S.E : SEM. III [MECH/AUTO]

STRENGTH OF MATERIALS

DEC. 2018

Q.1

a. A bar of 20 mm diameter is subjected to a pull of 50KN. The measured extension over a gauge length of 20cm is 0.1 mm and the change in diameter is 0.0035 mm calculate the Poisson's ratio and modulus of elasticity. [5]

Given: d= 20mm　　δd= 0.0035 mm

L= 20cm = 0.2m= 0.2×10^3mm　　δL= 0.1 mm

P= 50 KN = 50×10^3 N

$$\mu = \left(\frac{\delta d/d}{\delta l/l}\right) = \frac{0.0035/20}{0.1/0.2\times10^3} = 0.35$$

$$\delta L = \frac{PL}{AE}$$

$$E = \frac{PL}{A\delta L} = \frac{50\times10^3\times0.2\times10^3}{(\pi/4)\times(20)^2\times0.1} = \mathbf{318.31\times 10^3\ N/mm^2}$$

b. A short column 200mmX100mm is subjected to an eccentric load of 60 KN at an eccentricity of 40mm in the plane bisecting the 100mm side find maximum intensities of stresses at the base. [5]

Given: b= 200mm　　d= 100mm

P= 60 KN = 60×10^3 N

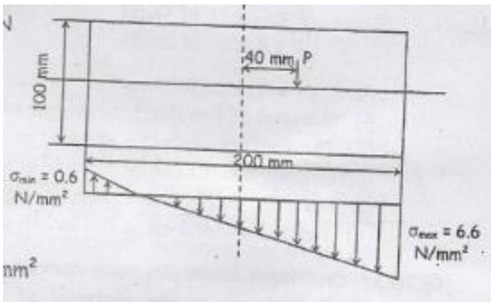
e= 40 mm

Direct stress,

$$\sigma_0 = \frac{P}{A} = \frac{P}{b\times d} = \frac{60\times10^3}{200\times100} = 3\ N/mm^2$$

Bending stress,

$$\sigma_b = \frac{M}{I_{yy}} = \frac{60\times10^3\times40}{\frac{100\times(200)^2}{6}} = 3.6\ N/mm^2$$



$\sigma_{max} = \sigma_0 + \sigma_b$ = 3+3.6 = **6.6 N/mm² (C)**

$\sigma_{min} = \sigma_0 - \sigma_b$ = 3-3.6 = -0.6 N/mm² = **0.6 N/mm² (T)**

c. A M.S. plate is 400mm long 200mm wide and 50mm thick is subjected to gradually tensile load 1200 KN , calculate : (i) proof resilience, (ii) modulus of resilience. Take E= 200X10^3 MPa. [5]

Given: L= 400mm b= 200mm t= 50mm

P= 1200 KN = 1200 X 10^3 N E= 200 X 10^3 MPa

For gradually increasing load,

$$\sigma = \frac{P}{A} = \frac{P}{b\times t} = \frac{1200\times 10^3}{200\times 50} = 120 \text{ N/mm}^2$$

$$U_{max} = \frac{\sigma^2}{2E}\times AL = \frac{\sigma^2}{2E}\times b.t\times L = \frac{(120)^2}{2\times 200\times 10^3}\times 200\times 50\times 400$$

$$=144 \text{ X } 10^3 \text{ Nmm}$$

Modulus of Resilence = $\frac{\sigma^2}{2E} = \frac{(120)^2}{2\times 200\times 10^3}$ = **0.036 N/mm^2**

d. State torsion formula explain meaning of each term. Also state assumptions made in theory of torsion. [5]

Torsional Formula:

$$\frac{T}{J} = \frac{G.\theta}{L} = \frac{\tau}{R}$$

Where,

T= Twisting moment J= Polar moment of Inertia

G= Modulus of rigidity θ= Angle of twist

L= Length of shaft τ = Shear Stress

R= Radius of shaft

Assumptions made in theory of torsion:

i> Material of the shaft is homogenous and isotropic.
ii> The shaft is perfectly straight and uniform in cross-section.
iii> Circular shaft remains circular after twisting.
iv> Plane shaft of shaft remains plane before and after twisting.
v> Twist is uniform along the length of shaft.

e. A cantilever beam 4m span carrying udl of 5KN/m and permissible bending stress in the material of beam is 15 N/mm^2. Design the section of beam if depth to width ratio is 2. [5]

