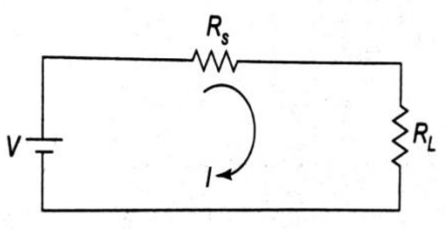
BEE QUESTION PAPER SOLUTION

(CBCGS DEC 16)

Q1] a) State Maximum power Transfer Theorem (2)

Solution:-

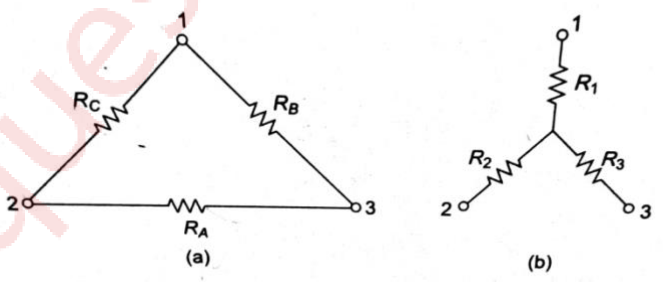
It states that 'The maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.'



The maximum power will be transferred to the load when load resistance is equal to the source resistance.

Q1] b)Derive the formula to convert a delta circuit into an equivalent star(4)

Solution:-



Referring to delta network shown,

The resistance between terminal 1 and 2 = $R_C \,||\, (R_A + R_B)$

$= \frac{R_C(R_A+R_B)}{R_A+R_B+R_C}$ ………………… (1)

Referring to the star network shown above , the resistance between terminals 1 and 2 = $R_1 + R_2$ ……………………….(2)

Since the two networks are electrically equivalent.

$$R_1 + R_2 = \frac{R_C(R_A+R_B)}{R_A+R_B+R_C} \quad \text{.....................(3)}$$

Similarly, $R_2 + R_3 = \frac{R_A(R_B+R_C)}{R_A+R_B+R_C}$ .... (4)

And $R_1 + R_3 = \frac{R_B(R_A+R_C)}{R_A+R_B+R_C}$ .... (5)

Subtracting Eq. (4) from Eq. (3)

$$R_1 - R_3 = \frac{R_B R_C - R_A R_B}{R_A+R_B+R_C} \quad \text{........................ (6)}$$

Adding Eq.(6) and Eq. (5)

$$R_1 = \frac{R_B R_C}{R_A+R_B+R_C}$$

Similarly, $R_2 = \frac{R_A R_C}{R_A+R_B+R_C}$

$$R_3 = \frac{R_A R_B}{R_A+R_B+R_C}$$

Thus, star resistor connected to a terminal is equal to the product of the two delta resistors connected to the same terminal divided by the sum of the delta resistor.

Q1] c) Define Average value and RMS value of an alternating quantity (4)

Solution:-

1. AVERAGE VALUE.

The average value of an alternating quantity is defined as the arithmetic means of all the values over one complete cycle.

In case of a symmetrical alternating waveform the average value over a complete cycle is zero. Hence ,in such a case the average value is obtained over half the cycle only.

The average value of any current (I) is given by,

$$I_{avg} = \frac{i_1+i_2 \ldots\ldots\ldots +i_n}{m}$$

The average value of any current i(t) over the specified interval $t_1 and\ t_2$ is expressed mathematically as,

$$I_{avg} = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} i(t)dt$$

2. RMS VALUE.

The RMS value of the alternating current can be represented as follows:-

$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 \ldots\ldots\ldots + i_m^2}{m}} = \sqrt{mean\ value\ of\ (i)^2}$$

The RMS value of any current i(t) over the specified interval $t_1 to\ t_2$ is expressed mathematically as,

$$I_{rms} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} i(t)dt}$$

The RMS value of an alternating current is of considerable importance in practice because the ammeters and voltmeters record the rms value of alternating current and voltage respectively.

Q1] d) Prove that power in a 3-phase delta connected system is 3 times that of a star connected system (4)

Solution:-

Let a balanced load be connected in star having impedance per phase as Z_{ph}.

