MECHANICAL ENGINEERING

STRENGTH OF MATERIALS

CBCGS(MAY- 2019) Q.P.Code: 39566

1.a) A material has Young's Modulus of 2 x 10^5 N/mm^2 and Poisson's Ratio of 0.32. Calculate the Modulus of Rigidity and Bulk Modulus of the material. (05)

E = 2 x 10^5 N/mm^2 μ = 0.32

E = 2G(1+ μ)

$\therefore$ **G** = $\frac{E}{2(1+\mu)} = \frac{2 \times 10^5}{2(1+0.32)}$ = **75.76 x 10^3 N/mm^2**

Also, E = 3K(1 - 2 μ)

$\therefore$ **K** = $\frac{E}{3(1+2\mu)} = \frac{2 \times 10^5}{2(1-2(0.32))}$ = **1.85 x 10^5 N/mm^2**

1.b) Derive the relationship between the rate of loading, shear force and bending moment in a beam. (05)

Relation between loading, shear force and bending moment:

Consider a small strip, PQ = Segment of beam se

Or, w = $\frac{\partial f}{\partial x}$ i.e load intensity = slope of SF curve

Also, F = $\int w.dx$ is the expression of SF given loading function w.

Now, equating the moments at Q,

M - w $\partial x \frac{\partial x}{2}$ – F. ∂x = M + ∂M

Or, F = -$\frac{\partial M}{\partial x}$ is the relation between BM and SF.

Important points: 1. From the above equation, M = - $\int F.dx$

2. +ve SF gives -ve BM and vice cersa

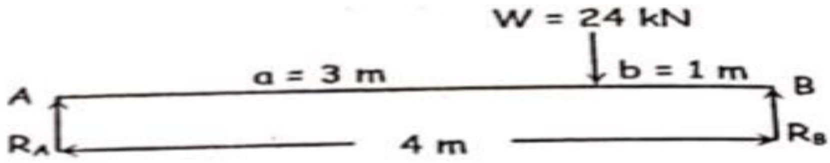
3. M= f(x). Hence, for M_{max}, $\frac{\partial M}{\partial x}$ = 0 i.e For max BM, SF =0

1.c) A simply supported beam of span 4m with EI constant throughout the span is subjected to a load of 24 kN at 3 m from left end support. Find total strain energy of the beam in bending. (05)

Support reactions,

$R_A = \frac{Wb}{L} = \frac{24 \times 1}{4} = 6$ kN

$R_B = \frac{Wa}{L} = \frac{24 \times 3}{4} = 18$ kN



Now, strain energy in bending is given by

$U = \int_0^L \frac{M^2 dx}{2EI}$

∴ Taking section at a distance x from A,

Origin	Portion	Moment	Limits
A	AC	6x	0–3
B	BC	18x	0–1

∴ Total strain energy,

$U = U_{AC} + U_{CB}$

$= \int_0^3 \frac{(6x)^2 dx}{2EI} + \int_0^1 \frac{(18x)^2 dx}{2EI} \quad = (\frac{36\,x^3}{6EI}) + (\frac{324\,x^3}{6EI})$

∴ U = $\frac{216}{EI}$ kN – m

1.(d) State the assumptions in the theory of pure bending and derive the formula, $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$ (05)

1.The material of the beam is homogeneous and isotropic.

2.The value of Young's Modulus of Elasticity is same in tension and compression.

3.The transverse sections which were plane before bending, remain plane after bending also.

4.The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.

5.The radius of curvature is large as compared to the dimensions of the cross-section.

6.Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

Let,

M = bending moment acting on beam

θ = Angle subtended at centre by the arc.

R = Radius of curvature of neutral layer *M′ N’.*

At any distance '*y*' from neutral layer *MN*, consider layer *EF*.

As shown in the figure the beam because of sagging bending moment. After bending, *A'B'*, *C'D’, M'N’ and E'F'* represent final positions of *AB*, *CD*, *MN* and *EF* in that order.

When produced, *A′ B′* and *C′ D′* intersect each other at the *O* subtending an angle θ radian at point *O*, which is centre of curvature.

As *L* is quite small, arcs *A′ C’, M′ N’, E′ F’ and B′ D’ can* be taken as circular.