Given: L = 4m w= 5kN/m σ_b= 15 N/mm^2

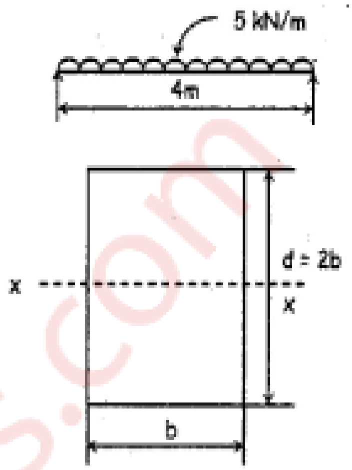
d/b = 2 i.e b= 0.5 d

M = $\frac{wL^2}{8} = \frac{5\times(4)^2}{8}$ = 10 kNm

I = $\frac{b.d^2}{12} = \frac{0.5d\times d^3}{12} = \frac{d^4}{24}$

$\frac{M}{I} = \frac{\sigma_b}{y}$

$\frac{10\times10^6}{(d^4/24)} = \frac{15}{(d/2)}$



d= 200mm

b= 0.5d = 0.5 X 200 = 100mm

f. State assumptions made in theory of bending also state bending formula. [5]

Assumptions made in theory of bending:

i> The material of beam is homogenous and isotropic.
ii> The beam is straight before loading.
iii> The beam is of uniform cross-section throughout its length.
iv> Transverse sections which are plane before loading remain plane even after loading.
v> Modulus of elasticity has some value in tension and compression.

Bending Formula = $\frac{\sigma_b}{y} = \frac{M}{I} = \frac{E}{R}$

Where,

σ_b= Bending stress

y= Distance of outer fibre from bending axis

M= Moment of resistance

I= Moment of inertia

E= Young's modulus of elasticity

R= Radius of curvature

Q.2

a. A wagon weighing 35KN is attached to a wire rope and moving down an incline plane at speed of 3.6 kmph. When the rope jams and wagon is suddenly brought to rest. If the

length of rope is 60m at the time sudden stoppage. Calculate the maximum instantaneous elongation produced diameter of rope is 40mm take E= 2.1 X 10^5 N/mm^2 [10]

Given: W= 35 KN = 35 X 10^3N V= 3.6 kmph= 1m/s

L= 60m = 60X10^3mm d= 40mm

E= 2.1X10^5N/mm^2

Kinetic energy of wagon =$\frac{1}{2} \times$ mv^2 = $\frac{1}{2}$ X $\frac{35\times10^3}{9.81}$X(1)^2

As the chain gets jammed, K.E of wagon is transformed into strain energy of rope

U= $\frac{\sigma^2}{2E} \times AL$ = $\frac{\sigma^2}{2\times2.1\times10^5} \times \frac{\pi}{4} \times (40)^2 \times 60 \times 10^3$ = 179.52 σ^2

Now, K.E = U

1783.9 X 10^3 = 179.52 σ^2

σ = 99.68 N/mm^2

δL = $\frac{\sigma.L}{E}$ = $\frac{99.68\times60\times10^3}{2.1\times10^5}$ = 28.48 mm

b. A compound tube consists of a steel tube of 140mm ID and 16mm OD and an outer brass tube of 160mm ID and 180mm OD. Both the tube are 1.5m in length. If the compound tube carries an axial compressive load of 900KN find its reduction in length also find stresses and the load carries by each tube. E_s = 200 GN/m^2 E_b = 1X10^5 N/mm^2 [10]

