(05)

KINEMATICS OF MACHINERY SOLUTION

SEM 4 (CBCGS-DEC 2018)

BRANCH-MECHANICAL ENGINEERING

Q 1) A) What are rigid and resistant bodies? Elaborate.

Solution:

i) Rigid body



If this a body & there are 2 points on this body A & B (as shown in above picture) & we started applying force on this body in different directions, this body try to deform it might elongate, compressed so this body can be deformable so the difference between point A & B will change. But in ideal condition of non-deformable you keep on applying forces it will not deform, not compress, not elongate and ofcourse no change in geometry at all and distance between any 2 points will not change, such non-deformable bodies is called rigid body.

ii) Resistant body

Now consider a vessel in which liquid is kept, consider 2 points A & B, if we just spilled this liquid outside the vessel the distance between point A & B definitely going to change even without applying force you can vary the distance between them.

But there will be the certain situation in which the distance between point A & B will not change let see how

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Consider a hydraulic lift (hydraulic lift is used to lift heavy weight just by a application of comparably very small force & it works on a principle of Pascal's law)

So in this case the fluid is under compression state & now if you choose any 2 point A & B in this fluid the distance between them will not change. So in this condition a non-rigid body is behaving like a rigid body, so such bodies are known as resistant body.

When a non-rigid body is act like a rigid body, it serves the purpose of rigid body for a particular case such bodies are known as resistant body.

Q 1) B) What is fundamental equation of steering gears? Which steering gear fulfill this condition?

Solution:

i) Fundamental equation of steering gears:



b = Wheel base, and

c = Distance between the pivots A and B of the front axle.

$$\cot \varphi - \cot \theta = c / b$$

This is the fundamental equation for correct steering. If this condition is satisfied, there will be no skidding of the wheels, when the vehicle takes a turn.

ii) Davis Steering Gear:

The Davis steering gear is an exact steering gear mechanism which fulfill the condition of fundamental equation of steering gears.

Q 1) C) Two points located along the radius of a wheel have velocities of 8 m/s and 14 m/s respectively. The distance between the points is 300 mm. What is radial distance of outer point from the centre. (05)

Solution:

$$d_{1} = \frac{\pi \times V_{1}}{60} = \frac{\pi \times 8}{60}$$

= 0.42 m = 418.87 mm
$$d_{2} = \frac{\pi \times V_{2}}{60} = \frac{\pi \times 14}{60}$$

= 0.73 m = 733.03 mm

Radial distance = $(d_1 + d_2)$ – distance between two points

=(418.87+733.03)-300

= 851.9 mm

Q 1) D) Define base circle, pitch circle, trace point, pitch curve and pressure angle. (05)

Solution:

i) Base circle:

It is the smallest circle that can be drawn to the cam profile.

ii) Pitch circle:

It is a circle drawn from the centre of the cam through the pitch points.

iii) Trace point:

It is a reference point on the follower and is used to generate the pitch curve.

iv) Pitch curve:

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It is the curve generated by the trace point as the follower moves relative to the cam.

v) Pressure angle:

It is the angle between the direction of the follower motion and a normal to the pitch curve.

Q 1) E) State and derive law of gearing.

Solution:

Consider the portions of the two teeth, one on the wheel 1 (or pinion) and the other on the wheel 2, as shown by thick line curves in Fig. Let the two teeth come in contact at point Q, and the wheels rotate in the directions as shown in the figure.

Let T T be the common tangent and MN be the common normal to the curves at the point of contact Q. From the centres O_1 and O_2 , draw O_1M and O_2N perpendicular to MN. A little



consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2.

Let v1 and v2 be the velocities of the point Q on the wheels 1 and 2 respectively. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.