Now, strain in layer *EF* because of bending can be given by *e* = (*E F* - *EF*)/*EF* = (*E F* - *MN*)/*MN*

As *MN* is the neutral layer, *MN* = *M′ N′*

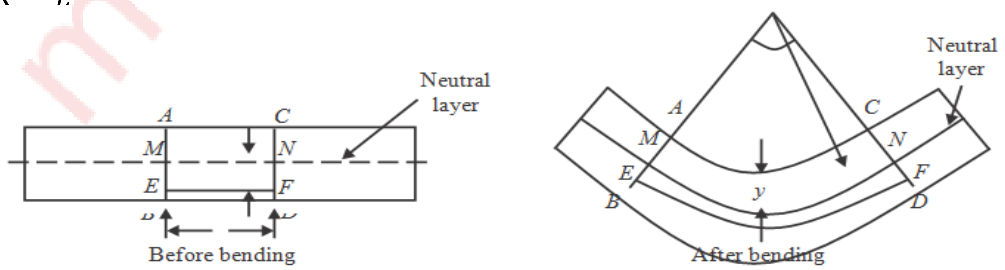
$$e = \frac{E'F' - M'N'}{M'N'} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y\theta}{R\theta} = \frac{y}{R} \qquad \text{.......}(i)$$

Let σ = stress set up in layer EF because of bending and E = Young's modulus of material of beam.

$$E = \frac{\sigma}{e} \text{ or } e = \frac{\sigma}{E} \qquad \text{.......(ii)}$$

Equate the equation (i) and (ii);

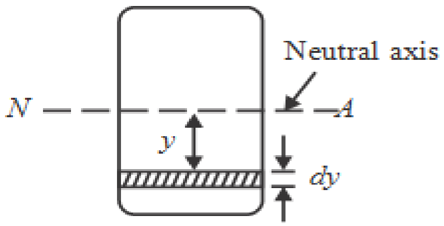
$$\frac{y}{R} = \frac{\sigma}{E} \qquad \text{....(iii)}$$



Before bending
(*a*)

After bending
(*b*)

$\sigma/y = \boldsymbol{E/R}$



At distance '*y*', let us consider an elementary strip of quite small thickness *dy*. We have already assumed that 'σ ' is bending stress in this strip.

Let *dA* = area of the elementary strip. Then, force developed in this strip = $\sigma.dA$.

Then the, elementary moment of resistance because of this elementary force can be
given by $dM = f.dA.y$

Total moment of resistance because of all such elementary forces can be given by

$$\int dM = \int \sigma \times dA \times y$$

$$M = \int \sigma \times dA \times y \qquad \text{....(iv)}$$

From the Equation (iii),

$$\sigma = y \times \frac{E}{R}.$$

By putting this value of σ in Equation (iv), we get

$$M = \int y \times \frac{E}{R} \times dA \times y = \frac{E}{R}\int dA \times y^2$$

But

$$\int dA \cdot y^2 = 1$$

where I = Moment of inertia of whole area about neutral axis N-A.

$$M = (E/R)\ .\ I$$
$$M/I = E/R$$
$$M/I = \sigma/y = E/R$$

Where;

M = Bending moment

I = Moment of Inertia about axis of bending that is I_{xx}

y = Distance of the layer at which the bending stress is consider

E = Modulus of elasticity of beam material.

R = Radius of curvature

1.(e) A short column of external diameter 400 mm and internal diameter 200 mm carries an eccentric load of 90 kN. Find the greatest eccentricity, which the load can have without producing tension on the cross section. (05)

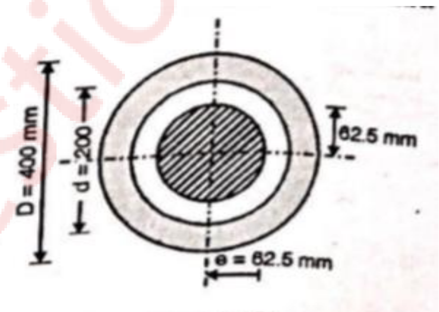
D = 400mm, d=200mm, P= 80kN = 80 x 10^3 N
For no tension condition
$\sigma_o - \sigma_b = 0$
$\therefore \sigma_o = \sigma_b$
$\therefore \frac{W}{A} = \frac{M}{Z} = \frac{W.e}{Z}$
$\therefore e \leq \frac{Z}{A}$
For circular section

$Z = \frac{\pi(D^4 - d^4)}{32D}$ $\quad$ $A = \frac{\pi(D^2 - d^2)}{4}$

$\therefore e \leq \frac{\frac{\pi(D^4-d^4)}{32D}}{\frac{\pi(D^2-d^2)}{4}} \leq \frac{D^2+d^2}{8D}$