Given: D_s= 160mm, d_s= 140mm

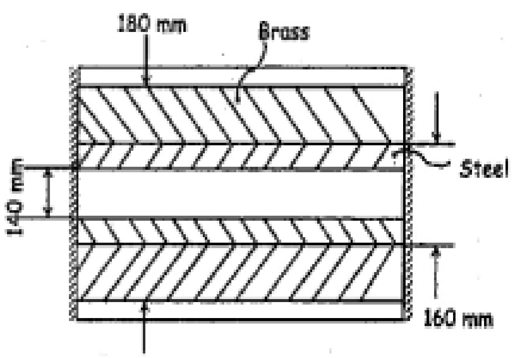
D_b= 180mm, d_b= 160mm

L= 1.5m = 1.5 X 10^3 mm

P= 900KN = 900X10^3 N

E_S= 2X10^5 N/mm^2

E_b= 1X10^5 N/mm^2



A_b= $\frac{\pi}{4}$[D_b^2-d_b^2] = $\frac{\pi}{4}$[180^2-160^2] = 5340.71 mm

A_s= $\frac{\pi}{4}$[D_s^2-d_s^2] = $\frac{\pi}{4}$[160^2-140^2] = 4712.4 mm^2

Now,

$$\frac{\sigma_S}{E_S} = \frac{\sigma_b}{E_b}$$

$$\frac{\sigma_s}{2\times10^2} = \frac{\sigma_b}{1\times10^2}$$

$$\sigma_s = 2\sigma_b$$

Also,

$P = P_s + P_b = \sigma_s.A_s + \sigma_b.A_b = 2.\sigma_b X4712.4 + \sigma_b X5340.71$

$900X10^3 = 14765.5\ \sigma_b$

σ_b = **60.95 N/mm^2**

$\sigma_s = 2.\sigma_b = 2X60.95$ = **121.91 N/mm^2**

$P_s = \sigma_s.A_s = 121.91X4712.4$ = **574.5X10^3 N**

$P_b = P - P_s = 900-574.5$ = **325.5 kN**

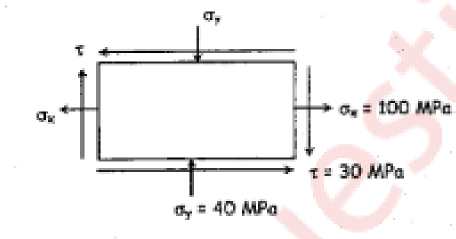
$\sigma_l = \frac{\sigma_s.L_s}{E_s} = \frac{121.91\times1500}{2\times10^5}$ = **0.941 mm**

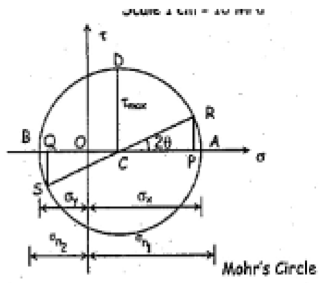
Q.3

a. At a certain point in a strained material, σ_x = 100MPa (T), σ_Y = 40MPa (C), and shear stress τ = 30 MPa. Locate the principle planes and evaluate the principal stresses. Also find the maximum shear stress and the plane carrying it. Use Mohr's circle method. [10]

Given: σ_x = 100 MPa (tensile) σ_y = 40 MPa (compressive)

τ = 30 MPa





Step 1: Select scale 1cm = 10 MPa. Select origin O, take σ_x = 10cm as OP and σ_y = 4cm as OD.

Step 2: Draw perpendicular at point P & Q such that PR = QS = τ = 3cm

Step 3: Join the point S & R, line SR cuts the horizontal axis at a point, mark it as C.

Step 4: Now C is a centre and take CR as a radius, draw a circle cutting horizontal axis at A & B.

Step 5: Measure OA & OB, which are major and minor principal stresses.

Principal stresses:

$\therefore$ Major Principal stresses, σ_{n1} = d (OA) X Scale = 10.6X10 =106 N/mm^2

Major Principal stresses, σ_{n2} = d (OB) X Scale = 4.6X10 = 46 N/mm^2

Angle ∠PCP represents two see of location of principal plane 'θ'