For a star-connected load

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{\sqrt{3}Z_{ph}} \quad \Rightarrow \quad I_{ph} = I_L = \frac{V_L}{\sqrt{3}Z_{ph}}$$

Now, $P_Y = \sqrt{3}V_L I_L cos\varphi = \sqrt{3} \times V_L \times \frac{V_L}{\sqrt{3}Z_{ph}} \times cos\varphi = \frac{V_L^2}{Z_{ph}} cos\varphi$

For a delta-connected load

$$V_{ph} = V_L$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L}{Z_{ph}} \quad \Rightarrow \quad I_{ph} = \sqrt{3}I_L = \sqrt{3}\frac{V_L}{Z_{ph}}$$

Now, $P_\Delta = \sqrt{3}V_L I_L cos\varphi = \sqrt{3} \times V_L \times \sqrt{3}\frac{V_L}{Z_{ph}} \times cos\varphi = 3\frac{V_L^2}{Z_{ph}} cos\varphi = 3P_Y$

$\boldsymbol{P_Y = \frac{1}{3} P_\Delta}$

Thus, power consumed by a balanced star-connected load is one third of that in the case of delta-connected load.

Q1] e) Explain the working principle of a single phase transformer (4)

Solution:-

When an alternating voltage V_1 is applied to a primary winding, an alternating current I_1 flows in it producing an alternating flux in the core. As per Faraday's laws of electromagnetic induction, an emf e_1 is induced in the primary winding.

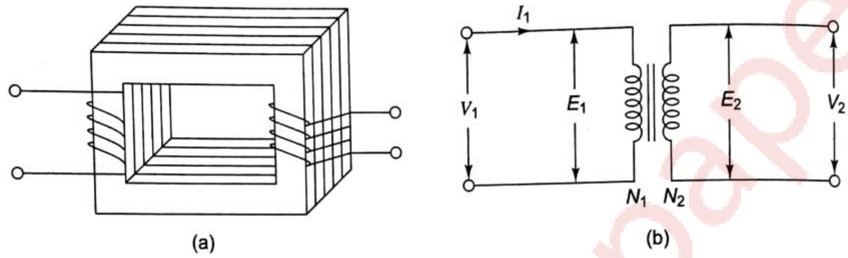


Fig. 6.5 *Working principle of a transformer*

$$e_1 = -N_1 \frac{d\varphi}{dt}$$

Where N_1 is the number of turns in the primary winding. The induced emf in the primary winding is nearly equal and opposite to the applied voltage V_1.

Assuming leakage flux to be negligible, almost the flux produced in primary winding links with the secondary winding. Hence, an emf e_2 is induced in the secondary winding.

$$e_2 = -N_2 \frac{d\varphi}{dt}$$

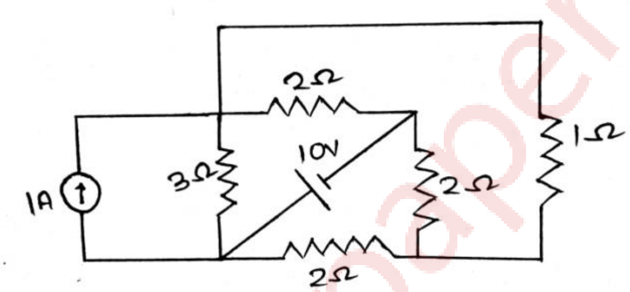
Where N_2 is the number of turns in the secondary winding. If the secondary circuit is closed through the load, a current I_2 flows in he secondary winding. Thus energy is transferred from the primary winding to the secondary winding.

Q1] f) What is the use of commutator in a DC machine. (2)

Solution:-

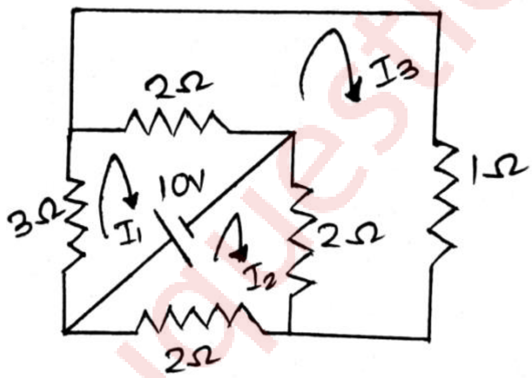
A commutator is used for collecting current from the armature conductors. It is made of a number of wedge- shaped segments of copper. These segment are insulated from each other by thin layers of mica. Each commutator segment is connected to the armature conductor. The commutator converts the ac current induced in the armature conductors into unidirectional current across the brushes .