$\therefore e \leq \frac{400^2+200^2}{8 \times 400}$

$\therefore$ e $\leq$ 62.5 mm



2.(a) For a beam loaded as shown in figure, calculate the value for UDL, w so that bending moment at C is 50 kNm. Draw the shear force and bending moment diagrams for the beam for the calculated value of w. Locate the point of contraflexure, if any. (12)

BM_C = 50 kN-m

$\sum M_A = 0$

w x 4 x $\frac{4}{2}$ – 8R_B + 20 x 10 = 0

$R_B = 25 + w$

Also, $\sum F_y = 0$

$R_A + R_B = 4w + 20$

$\therefore\ R_A = 3w - 5$

Now, taking moment about point C,

$BM_C = 4\ R_A = w \times 4 \times \frac{4}{2}$

$50 = 4(3w - 5) - 8w$

$\therefore w = 17.5\ kN/m$

$\therefore R_A = 47.5\ kN \qquad R_B = 48.5\ kN$

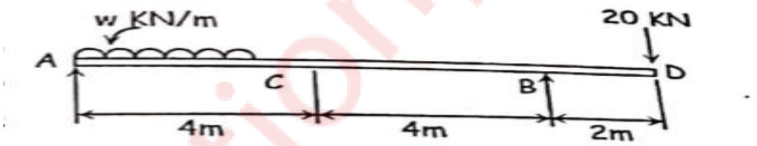
S.F calculations:

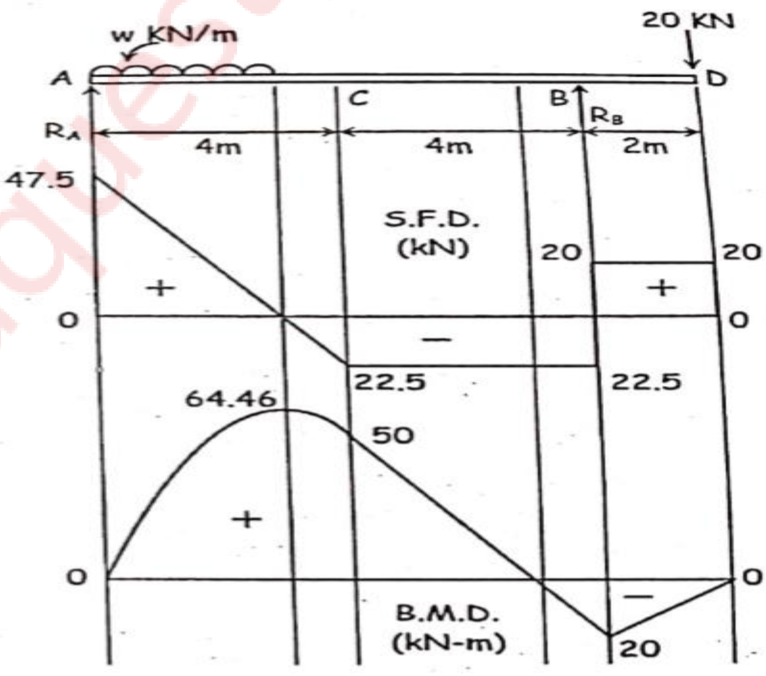
$SF_{AL} = 0$

$SF_{AR} = 3 \times 17.5 - 5 = 47.5\ kN$

$SF_{CL} = 47.5 - 17.5 \times 4 = --22.5\ kN$

$SF_{CR} = -22.5\ kN$





SF_{BL} = -22.5 kN

SF_{BR} = -22.5 + (25 + w) = 20 kN

SF_{DL} = 20 kN

SF_{DR} = 20 – 20 = 0

Point of maximum bending moments(x): using similarity of triangle,

$\frac{47.5}{x} = \frac{22.5}{4-x}$

$\therefore$ x = 2.714 m

BM calculation:

BM_A = 0

BM_C = 50 kNm

BM_B = -20 x 2 = -40 kNm

BM_D =0

$BM_x = R_A$ x 2.714 – w x 2.714 x $\frac{2.714}{2}$ = 64.46 kNm

Point of contraflexure(x'):

Taking moment about point E,

$BM_E = R_B$ x x' – 20 x (x' +2)

0 = 42.5 x x' – 20 x' – 40

$\therefore$ x' = 1.78 m

Point of contraflexure lies at 1.78 m to the left of point B.