$$2\theta = \angle PCR = 23° \qquad \therefore \theta_1 = 11.5°$$

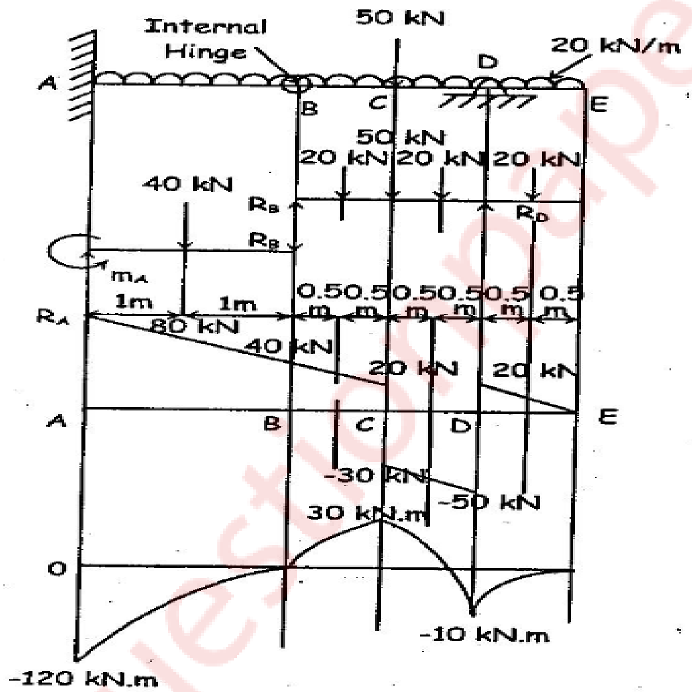
$$\theta_2 = \theta_1 + 90° = 101.5°$$

Draw a perpendicular on horizontal axis at a centre (CD),

Maximum shear stress = d (CD) X Scale

$\tau_{max} = 7.5 \times 10$ = **75 N/mm²**

b. Draw SF and BM diagram for beam shown with B as internal hinge. [10]



i> Considering FBD of beam BCDE,

$\therefore \Sigma F_y = 0$

$\therefore R_b - 20 - 50 - 20 + R_d - 20 = 0$

$\therefore R_b + R_d = 110$

$\Sigma M_B = 0$

$(20 \times 2.5) + (R_d \times 2) - (20 \times 1.5) - (50 \times 1) - (20 \times 0.5) = 0$

$R_d = 70$ KN

From equation (1),

$\therefore R_b = 40$ KN

ii> Considering FBD of beam AB,

$\Sigma M_A = 0$

$M_A - (40 \text{ X}1) - (40 \text{ X}2) = 0$

$M_A = 120$ KNm

& $\Sigma F_y = 0$

$\therefore -R_b - 40 + R_a = 0$

$R_a = 80$ KN

iii> SF calculations:

$SF_E = 0$ KN

$SF_D = 0 + 20 - 70 = -50$ KN

$SF_C = -50 + 20 + 50 = 20$ KN

$SF_B = 20 + 20 = 40$ KN

$SF_A = 40 - 40 = 0$ KN

iv> BM Calculation:

$BM_E = 0$ KNm

$BM_D = -(20 \times 0.5) = -10$ KNm

$BM_C = -(20 \times 1.5) + (70 \times 1) - (20 \times 0.5) = 30$ KNm

$BM_B = 0$ KNm

$BM_{RofA} = -(20 \times 4.5) + (70 \times 4) - (20 \times 3.5) - (50 \times 3) - (20 \times 2.5) - (40 \times 1) = -120$ KNm

$BM_{LofA} = -120 + 120 = 0$ KNm

Q.4

a. A hollow shaft of diameter ratio 3/8(d_i to d_o) is to transmit 375KW power at 100rpm, the maximum torque being 20% greater than the mean the shear stress is not to exceed than 60 N/mm^2 and twist in a length of 4m not to exceed 2°, calculate it's external and internal diameter which would satisfy both the above condition take G = 0.85X10^5 N/mm^2. [10]

Given: Hollow Shaft,

$$\frac{d_i}{d_o} = \frac{3}{8} = 0.375$$

P= 375 KW = 375 x 10^3 W

N= 100 r.p.m

$T_{max} = 1.2\, T_{mean}$ $\tau = 60$ N/mm^2 L = 4m = 4 X 10^3 mm

$\theta = 2° = 0.0349 rad$ G= 0.85 X 10^5 N/mm^2

$P = \frac{2\pi N T_{mean}}{60}$

375 X 10^3 = $\frac{2\pi \times 100 \times T_{mean}}{60}$

T_{mean}= 35.81 X 10^3 Nm

T_{max}= 1.2 T_{mean} = 1.2 X 35.81 X 10^3 = 42.972 X 10^3 Nm = 42.972 X 10^6 Nmm

Strength condition:

$T_{max} = \frac{\pi}{16} \times (\frac{{d_0}^4 - {d_i}^4}{d_o}) \times \tau$

42.972 X 10^6 = $\frac{\pi}{16} \times (\frac{{d_0}^4 - (0.375 d_o)^4}{d_o}) \times 60$

42.972 X 10^6 = $\frac{\pi}{16} \times 0.9802.{d_0}^3 \times 60$

d_o = 154.96 mm

d_i = 0.375 d_o = 0.375 X 154.96 = 58.11 mm

Stiffness condition:

$$\frac{G.\theta}{L} = \frac{T_{max}}{J}$$

$$\frac{0.85 \times 10^5 \times 0.0349}{4 \times 10^3} = \frac{42.972 \times 10^6}{(\pi/32) \times [{d_0}^4 - {d_i}^4]}$$