Q2] a) Obtain current through 1Ω resistance by using Super position theorem, in given diagram. (10)



Solution:-

1. When 10V is active :-



Applying mesh analysis:-

Mesh 1

$-10 + 3I_1 + 2(I_1 - I_3) = 0$

$3I_1 + 2I_1 - 2I_3 = 10$

$5I_1 - 2I_3 = 10$ ……………………(1)

Mesh 2

$-10 + 2(I_2 - I_3) + 2I_2 = 0$

$2I_2 - 2I_3 + 2I_2 = 10$

$4I_2 - 2I_3 = 10$ ……………………(2)

Mesh 3

-2$(I_3 - I_2)$ $-2(I_3 - I_1) + I_3 = 0$

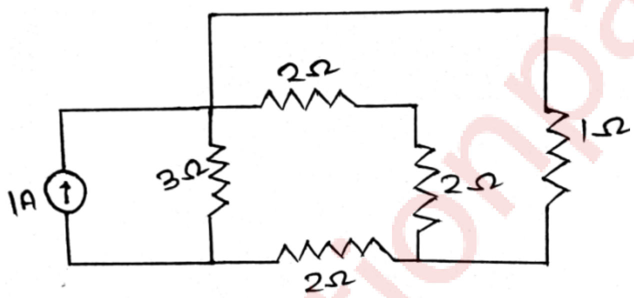
$-2I_3 + 2I_2 - 2I_3 + 2I_1 + I_3 = 0$

$2I_1 + 2I_2 - 3I_3 = 0$ ……………………(3)

From equation (1), (2) and (3), we get

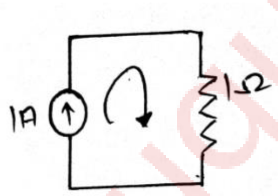
$\mathbf{I' = 7.5 \downarrow A}$

2. When 1A is active :-



Modified circuit

$\mathbf{I'' = 1 \uparrow A}$



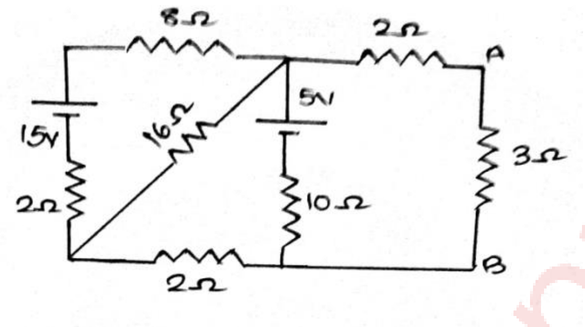
$\mathrm{I}(1\Omega) = -7.5 + 1 = -6.5$

$\mathbf{I(1\Omega) = 6.5 \downarrow A}$

Current passing through 1Ω is 6.5A

Q2] b) A coil is connected across a non-inductive resistor of 120Ω. When a 240V, 50Hz supply is applied to this circuit the coil draws a current of 5A and the total current is 6A. Determine the power and the power factor of (i) the coil. (ii) the whole circuit . (10)

Q3] a) Obtain Norton's equivalent circuit of the network , across the terminal A and B (10)



Solution:-

1. Calculation of I_N

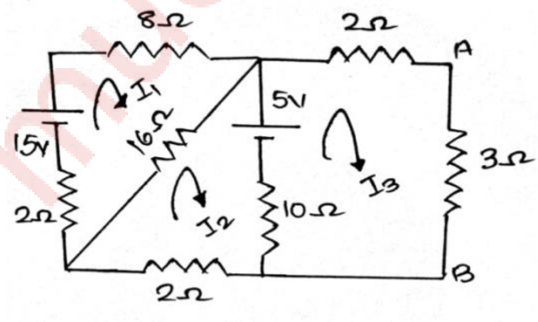
Replacing the 3Ω by a short circuit

Applying KVL to mesh 1

$-15 + 8I_1 + 16(I_1 - I_2) + 2I_1 = 0$

$8I_1 + 16I_1 - 16I_2 + 2I_1 = 15$

$26I_1 - 16I_2 = 15$ ……………………..(1)