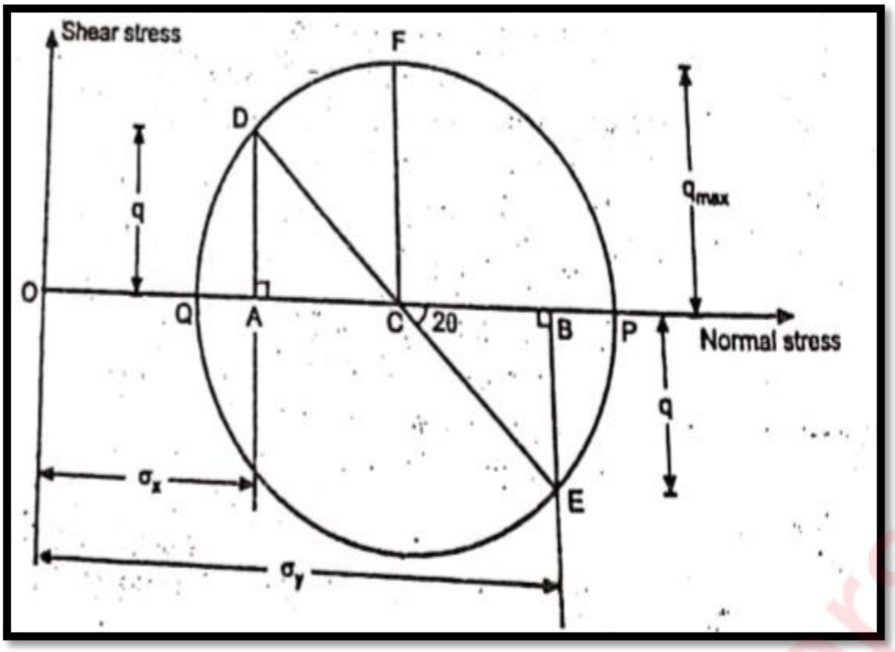
2.(b) An elemental cube is subjected to tensile stresses of 30 N/mm^2 acting on two mutually perpendicular planes and a shear stress of 10 N/mm^2 on these planes. Draw the Mohr's circle of stresses and hence or otherwise determine the magnitudes and directions of principal stresses and also the greatest shear stress. (08)

$\sigma_x = \sigma_y$ = 30 N/mm^2

τ = 10 N/mm^2

From diagram,

σ_{N1} = Major principle stress = L(OP) * Scale = 4*10 = 40 N/mm^2

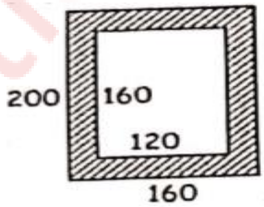


σ_{N2} = Minor principle stress = L(OQ) * Scale = 1*10 = 40 N/mm^2

τ_{max} = Radius of Mohr's Circle * Scale = 1.5 * 10 = 15 N/mm^2

$\theta_1 = 90^0$, $\theta_2 = 180^0$

3.(a) A box beam supports the loads as shown in figure. Compute the maximum value of P that will not exceed bending stress σ = 8 MPa or shear stress τ = 1.2 MPa for section between the supports. Also, draw shear stress distribution diagram at a section where shear force is maximum. (12)



σ = 8 MPa τ = 1.2 MPa

$\sum M_A = 0$

P x 2 – R_B x 4 + 4000 x 6 = 0

$R_B = \frac{P}{2} + 6000$

Also, $\sum F_Y = 0$

$R_A - P + R_B - 4000 = 0$

$R_A = \frac{P}{2} - 2000$

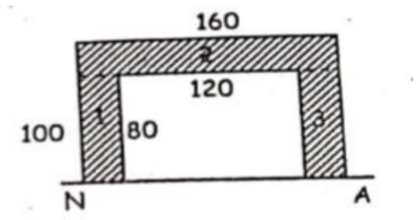
SF calculation:

$SF_{AL} = 0$ $\qquad$ $SF_{AR} = \frac{P}{2} - 2000$

$SF_{CL} = \frac{P}{2} - 2000$ $\qquad$ $SF_{CR} = \frac{P}{2} - 2000 - P$

$SF_{BL} = -\frac{P}{2} - 2000$ $\qquad$ $SF_{BR} = -\frac{P}{2} - 2000 + \frac{P}{2} + 6000$

$SF_{DL} = 4000$ $\qquad$ $SF_{DR} = 4000 - 4000 = 0$



BM Calculation:

