\partial}{\partial y}(x^3 - 3xy^2) = -(-6xy) = 6xy$$

(ii) Check for continuous and irrotational flow

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(-3x^2 + 3y^2) = -6x, \quad \frac{\partial u}{\partial x} = 6y$$

$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(6xy) = 6x, \quad \frac{\partial v}{\partial y} = 6y$$

Continuity equation, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -6x + 6x = 0$

Flow is continuous,

For irrotational flow,

$$w_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right] = 0$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \ 6y = 6y$$

The flow is irrotational.

Q 6) A) Two reservoirs are connected by three pipes in series.

F	Length	Diameter	f
1	**500 m**	**30 cm**	**0.02**
2	**200 m**	**10 cm**	**0.025**
3	**100 mm**	**10 cm**	**0.03**

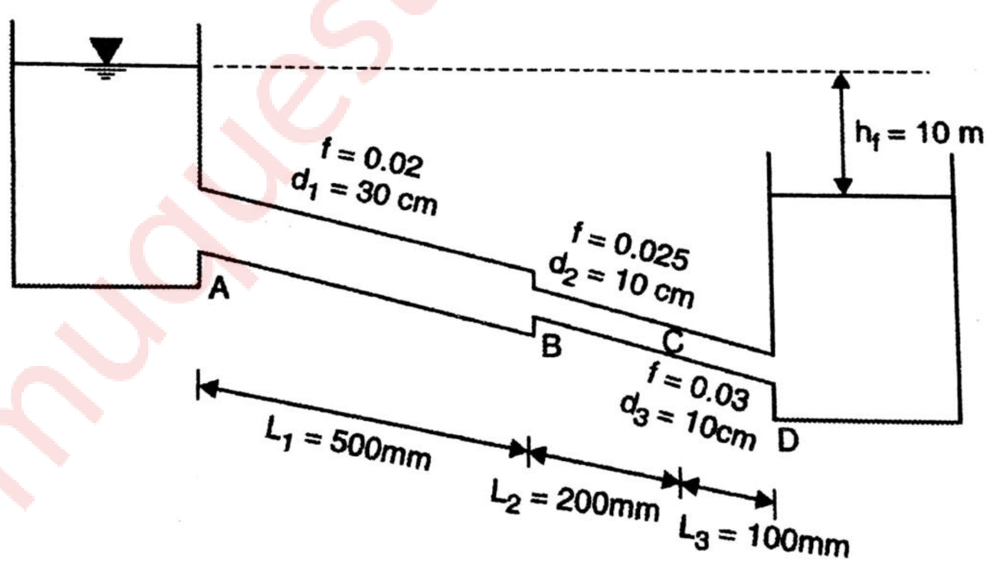
Calculate the discharge through them if the elevation difference of the levels is in the reservoirs is 10 m considering minor losses. (10)

Solution:

Area of pipe

$$A_1 = \frac{\pi}{4}(d_1)^2 = A_1 = \frac{\pi}{4}(0.3)^2 = 0.0707 \text{ m}^2$$

$$A_2 = \frac{\pi}{4}(d_2)^2 = A_1 = \frac{\pi}{4}(0.1)^2 = 7.854 \times 10^{-3} \text{ m}^2$$



Using continuity equation

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

$$V_2 = \left(\frac{A_1}{A_2}\right) V_1$$

$$= \left(\frac{0.0707}{7.854\times10^{-3}}\right) V_1 = 9V_1$$

$$V_2 = V_3 = 9V_1$$

Head loss for pipe in series (neglecting minor losses)

$$h_f = h_{f1} + h_{f2} + h_{f3}$$

$$= \left(\frac{flv^2}{2gd}\right)_1 + \left(\frac{flv^2}{2gd}\right)_2 + \left(\frac{flv^2}{2gd}\right)_3$$

$$= \frac{0.02\times500\ V_1^2}{2\times9.81\times0.3} + \frac{0.025\times200\ (9V_1)^2}{2\times9.81\times0.1} + \frac{0.03\times100\ (9V_1)^2}{2\times9.81\times0.1}$$

$$= 1.699\ V_1^2 + 206.42\ V_1^2 + 123.85\ V_1^2$$

$$10 = 331.97\ V_1^2$$

$$V_1 = \sqrt{\frac{10}{331.97}} = 0.174$$

Discharge $\quad Q = A_1V_1 = 0.0707 \times 0.174$

$$= 0.0123\ m^2/s$$

$= 12.3$ Lps.

Q 6) B) Write short notes: (Any two) **(10)**

(I) Moody's diagram

Solution:

(i) L.F moody's diagram is a graph plotted to find darcy – Weisbach friction factor for commercial pipe. The diagram is plotted in the form of frictional factor verses Reynold's number R_e curves for various value of relative roughness (R/k).