 \therefore v1 cos α = v2 cos β

 $(\omega_1 \times 0_1 Q) \cos \alpha = (\omega_2 \times 0_2 Q) \cos \beta$

$$(\omega_1 \times O_1 Q) \ \frac{O_1 M}{O_1 Q} = (\omega_2 \times O_2 Q) \ \frac{O_2 N}{O_2 Q} \ \text{or} \omega_1 \times O_1 M = \omega_2 \times O_2 N$$

 $\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M}$

Also from similar triangles O_1MP and O_2NP ,

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

Combining equations (i) and (ii), we have

$$\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P}$$

From above, we see that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the centres $O1_1$ and O_2 , or the common normal to the two surfaces at the point of contact Q intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities.

Therefore in order to have a constant angular velocity ratio for all positions of the wheels, the point P must be the fixed point (called pitch point) for the two wheels. In other words, the common normal at the point of contact between a pair of teeth must always pass through the pitch point. This is the fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels. It is also known as law of gearing.

Q 2) A) A pinion of 120 mm pitch circle diameter and having 20 involute teeth drives a rack. The addendum of both pinion and rack is 6 mm. Determine the least value of pressure angle to avoid interference. With this value of pressure angle, find the length of the arc of contact and the minimum number of teeth in contact at a time. (10)

Solution.

Given : T = 20; d = 120 mm or r = OP = 60 mm; LH = 6 mm

...(Refer fig.)



Least pressure angle to avoid interference :

Let ϕ = Least pressure angle to avoid interference.

We know that for no interference, rack addendum,

LH =
$$rsin^2 \phi$$
 or $rsin^2 \phi = \frac{LH}{r} = \frac{6}{60} = 0.1$

or

$$\therefore \sin \varphi = 0.3162$$

$$\varphi = 18.435^\circ \times \frac{\pi}{180} = 0.322$$
 rad

Length of the arc of contact:

We know that length of the path of contact,

$$KL = \sqrt{(OK)^2 - (OL)^2}$$

= $\sqrt{(OP + 6)^2 - (OP \cos \phi)^2}$
= $\sqrt{(60 + 6)^2 - (60 \cos 0.322)^2}$
= $\sqrt{4356 - 3239.46} = 33.41 \text{ mm}$

∴ Length of the arc of contact

$$= \frac{\text{Length of the path of contact}}{\cos \varphi} = \frac{33.41}{\cos 0.322} = 35.22 \text{ mm}$$

Minimum number of teeth:

We know that circular pitch,

 $pc = \pi d / T = \pi \times 120 / 20 = 18.84 \text{ mm}$

and the number of pairs of teeth in contact

$$=\frac{\text{Length of the arc of contact}}{\text{circular pitch (P_c)}} = \frac{35.22}{18.84} = 1.87$$

: Minimum number of teeth in contact

= 2 or one pair

Q 2) B) What is the effect of centrifugal tension on the power transmitted?

Solution:

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as the slack sides. The tension caused by centrifugal force is called centrifugal tension. At lower belt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher belt speeds (more than 10 m/s), its effect is considerable and thus should be taken into account.

When the centrifugal tension is taken into account, then total tension in the tight side,

$$T_{t1} = T_1 + T_C$$

and total tension in the slack side,

$$T_{t2} = T_2 + T_0$$

...(in watts)

Power transmitted,

$$P = (T_{t1} - T_{t2}) v$$

 $= [(T_1 + T_C) - (T_2 + T_C)]v = (T_1 - T_2)v \qquad ...(same as before)$

Thus we see that centrifugal tension has no effect on the power transmitted.

Q 3) A) In an open belt drive, the diameters of the larger and smaller pulley are 1.2 m and 0.8 m respectively. The smaller pulley rotates at 320 rpm. The centre distance between the shaft is 4 m. When stationary, the initial tension on the belt is 2.8 KN. The mass of belt is 1.8 kg/m and the coefficient of friction between the belt and pulley is 0.25. Determine the power transmitted. (10)

Solution:

Given :
$$x = 4 \text{ m}$$
; $d_1 = 1.2 \text{ m}$; $N_2 = 320 \text{ rpm}$; $d_2 = 0.8 \text{ m}$; $\mu = 0.25$;