$BM_A = BM_D = 0$

$BM_C = (\frac{P}{2} - 2000) \text{ x } 2000 = 1000P - 4000000$

$BM_B = 4000 \text{ x } 2000 = 8000000$

Now, $I = \frac{BD^3 - bd^3}{12} = \frac{160 \text{ x } 200^3 - 120 \text{ x } 160^3}{12} = 65.71 \text{ x } 10^6 \text{ mm}^4$

$y = \frac{200}{2} = 100$ mm

$\sigma = \frac{M_{max} \text{ x } y}{I}$

$\therefore P = 9256.8$ N

Also, $A\bar{y} = A_1y_1 + A_2y_2 + A_3y_3$

$= (80 \text{ x } 20 \text{ x } 40) + (160 \text{ x } 20 \text{ x } 90) + (80 \text{ x } 20 \text{ x } 40)$

$= 416000 \text{ m}^3$

Now, $\tau = \frac{SF_{max}(A\bar{y})}{Ib}$

Putting all the values we get,

P = 9256.8 N

For safe stress, selecting **P = 9256.8 N**

Shear stress distribution:

Shear stress at top/bottom fibre = 0

S = 2628.4 N

Shear stress at 80 mm above/below NA taking b = 160 mm

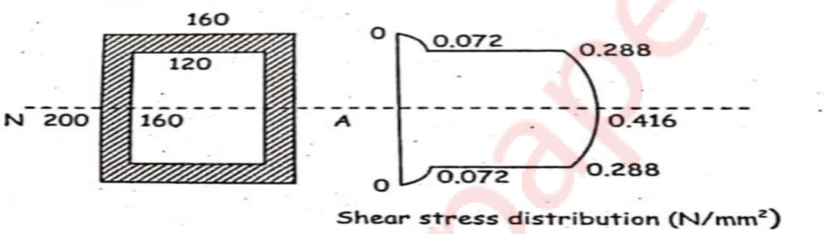
$\tau_{80,1} = \frac{2628.4 \text{ x } 160 \text{ x } 20 \text{x } 90}{65.71 \text{ x } 10^6 \text{ x } 160} = 0.072 \text{ N/mm}^2$

Shear stress at 80 mm above/below NA taking b = 40 mm

$\tau_{80,2} = \frac{2628.4 \text{ x } 160 \text{ x } 20 \text{x } 90}{65.71 \text{ x } 10^6 \text{ x } 40} = 0.288 \text{ N/mm}^2$

Shear stress at NA

$\tau_{max} = \frac{2628.4 \text{ x } 416000}{65.71 \text{ x } 10^6 \text{ x } 40} = 0.416 \text{ N/mm}^2$



Shear stress distribution (N/mm²)

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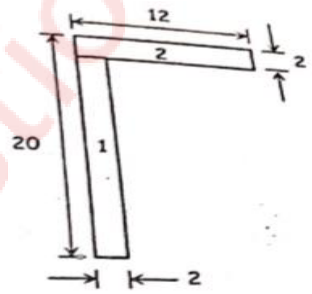
3. (b) Find the principal moments of inertia and directions of principal axes for the angle section shown. All dimensions are in cm. (08)



$A_1 = 18 \text{ x } 2 = 36 \text{ cm}^2$

$A_2 = 12 \text{ x } 2 = 24 \text{ cm}^2$

$y_1 = 9$ cm $\qquad x_1 = 1$ cm

$y_2 = 19$ cm $\qquad x_2 = 6$ cm

$\overline{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} = 3$ cm

$\overline{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = 13$ cm

$I_{XX} = I_{XX1} + I_{XX2}$

$= \frac{b_1 d_1^3}{12} + A_1(\bar{y} - y_1)^2 + \frac{b_2 d_2^3}{12} + A_1(\bar{y} - y_2)^2$

$= 2420\ cm^4$

$I_{YY} = I_{YY1} + I_{YY2}$

$= \frac{d_1 b_1^3}{12} + A_1(\bar{y} - y_1)^2 + \frac{d_2 b_2^3}{12} + A_1(\bar{y} - y_2)^2$

$= 660\ cm^4$

$I_{XY} = \sum A_i\ (x_i - \bar{x})(y_i - \bar{y}) \qquad = 720\ cm^4$

Principle Moment of Inertia:

$I_{max,min} = \frac{I_{xx} + I_{yy}}{2} \pm \sqrt{(\frac{I_{xx} + I_{yy}}{2})^2 + {I_{xy}}^2}$