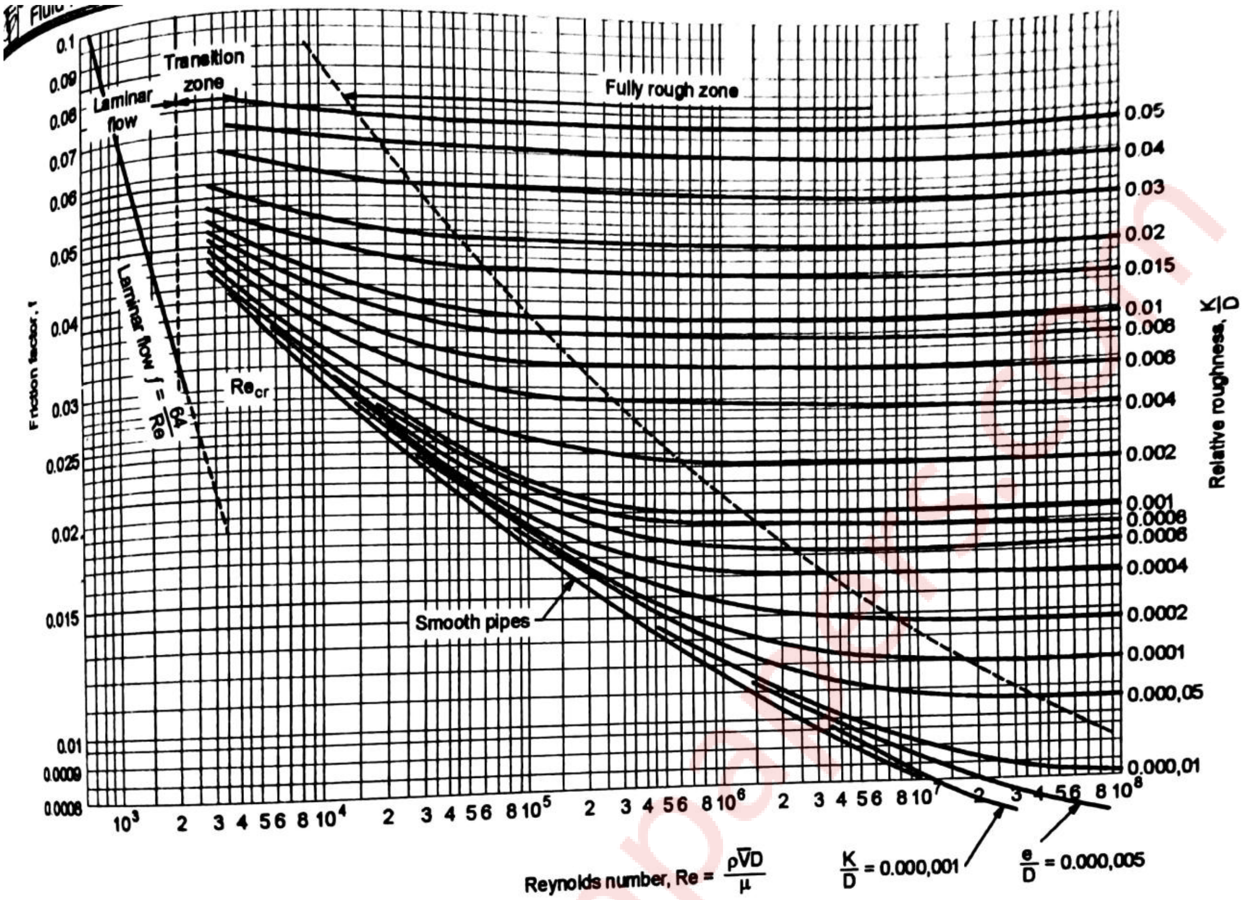


FIG : MOODY'S CHART

(ii) It is similar to Nikuradse's plot except for the transition regions. Moody's has plotted curve for equation,

$$\frac{1}{\sqrt{f}} - 2\log_{10}\left(\frac{R}{k}\right) = 1.74 - 2\log_{10}\left(1 + \frac{R/k}{R_e\sqrt{f}}\,18.7\right)$$

Which helps to determine friction factor f from the curve if the numerical values of $\frac{R}{k}$ and R_e of flow is known.

(iii) The value of equivalent sand grain roughness k is depends on the condition of material. As the pipe become older the roughness increase due to corrosion.

(ii) Lift force on circulating cylinder in uniform flow.

Solution:

A body wholly immersed in a real fluid may be subjected to following force due to relative motion between the body and the fluid as shown in fig. :

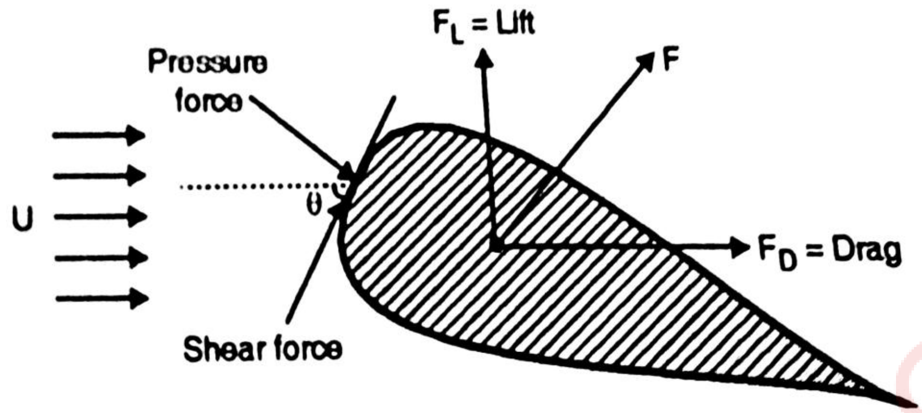


FIG : **LIFT AND DRAG**

(i) Lift force: The component of force at right angles to the direction of flow is called the lift force, F_L.