${d_o}^4 - (0.375 d_o)^4$ = 590.202 X 10^6

d_o = 156.65 mm

d_i = 0.375 d_o = 0.375 X 156.65 = 58.74 mm

b. A cylindrical shell 1m in diameter and 3m long has a thickness of 10mm if it is subjected to an internal pressure of 3N/mm². Calculate change in length, change in diameter, change in volume. Take E= 210KN/mm² and μ = 0.3. [10]

Given: Cylindrical shell,

d = 1m = 1000mm   L = 3m = 3000mm   t= 10 mm

P = 3 N/mm²   E = 210 N/mm² = 210 X 10^3 N/mm²

μ = 0.3

Circumferential strain,

$e_h = \frac{p.d}{4tE}(2-\mu) = \frac{3 \times 1000}{4 \times 10 \times 210 \times 10^3}(2-0.3)$ = 6.0714 X 10^{-4}

Longitudinal strain,

$e_L = \frac{p.d}{4tE}(1-2\mu) = \frac{3\times1000}{4\times10\times210\times10^3}(1-2X0.3) = 1.428 \text{ X } 10^{-4}$

Volumetric strain,

$e_V = \frac{p.d}{4tE}(5-4\mu) = \frac{3\times1000}{4\times10\times210\times10^3}(5-4X0.3) = 1.3571 \text{ X } 10^{-3}$

Change in length,

$\delta L = e_L \text{ X } L = 1.428 \text{ X } 10^{-4} \text{ X } 3000 =$ **0.4284 mm**

Change in diameter,

$\delta d = e_h \text{ X } d = 6.0714 \text{ X } 10^{-4} \text{ X } 1000 =$ **0.6071 mm**

Change in volume,

$\delta V = e_V \text{ X } V = e_v \times \frac{\pi}{4}\ d^2 \times L$

$= 1.3571\times 10^{-3} \times \frac{\pi}{4}\ (1000)^2 \times 3000 =$ **3.1976 X 10^6 mm^3**

Q.5

a. Find the stresses in the wire of the system made of two copper wire and one steel wire of equal length and 65mm^2 cross sectional area the load of 18KN is attached to it. The temperature of the system rises by 10°C assume α_c = 16X10^{-6} /°C, E_c = 110KN/mm^2

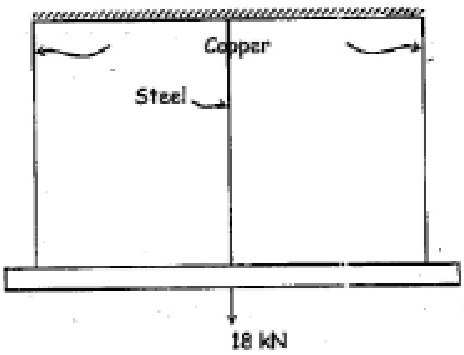
α_s = 12X10^{-6} /°C, E_s = 210KN/mm^2 [10]

Given: $A_S = A_C = 65mm^2$ P= 18 KN = 18 X 10^3 N

$\Delta t = 10°C$

$\alpha_C = 16 \text{ X } 10^{-6}$ /°C $E_C = 110 \text{ KN/mm}^2 = 110 \text{ X } 10^3 \text{ N/mm}^2$

$\alpha_S = 12 \text{ X } 10^{-6}$ /°C $E_s = 210$ KN/mm^2 = 210 X 10^3 N/mm^2



Stresses due to load of 18KN:

$\delta L_S = \delta L_c$

$\frac{\sigma_{s1}}{E_S} = \frac{\sigma_{c1}}{E_C}$ [$\because L_S=L_C$]

$\frac{\sigma_{s1}}{210\times10^3} = \frac{\sigma_{c1}}{110\times10^3}$

σ_{s1}=1.909σ_{c1}

Also, $P = 2P_C + P_S$

$= 2\,\sigma_{c1}A_C + \sigma_{s1}A_s$

$18 \times 10^3 = 2\,\sigma_{c1} \times 65 + 1.909\sigma_{c1} \times 65$

$\sigma_{c1} = 70.84\ \text{N/mm}^2$

$\sigma_{s1} = 1.909 \times 70.84 = 135.24\ \text{N/mm}^2$

Stresses due to rise in temperature:

Since, $\alpha_c > \alpha_s$

Compression in copper = Tension in steel

i.e. $2\sigma_{c2}A_C = \sigma_{s2}A_s$

$2\sigma_{c2} = \sigma_{s2}$

Now, $\frac{\sigma_{s2}}{E_S} + \frac{\sigma_{c2}}{E_C} = (\alpha_c - \alpha_s)\Delta t$

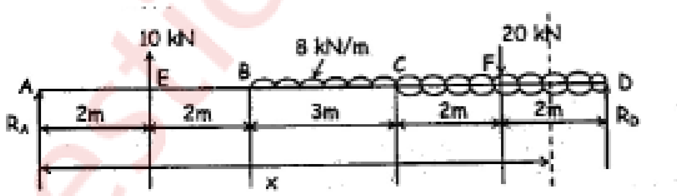
$$\frac{2\sigma_{c2}}{210\times10^3} + \frac{\sigma_{c2}}{110\times10^3} = (16 - 12)\times10^{-6}\times10$$

$\sigma_{c2} = 2.15\ \text{N/mm}^2$

$\sigma_{s2} = 2\sigma_{c2} = 2\times2.15 = 4.3\ \text{N/mm}^2$

$\sigma_{sT} = \sigma_{s1} + \sigma_{s2} = 135.24+4.3 =$ **$139.54\ \text{N/mm}^2$**

$\sigma_{cT} = \sigma_{c1} + \sigma_{c2} = 70.84+2.15 =$ **$72.99\ \text{N/mm}^2$**



b. Determine the deflection at B and slope at D for simply supported beam as shown. Also find the max deflection and its location, take E= 200 GN/m^2 and I=300X10^8 mm^4. [10]

$EI = 200 \times 10^3 \times 300 \times 10^8 = 6 \times 10^{15}\ \text{Nmm}^2 = 6 \times 10^6\ \text{KNm}^2$

Now, $\Sigma f_y = 0$

$R_A + 10 – 8X3 -20 + R_D = 0$

$R_A + R_D = 34$ KN

Also, $\Sigma M_A = 0$

$-10X2 + (8X3)X5.5 +20X9 – R_DX11 =0$

$R_D = 26.55$ KN

$R_A = 7.45$ KN

Applying Macaulay's method,

$M_{XX} = EI.\frac{d^2y}{dn^2}$

$= 7.45x + 10(x-2) - 8\frac{(x-4)^2}{2} + 8\frac{(x-7)^2}{2} - 20(x-9)$

On Integrating,

$EI.\frac{dy}{dn} = 7.45\frac{x^2}{2} + c_1 + 10\frac{(x-2)^2}{2} - 8\frac{(x-4)^3}{6} + 8\frac{(x-7)^3}{6} - 20\frac{(x-9)^2}{2}$

Again, on integrating,

$EI.y = 7.45\frac{x^3}{6} + c_1x + c_2 + 10\frac{(x-2)^3}{6} - 8\frac{(x-4)^4}{24} + 8\frac{(x-7)^4}{24} - 20\frac{(x-9)^3}{6}$

Applying boundary conditions,

When x=0, y=0

$c_2 = 0$ and when x=11, y=0

$0 = \frac{7.45}{6}(11)^3 + 11\, c_1 + \frac{10}{6}(9)^3 - \frac{8}{24}(7)^4 + \frac{8}{24}(4)^4 - \frac{20}{6}(2)^3$

$C_1 = -193.27$

Definition at B, i.e. at x=4m

$EI.y_b = 7.45X\frac{4^3}{6} - 193.27(4) + \frac{10}{6}(2)^3$

$y_b = \frac{1}{6\times10^6}X -680.28 = -1.134 X 10^{-4}$ m = 0.1134 mm

Slope at D, i.e. at x= 11m

$EI.(\frac{dy}{dx})_D = \frac{7.45}{3} \times (11)^2 - 193.27 + \frac{10}{2}(9)^2 - \frac{8}{6}(7)^3 - \frac{8}{6}(4)^3 - \frac{20}{2}(2)^2$

$\theta_D = \frac{1}{6\times 10^6} \times 100.213 = 1.67\times 10^{-5}$ rad

Maximum deflection and its location:

Maximum deflection occurs where slope is zero i.e. between B & C.