 $m = 1.8 \text{ kg} / m : \mu = 0.3$; $T_0 = 2.8 \text{ KN} = 2800 \text{ N}$

We know that velocity of the belt,

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...(i)

$$\mathbf{V} = \frac{\mathbf{\pi} \times \mathbf{d}_2 \times \mathbf{N}_2}{60} = \frac{\mathbf{\pi} \times 0.8 \times 320}{60}$$

$$V = 13.4 \text{ m/s}$$

and centrifugal tension, $T_{C}\ =\ m.\,v^{2}\ =\ 1.8\ (13.4)^{2}\ =\ 323.2\ N$

Let T_1 = Tension in the tight side, and

 T_2 = Tension in the slack side.

We know that initial tension (T_0) ,

$$2800 = \frac{T_1 + T_2 + 2T_c}{2} = \frac{T_1 + T_2 + 2 \times 323.2}{2}$$
$$T_1 + T_2 = 2800 \times 2 - 2 \times 323.2 = 4953.6 \text{ N}$$

For an open belt drive,

$$\sin \alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{1.2 - 0.8}{2 \times 4} = 0.05 \text{ or } \alpha = 2.86^{\circ}$$

: Angle of lap on the smaller pulley,

$$\theta = 180^{\circ} - 2\alpha = 180^{\circ} - 2 \times 2.86^{\circ} = 174^{\circ}$$

$$= 174^{\circ} \times \pi / 180 = 3.04 \text{ rad}$$

We know that

2.3 log
$$\left|\frac{T_1}{T_2}\right| = \mu. \theta = 0.25 \times 3.04 = 0.76$$

log $\left|\frac{T_1}{T_2}\right| = \frac{0.76}{2.3} = 0.3304 \text{ or } \frac{T_1}{T_2} = 2.07$...(ii)

...(Taking antilog of 0.3304)

From equations (i) and (ii),

$$T_1 = 3340 \text{ N}$$
; and $T_2 = 1613 \text{ N}$

∴ Power transmitted,

$$P = (T_1 - T_2)V$$

= (3340 - 1613) × 13.4
= 23141.8 w = 23.1 KW

Q 3) B) Use the following data of cam which is a knife-edge follower is raised with uniform acceleration and deceleration and is lowered with simple harmonic motion: least radius of

cam= $60 \ mm$, lift of follower= $45 \ mm$, angle of ascent = 60° , dwell between ascent and descent = 40° Angle of descent= 70°

If cam rotates at 180 rpm, determine maximum velocity and acceleration during ascent and descent. (10)

Solution:

Given:

 ${\rm Cam\ speed\ N}=180\ {\rm rpm}$

i) During ascent,

$$v_{max} = \frac{4h\omega}{{\phi_a}^2} \cdot \theta$$

When $\theta = \varphi_{a/2}$

$$v_{max} = 2h \frac{\omega}{\phi_a} = 2 \times 0.045 \times \frac{18.85}{60\pi/180} = 1.62 \ m/s$$

Acceleration is constant,

$$f_{\text{uniform}} = \frac{4 \text{ h} \omega^2}{(\phi_a)^2} = \frac{4 \times 0.045 \times (18.85)^2}{\left(\frac{60\pi}{180}\right)^2}$$
$$= 58.32 \text{ m/s}^2$$



ii) During descent it is a simple harmonic motion

$$\mathbf{v} = \frac{h}{2} \frac{\pi \omega}{\phi_{\mathrm{d}}} \sin \frac{\pi \theta}{\phi_{\mathrm{d}}}$$

maximum value is at $\theta = \phi_d/2$

$$v_{\text{max}} = \frac{h}{2} \frac{\pi \omega}{\phi_{\text{d}}} = \frac{0.045}{2} \times \frac{\pi \times 18.85}{70\pi/180}$$
$$= 1.09 \ m/s$$