$\therefore$ $\mathbf{I_{max}} = 1540 + 80\sqrt{202}$ = $\mathbf{2677.01\ cm^4}$

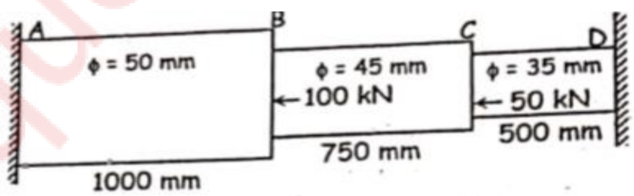
$\mathbf{I_{min}} = 1540 - 80\sqrt{202}$ = $\mathbf{402\ cm^4}$

Location of principle points

$\tan 2\theta_1 = \frac{2I_{xy}}{I_{yy} - I_{xx}} = \frac{-9}{11}$

$\therefore \boldsymbol{\theta_1} = \mathbf{70.35^0}$ and $\boldsymbol{\theta_2} = \theta_1 + 90 = \mathbf{160.35^0}$

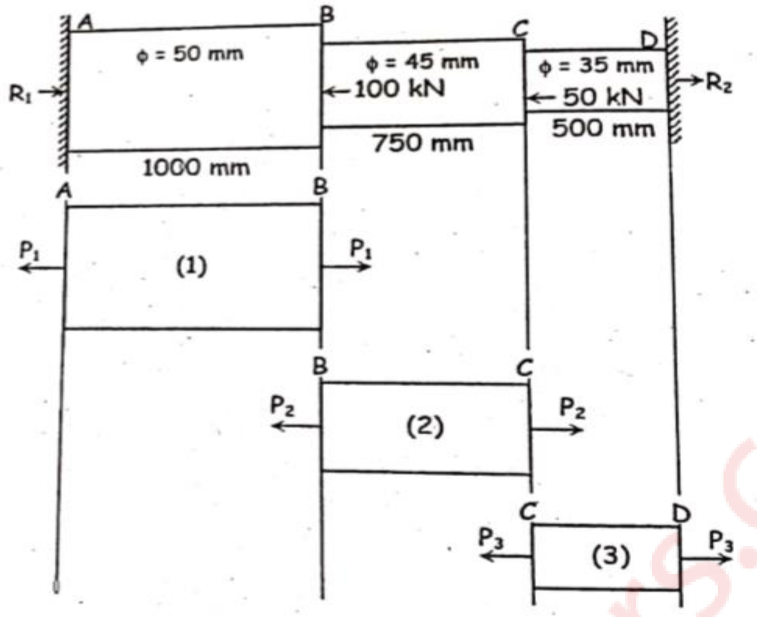
4.(a) A stepped round bar ABCD is fixed to unyielding support at sections A & D as shown in the figure. It is subjected to axial loads at sections B and C. Determine stresses in each portion of the bar and deflections of sections B and C. Take E = 200GN/m² **(10)**



$E = 200 \times 10^3$ MPa

$P_2 - P_1 = 100$

And $P_3 - P_2 = 50$



Net deformation = 0

$\therefore\ \delta L_{AB} + \delta L_{BC} + \delta L_{CD} = 0$

$\frac{P_1 L_1}{A_1 E} + \frac{P_2 L_2}{A_2 E} + \frac{P_3 L_3}{A_3 E} = 0$

Putting the values, we get

$\therefore P_2 = 22.786$ kN

$P_1 = -77.214$ kN

$P_3 = 72.786$ kN

$\sigma_{AB} = \frac{77.214 \times 10^3}{\left(\left(\frac{\pi}{4}\right) \times 50^2\right.} = 39.32\ N/mm^2$ (C)

$\sigma_{BC} = \frac{22.786 \times 10^3}{\left(\left(\frac{\pi}{4}\right) \times 45^2\right.} = 14.33\ N/mm^2$ (T)

$\sigma_{AB} = \frac{77.786 \times 10^3}{\left(\left(\frac{\pi}{4}\right) \times 35^2\right.} = 75.65\ N/mm^2$ (T)

$\delta L_C = \frac{P_3 L_3}{A_3 E} = \mathbf{0.189\ mm}$

$\delta L_B = \delta L_C + \frac{P_2 L_2}{A_2 E} = \mathbf{0.2427\ mm}$

4.(b) A cylindrical vessel of 1.5 m diameter and 4 m long is closed at the ends by a rigid plate. It is subjected to an internal pressure of 3 N/mm². If

maximum circumferential stress is not to exceed 150 N/mm^2, find the thickness of the shell. Find change in diameter, length and volume of the shell.