$\therefore\ 0 = \frac{7.45}{3}x^2 - 193.27 - \frac{10}{2}(x-2)^2 - \frac{8}{6}(x-4)^3$

$= \frac{7.45}{3}x^2 - 193.27 - \frac{10}{2}(x^2-4x+4) - \frac{8}{6}(x^3-12x^2+48x-64)$

$\therefore x = 5.7$ m

$\therefore$ Deflection at x = 5.7 m

$y_{max} = \frac{1}{6\times10^6}\left[\frac{7.45}{6}\times(5.7)^3 - 193.27(5.7) + \frac{10}{6}(3.7)^3 - \frac{8}{24}(1.7)^4\right]$

$= -1.31 \text{ X } 10^{-4}$ m

$= -0.13167$ mm

Q.6

a. A hollow cylindrical CI column is 4m long with both ends fixed, determine the minimum diameter of the column if it has to carry a safe load of 250 KN with a FOS of 5. Take internal diameter as 0.8 times the external diameter E= 200 GN/m². [10]

Given: Hollow column, $L = 4\text{m} = 4\text{X}10^3$ mm

Both ends fixed, $P_{safe} = 250\text{KN} = 250\text{X}10^3$ N

FOS = 5 $d_i = 0.8\ d_o$

$E = 200\ \text{GN/m}^2 = 200\text{X}10^3\ \text{N/mm}^2$

Since, both ends are fixed,

$L_e = \frac{L}{2} = \frac{4\times10^3}{2} = 2\text{X}10^3$ mm

Now, FOS = $\frac{P_E}{P_{safe}}$

$5 = \frac{P_E}{250\times10^3}$

$P_E = 1.25\text{X}10^6$ N

Also, $I = \frac{\pi}{64}[d_o^4 - d_i^4] = \frac{\pi}{64}[d_o^4 - (0.8d_o)^4] = 0.02898\ d_o^4$

By Euler's formula,

$P_E = \frac{\pi^2 EI}{Le^2}$

$$1.25X10^6 = \frac{\pi^2 \times 200 \times 10^3 \times 0.02898 do^4}{(2\times10^3)^2}$$

d_o = 96.69 mm

d_i = 0.8 d_o = 0.8X96.69 = 77.35 mm

b. A simply supported beam of length 3m and cross section of 100mmX200mm carrying a udl of 4KN/m neglecting the weight of beam find:

(i) Max, bending stress in the beam.

(ii) Max, shear stress in the beam.

(iii) The shear stress at point 1m to the right of the left support and 25 mm below the top surface of the beam. **[10]**

Given: L= 3m b= 100mm d= 200 mm

w= 4KN/m

Max. shear force, S= $\frac{wl}{2} = \frac{4\times3}{2}$ = 6 KN

Max. bending moment, M= $\frac{wl^2}{8} = \frac{4\times3^2}{8}$ = 4.5 KN.m

y = $\frac{d}{2} = \frac{200}{2}$ = 100mm

I = $\frac{bd^3}{12} = \frac{100\times200^3}{12}$ = 66.67X10^6 mm^4

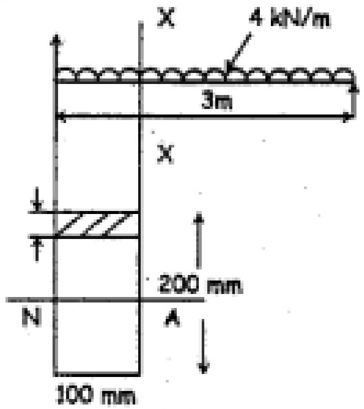
i> Max. bending stress in the beam:

$$\frac{\sigma_b}{y} = \frac{M}{I}$$

$$\sigma_b = \frac{M.y}{I} = \frac{4.5\times10^6\times100}{66.67\times10^6}$$ = **6.75 N/mm²**

ii> Max. shear stress:

$$\tau_{max} = 1.5 \times \frac{s}{b.d} = 1.5 \times \frac{6\times10^3}{100\times200}$$ = **0.45 N/mm²**



iii> Shear stresses at a point 1m to the right of left support and 25mm below the top surface :

S_{XX} = 6-4X1 = 2KN

A= 100X25 = 2500 mm^2

$\bar{y}$ = 75 + $\frac{25}{2}$ = 87.5 mm

$$\tau = \frac{S_{XX} A\bar{y}}{I.b}$$

$$= \frac{2\times10^3\times2500\times87.5}{66.67\times10^6\times100}$$

= **0.066 N/mm²**