Acceleration variation is given by

$$f = \frac{h}{2} \left(\frac{\pi \, \omega}{(\phi)}\right)^2 \, \cos \frac{\pi \theta}{\phi}$$

it is max at $\theta = 0$ *i.e.*

$$f_{max} = \frac{h}{2} \left(\frac{\pi \,\omega}{(\phi)}\right)^2 = \frac{0.045}{2} \times \left(\frac{\pi \times 18.85}{70\pi/180}\right)^2$$
$$= 52.86 \,\mathrm{m/s^2}$$

Q 4) A) In a reduction gear shown in fig. 1, the input S has 24 teeth. P and C constitute a compound planet having 30 and 18 teeth respectively. If all gears are of the same pitch, find ratio of reduction gear. Assume A to be fixed. (10)



Solution:

Given: $t_P = 30$; $t_C = 18$; $t_S = 24$; $N_A = 0$;

Referring to fig. we have

Since number of teeth are proportional to their pitch circle diameters, it follows that

$t_{A} = 2 \left[\frac{t_{S}}{2} + t_{P} \right] = 2 \left[\frac{24}{2} + 30 \right] = 84$							
$t_D = 2 \left[\frac{t_S}{2} + \frac{t_P}{2} + \frac{t_C}{2} \right] = 2 \left[\frac{24}{2} + \frac{30}{2} + \frac{18}{2} \right] = 72$							
Action	Arm a	S	P/C	А	D		
'a' fixed,	0	+1	24	24 30	24 18		
S + 1 rev			$-\frac{30}{30}$	$-\frac{1}{30}$ $^{-}\overline{84}$	$-\frac{1}{30}$ 72		
'a' fixed,	0	x	4 <i>x</i>	2x	<u>_ x</u>		
S + x rev			5	7	5		
Add y	Y	y + x	$v - \frac{4x}{\overline{x}}$	$y - \frac{2x}{-}$	$y - \frac{x}{r}$		
			- <u>5</u>	7	5		

From given conditions,

$$N_A = y - \frac{2x}{7} = 0$$
 or $y = \frac{2x}{7}$

$$\frac{N_{\rm S}}{N_{\rm D}} = \frac{y+x}{y-\frac{x}{5}} = \frac{\frac{2x}{7}+x}{\frac{2x}{7}-\frac{x}{5}} = \frac{9}{7} \times \frac{35}{3} = 15$$

Q 4) B) A uniform 50-kg crate rest on a horizontal surface for which coefficient of friction is 0.2. Determine the crate acceleration if a force of P = 600 N is applied to the crate is shown in fig.2 (10)



Solution:

The horizontal forces acting on the crate are

i) applied pushing force (directed in the positive direction)

ii) retarding friction force fk (directed in the negative direction)

The equation for the net horizontal force is thus

$$\sum Fx = \overbrace{F_{applied}}^{positive} - \overbrace{f_k}^{negative}$$

The equation for the friction force f_k is

$$f_k = \mu_k n$$

- μ_k is the coefficient of kinetic friction
- n is the normal force exerted by the horizontal surface, equal to mg

Thus, we can also write the net force equation as

$$\sum F_x = F_{applied} - \mu_k mg$$

According to Newtons second law, the acceleration ax is given by

$$\sum F_x = ma_x$$

or

$$a_z = \frac{\sum F_x}{m}$$

Plugging in $F_{applied} - \mu_k mg$ for $\sum F_x$, we have

$$a_x = \frac{F_{applied} - \mu_k mg}{m}$$

We're given in the problem:

$$F_{applied} = 600 N$$
$$\mu_k = 0.2$$
$$m = 50.0 kg$$
and g = 9.81 m/s²

Plugging these in:

$$a_x = \frac{600 - 0.2 \times 50 \times 9.81}{50}$$
$$= 10.03 \ m/s^2$$

Q 5) A) A toggle mechanism shown in fig.3. Find the velocities of the slider by

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i) relative velocity method

ii) Instantaneous centre method



Solution:

. By relative velocity method :

 $N_{AO} = 50 \text{ r. p. m. or } \omega_{AO} = 2 \pi \times 50/60 = 5.23 \text{ rad/s}$