Assume E = 2 x 10^5 N/mm^2 and Poisson's ratio = 0.25 (10)

d = 1.5 m = 1500 mm L = 4m = 4000 mm p = 3 N/mm^2

σ_C = 150 N/mm^2 E = 2 x 10^5 N/mm^2 $\mu = 0.25$

$\sigma_C = \frac{pd}{2t}$ $\therefore$ **t = 15 mm**

$\sigma_L = \frac{pd}{4t}$ $\therefore$ σ_L = 75 N/mm^2

$e_c = \frac{1}{E}(\sigma_C - \mu\,\sigma_L) = \mathbf{6.5625 \times 10^{-4}}$

$e_L = \frac{1}{E}(\sigma_L - \mu\,\sigma_C) = \mathbf{1.875 \times 10^{-4}}$

$e_v = 2e_c + e_L = \mathbf{1.5 \times 10^{-3}}$

Now, $e_c = \frac{\delta d}{d}$ $\therefore \delta d =$ **0.984 mm**

$e_L = \frac{\delta l}{l}$ $\therefore \delta l =$ **0.75 mm**

$e_V = \frac{\delta V}{V}$ $\therefore \delta V =$ **10.6 x 10^6 mm^3**

5.(a) Determine the diameter of a solid steel shaft that will transmit 150 kW at a speed of 3 rev/sec, if the allowable shearing stress is 85 MPa. Also, determine the diameter of a hollow steel shaft, whose inside diameter is 3/4th of its outside diameter for the same conditions. What is the ratio of angle of twist per unit length for these two shafts? (10)

(i) Solid shaft

P = 150 kW N = 3 rps τ = 85 MPa

P = 2πNT

$\therefore$ T = 7.96 x 10^6 N-mm

Now, $T = \frac{\pi}{6} \times d^3 \times 85$

$\therefore$ d = 78.13 mm

(ii) Hollow shaft

$d_i = \frac{3}{4} D_0$

$T = \frac{\pi}{6} \times \frac{D_0^4 - d_i^4}{D_0} \times \tau$

∴ D_0 = 88.69 mm

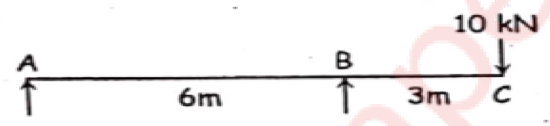
And d_i = 66.52 mm

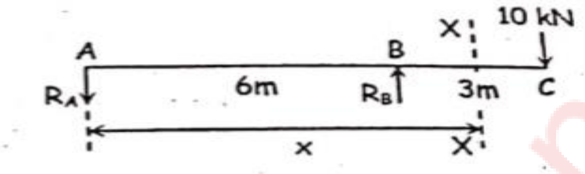
Now, $\theta = \frac{TL}{GJ}$

$\theta_{solid} = \frac{TL}{GJ_{solid}}$ $\theta_{hollow} = \frac{TL}{GJ_{hollow}}$

$\frac{\theta_{solid}}{\theta_{hollow}} = \frac{J_{hollow}}{J_{solid}}$ = **1.135**

5.(b) An overhanging beam ABC is loaded as shown in the figure. Find the slopes over each support and the deflection at the right end. Take E= 2 x 10^5 N/mm^2 and I = 5 x 10^8 mm^4 (10)





E = 2 x 10^5 N/mm^2 I = 5 x 10^8 mm^4

$\sum M = 0$

$- R_B \times 6 + 10 \times 9 = 0$

∴ R_B = 15 kN

$\sum F = 0$

∴ $R_A + R_B - 10 = 0$

∴ R_A = 5 kN

Using Macaulay's method taking section X-X at a distance x from point A

$EI \frac{d^2y}{dx^2} = -5x| + 15(x-6)$ ……(1)

Slope Equation

$EI \frac{dy}{dx} = C_1 - \frac{5x^2}{2} | + \frac{15\,(x-6)^2}{2}$ …….(2)

Deflection equation

$EI.y = C_1x + C_2 -0.833x^2| +2.5(x-6)^3$ ….(3)

Applying boundary conditions,

(i) At x = 0, y = 0 , from eq.(3) $C_2 = 0$
(ii) At x = 6, y = 0 , from eq.(2) $C_1 = 30$

To find **deflection** at point C, substituting x=9 in deflection equation

$\therefore$ y = -2.7 mm

(ii) To find **slope** at end A

Substituting x= 0 in slope equation.