Applying KVL to mesh 2

$5 + 10(I_2 - I_3) + 2I_2 + 16(I_2 - I_1) = 0$

$10I_2 - 10I_3 + 2I_2 + 16I_2 - 16I_1 = -5$

$-16I_1 + 28I_2 - 10I_3 = -5$ ……………………..(2)

Applying KVL to mesh 3

$-5 + 10(I_3 - I_2) + 2I_3 = 0$

$10I_3 - 10I_2 + 2I_3 = 5$

$-10I_2 + 12I_3 = 5$ ……………….(3)

From (1), (2) and (3) we get,

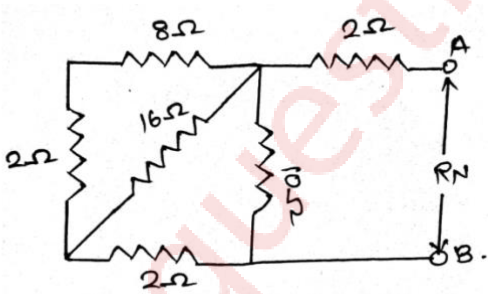
$I_1 = 1.103$A

$I_2 = 0.855$A

$I_3 = 1.129$A

$\boldsymbol{I_N = 1.129}$**A**

2. Calculation of R_N



Replacing voltage sources by short circuits

8Ω + 2Ω = 10Ω

(10Ω || 16Ω) = 6.1533Ω

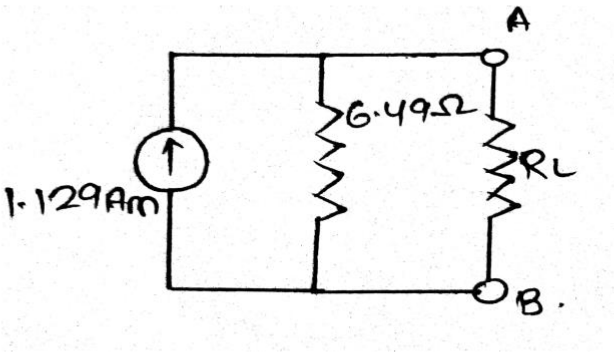
6.15Ω + 2Ω = 8.15Ω

8.15Ω || 10Ω = 4.49Ω

4.49Ω || 2Ω = 6.46Ω

$R_N = 6.49\Omega$

3. Norton's Equivalent circuit.



Q3] b) A series RLC circuit if ω_o is the resonant frequency ω_1 and ω_2 are the half power frequencies, prove that $\omega_o = \sqrt{\omega_1 \, \omega_2}$ (5)

Solution:-

The impedance is purely resistive i.e. the LC series combination acts like a short circuit and the entire voltage is across R. voltage and current are in phase and therefore the power factor is unity. The magnitude of the transfer function is minimum. The inductor voltage and capacitor voltage can be much more than the source voltage. The frequency response of the circuit's current magnitude:

$$I = |I| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

The average power dissipated by the RLC circuit.

$$P(\omega) = \frac{1}{2} I^2 R$$

The highest power dissipated happens at the resonance when current pak of $I = \frac{V_m}{R}$

$$P(\omega_o) = \frac{1}{2}\frac{V_m}{R}$$

Lets assume that half power is dissipated at frequencies of ω_1 and ω_2 :

$$P(\omega_1) = P(\omega_2) = \frac{(V_m/\sqrt{2})^2}{2R} = \frac{V_m^2}{4R}$$

The half-power frequencies can be obtained by setting Z equal to $\sqrt{2}$R

$$\sqrt{2}R = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$\omega_1 = -\frac{R}{2L}\sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L}\sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