Since the crank length OA = 30 mm = 0.03 m, therefore velocity of A with respect to O or velocity of A (because O is a fixed point),

 $v_{AO} = v_A = \omega_{AO} \times OA = 5.23 \times 0.03 = 0.1569 \text{ m/s}$

... (Perpendicular to OA)

1. Velocity of slider C

First of all draw the space diagram, to some suitable scale, as shown in Fig. (a). Now the velocity diagram, as shown in Fig. (b), is drawn as discussed below :



FIG(B): VELOCITY DIAGRAMW

1. Draw vector oa perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O or velocity of A (i.e. v_{AO} or v_A ,) such that

vector oa = v_{AO} = v_A = 3.4 m/s

2. Since point B moves with respect to A and also with respect to C, therefore draw vector ab perpendicular to AB to represent the velocity of B with respect to A i.e. v_{BA} and draw vector cb perpendicular to QB to represent the velocity of B with respect to C, i.e. v_{BC} . The vectors ab and cb intersect at b.

3. From point b, draw vector bd perpendicular to BD to represent the velocity of C with respect to B i.e. v_{CB} , and from point c draw vector cd parallel to the path of motion of the slider C (which is along QD) to represent the velocity of D, i.e. v_{C} . The vectors bd and cd intersect at c. By measurement, we find that velocity of the slider C,

 v_D = vector oc = $1.5 \times 157.07 = 235.6$ mm/s = 0.23 m/s

By Instantaneous centre method :

Speed of crank OA,

$$N_{AO} = 180 \text{ r. p. m. or } \omega_{AO} = 2 \pi \times 180/60 = 18.85 \text{ rad/s}$$

Velocity of link AO is,

 $v_{AO} = v_A = \omega_{AO} \times OA = 5.23 \times 0.03 = 1569 \text{ m/s}$

The No. of instantaneous centres are,

$$n = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$



Link	1	2	3	4	5.	6
ICR's	12	23	34	(45)	66	
	13	24	35	46		
	14	25	36			
	15	26	-			
	16					



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$$\frac{\omega_3}{\omega_4} = \frac{I_{14}I_{34}}{I_{13}I_{34}}$$
$$\frac{1.57}{\omega_4} = \frac{50}{90}$$
$$\omega_4 = 2.82 \text{ cm/s}$$

Angular Velocity ratio for link 4 and 5 is

$$\frac{\omega_4}{\omega_5} = \frac{I_{15}I_{45}}{I_{14}I_{45}}$$
$$\frac{2.82}{\omega_5} = \frac{50}{50}$$
$$\omega_5 = 2.82 \text{ mm/s}$$

Velocity of slider C

$$v_{\rm C} = \omega_5 \times (I_{15}I_{56}) = 2.82 \times 50 = 141 \, {\rm mm/s} = 0.141 \, m/s$$

Q 5) B) Explain self energizing and self locking brake.

Solution:

i) Self energizing brake:

1) The frictional force helps to apply the brake. Such type of brakes are said to be self energizing brakes.

2)
$$P = \frac{R_n(x-\mu.a)}{l}$$
 $R_n x - \mu.R_n.a = P.l$

 $R_n. x = P.l + \mu. R_n. a$

From the above equation we can say that moment of frictional force (μ, R_n, a) adds to the moment force (p, l). In other word frictional force helps to apply brake. Such types of brakes are called self energizing brakes.

ii) Self locking brakes :

1) When the frictional force is great enough to apply the brake with no external force, then the brake is said to be self-locking brake

2)
$$R_n = \frac{P.l}{x-\mu.a}$$
 $P = \frac{R_n(x-\mu.a)}{l}$

In above equation if $x \le \mu$. *a*, the effort P become negative or zero. Thus no effort is required to apply brakes and hence brake is self locking.