(ii) When a free sream approaches the body along the axis of symmetry, the force acting on the body is only the drag force, in the direction of flow and there is no lift force.

(iii) The production of lift force requires asymmetric of flow, while drag force exists always.

(iv) It is possible to create drag without lift but impossible to create lift without drag.

(v) If the axis of the body is parallel to fluid flow, lift force is zero.

$$F_L = C_L \times \frac{1}{2} \rho U^2 . A$$

(iii) Compressible flow through convergent divergent nozzle.

Solution:

(i) Convergent – divergent nozzle is also known as laval nozzle.

(ii) In this nozzle, subsonic flow induces in the converging section, critical or transonic conditions in the throat and supersonic flow in the diverging section.

(iii) it is used to expand a flow from a given reservoir condition to a controlled back condition.

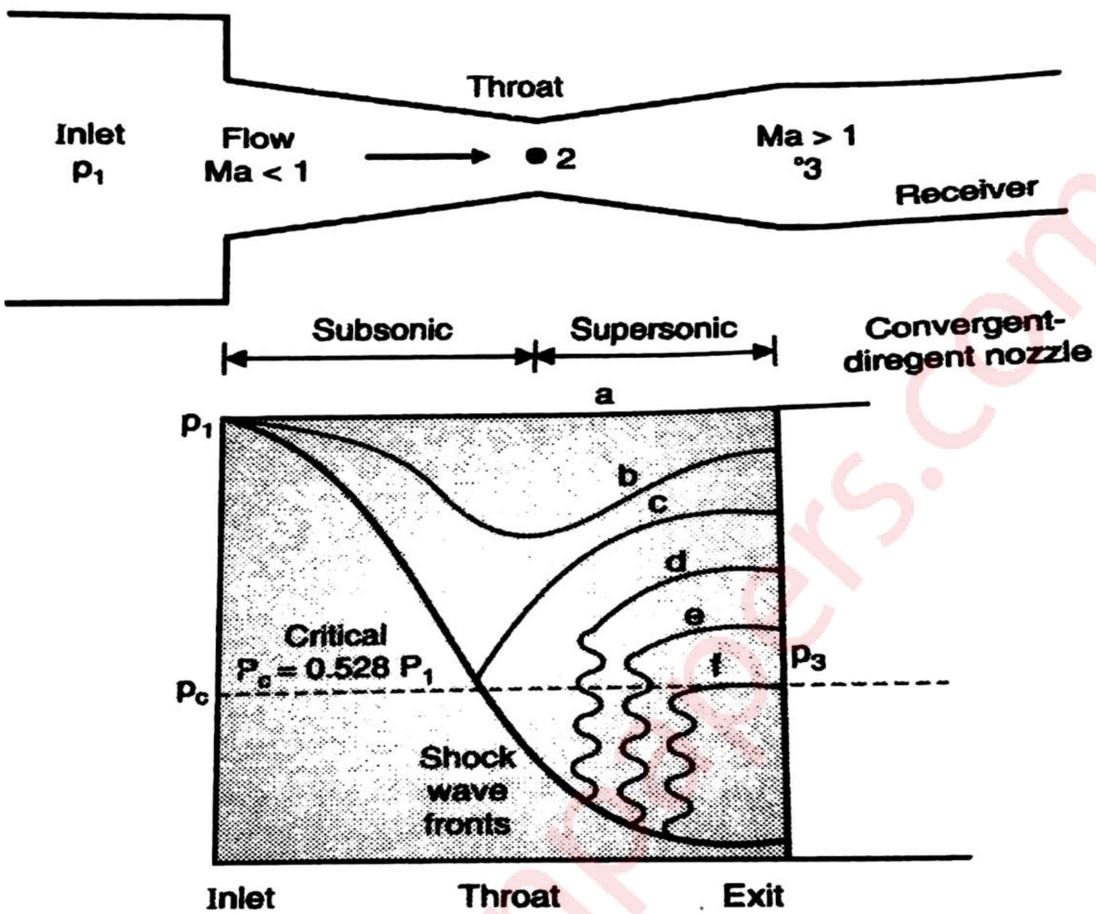


FIG : CONVERGENT-DIVERGENT NOZZLE

(iv) It is used to accelerate a hot, pressurized gas passing through it to a higher supersonic speed on the axial direction, by converting the heat energy of flow into kinetic energy.

(v) This laval nozzle is used in steam turbine and rocket engine nozzles.