$\therefore \frac{dy}{dx}$ **= 0.0003 radians**

To find slope at end B

Substituting x= 6 in slope equation.

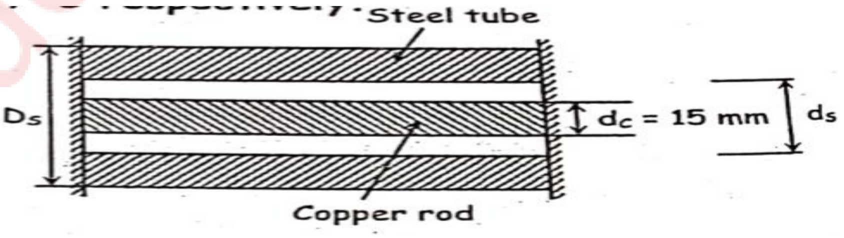
$\therefore \frac{dy}{dx}$ **= -0.0006 radians**

6.(a) A steel tube of 30 mm external diameter and 20 mm internal diameter encloses a copper rod of 15 mm diameter to which it is rigidly fixed at each end. If at a temperature of 10^0C, there is no longitudinal stress, calculate the stresses in the rod and tube when the temperature is raised to 200^0C. Take E for steel and copper as 2.1×10^5 N/mm^2 and 1×10^5 N/mm^2. The value of coefficient of linear expansion for steel and copper is given as $11 \times 10^{-6}/^0$C and $18 \times 10^{-6}/^0$C. (10)

$D_s = 30$ mm $d_s = 20$ mm $d_c = 15$ mm

$T_1 = 10^0$C $T_2 = 200^0$ C

$E_S = 2.1 \times 10^5$ N/mm^2 $E_C = 1 \times 10^5$ N/mm^2



$\alpha_s = 11 \times 10^{-6}/^0$C $\alpha_C = 18 \times 10^{-6}/^0$C

Since $\alpha_C > \alpha_s$ compression will be produced in copper and tension in steel.

$\therefore \sigma_C A_C = \sigma_S A_S$

$\therefore \sigma_C \times \frac{\pi}{4} \times d_c^2 = \sigma_S \times \frac{\pi}{4} \times [D_s^2 - d_s^2]$

$\therefore \sigma_C = 2.22\, \sigma_S$

Also, $(\alpha_C - \alpha_S)\, \Delta T = \frac{\sigma_C}{E_C} + \frac{\sigma_S}{E_S}$

$(18 \times 10^{-6} - 11 \times 10^{-6})\,(200 - 10) = \frac{2.22\sigma_S}{1 \times 10^5} + \frac{\sigma_S}{2.1 \times 10^5}$

$\therefore \sigma_S = 49.33$ N/mm^2(Tensile)

And $\boldsymbol{\sigma_C} = 2.22 \times 49.33 = 109.51$ N/mm^2(Compressive)

6.(b) From the following data, determine the thickness of cast iron column. Assume both ends of the column are fixed.

Length of the column = 3 m   Factor of safety =5

External diameter = 200 mm   Ultimate compressive stress = 570 N/mm^2

Safe working load = 600 kN   Rankine constant = 1/1600   (10)

Both ends fixed

L = 3m = 3000mm   D = 200 mm

P_{safe} = 600 kN = 600 x 10^3 N   FOS = 5

σ_C = 570 N/mm^2 $\alpha = 1/1600$

$$FOS = \frac{Crippling\ load}{Safe\ load} = \frac{P_C}{P_{Safe}}$$

$$\therefore 5 = \frac{P_C}{600 \times 10^3}$$

$\therefore P_C = 3000 \times 10^3$ N

Now, Area(A)$= \frac{\pi}{4}(D^4 - d^4) = \frac{\pi}{64}(D^2 + d^2)\,(D^2 - d^2)$

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}(D^2 + d^2)\,(D^2 - d^2)}{\frac{\pi}{4}(D^2 - d^2)}}$$

$$K^2 = \frac{(D^2 + d^2)}{16}$$

$L_e = \frac{L}{2} = 1500$ mm

Using Rankine's formula,

$P_c = \frac{\sigma_c \cdot A}{1+\alpha(\frac{Le}{k})^2}$

Putting the values of respective terms we get,

$d^2 = 30936.8$

$\therefore d = 175.89$ mm