Therefore, condition for self-locking is,

$x \leq \mu.a$

Q 6) A) In the toggle mechanism as shown Fig. 3, the crank OA rotates at 210 rpm counterclockwise increasing at the rate of 60 rad/ s^2 . For the given configuration, determine i) velocity of slider D and angular velocity of link BD ii) acceleration of slider D and angular acceleration of link BD.

Solution:

$$v_a = \frac{2\pi \times 210}{60} \times 0.2 = 4.4 \, m/s$$

Complete the velocity diagram as follows (Fig. b):



1) Take the vector oa representing v_a .

2) Draw lines ab $\perp AB$ through a and qb $\perp QB$ through q, the intersection locates the point b.

3) Draw the line $bd \perp BD$ through b and a horizontal line through q or g to represent the line of motion of the sliderD. The intersection of the two lines locates the point d.

Velocity of slider D = gd = 2.54 m/s

Angular velocity of BD = bd/BD = 3.16/0.5 = 6.32 rad/s

Set the following vector table:

For acceleration diagram, adopt the following steps:

i) Take the pole point o_1 or c_1 (fig c).

ii) Starting from o_1 , take the first vector o_1a_0 . To the first vector, add the second vector. Thus, the total acceleration o_1a_1 of A relative to O is obtained.

iii) Take the third vector and place its tail at q_1 and through its head draw a perpendicular line to have the fourth vector.

iv) Take the fifth vector and place its tail at a_1 Through its head draw a perpendicular line to add the sixth vector.

v) The intersection of lines of the fourth and sixth vectors locates the point b_1 .

vi) Take the seventh vector and put its tail at b_1 . Through its head, draw a perpendicular line to add the eight vector.

vii) For ninth vector, draw a line through g_1 parallel to the slider motion.

viii) The intersection of lines of the eighth and ninth vectors locates the point d_1 .

Acceleration of slider $D = d_1g_1 = 16.4 m/s^2$

Angular acceleration of $BD = \frac{b_d d_1}{BD} = 5.46/0.5 = 109.2 \ rad/s^2$

SN	Vector	Magnitude (m/s^2)	Direction	Sense
1	f_{ao}^c or $o_1 a_0$	$(0a)^2 - (4.4)^2 = 0.00$	∥ <i>0A</i>	$\rightarrow 0$
		$\frac{1}{0A} = \frac{1}{0.2} = 96.8$		
2	f_{ao}^t or a_0a_1	$\alpha \times OA = 60 \times 0.2 = 12$	$\perp OA$	-
3	f_{bq}^c or $q_1 b_q$	$(bq)^2 - (3.39)^2 - 20.2$	∥ <i>BQ</i>	$\rightarrow Q$
		$\frac{1}{BQ} = \frac{1}{0.3} = 38.3$		
4	qf_{bq}^t or b_qb_1	-	$\perp BQ$	-
5	f_{ba}^{c} or $a_{1}a_{b}$	$(ab)^2 - (1.54)^2 - 5.02$	∥ <i>AB</i>	$\rightarrow A$
		AB = 0.4 = 5.93		
6	f_{ba}^t or $a_b a_1$	-	$\perp AB$	-
7	\int_{db}^{c} or $b_1 b_d$	$(bd)^2 - (3.16)^2 - 20$	<i>BD</i>	$\rightarrow B$
		$\frac{1}{BD} = \frac{1}{0.5} = 20$		
8	f_{db}^t or $b_1 d_1$	-	$\perp BD$	-
9	f_{dg}^t or $g_1 d_1$	_	To slider	-
			motion	

Q 6) B) A fixed gear having 200 teeth meshes with a pinion having 50 teeth. The two are connected by an arm. What is the number of turn made by pinion for one complete revolution of the arm about center of thr gear?

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Solution:

Given:

Number of teeth on gear, T = 200

Number of teeth on pinion, t = 50

 $Velocity ratio = \frac{Number of teeth on pinion}{Number of teeth on gear}$

$$=\frac{50}{200}=0.25$$

Number of turns on pinion = $V.R \times t$

 $= 0.25 \times 50$

= 12.5 turns per revolution