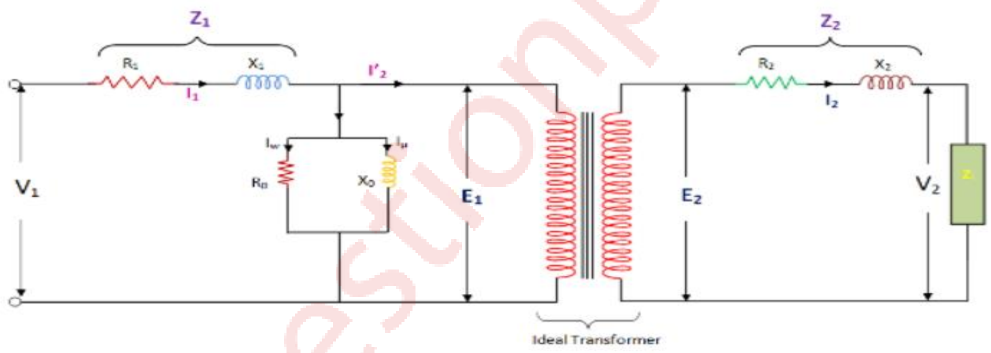
$\omega_o = \sqrt{\omega_1\,\omega_2}$ ....(hence proved)

Q3] c) Derive the equivalent circuit of a 1- phase transformer (5)

Solution:-

Equivalent circuit diagram of a transformer is basically a diagram which can be resolved into an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the winding.

The equivalent circuit diagram of transformer is given below:-



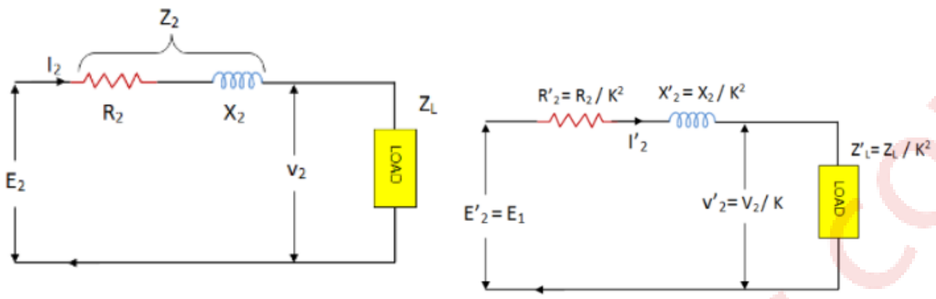
Equivalent Circuit diagram of Transformer

(fig-1)

Where,
R_1 = Primary Winding Resistance.
R_2= Secondary winding Resistance.
I_0= No-load current.
I_μ = Magnetizing Component,
I_w = Working Component,
This I_μ & I_w are connected in parallel across the primary circuit. The value of E_1 (Primary e.m.f) is obtained by subtracting vectorially I_1 Z_1 from V_1 . The value of X_0= E_1 / I_0 and R_0 = E_1 /I_w. We know that the relation of E_1 and E_2 is E_2 /E_1 = N_2 /N_1= K , (transformation Ratio)

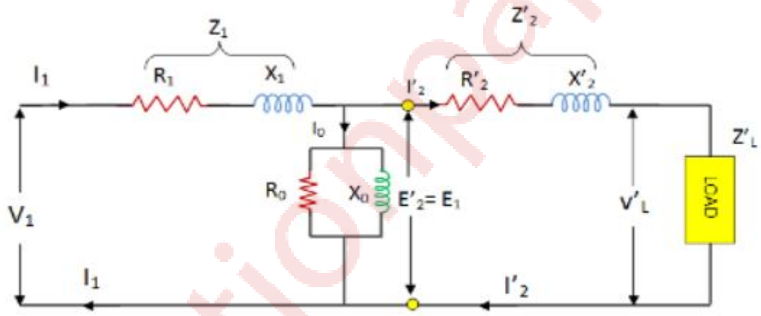
From the equivalent circuit , we can easily calculate the total impedance of to transfer voltage, current, and impedance either to the primary or the secondary.

The secondary circuit is shown in fig-1. and its equivalent primary value is shown in fig-2



(fig-2)

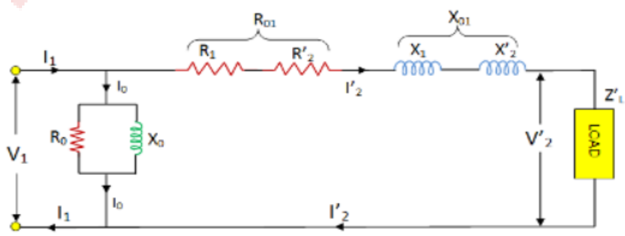
The total equivalent circuit of the transformer is obtained by adding in the primary impedance as shown in – Fig-3



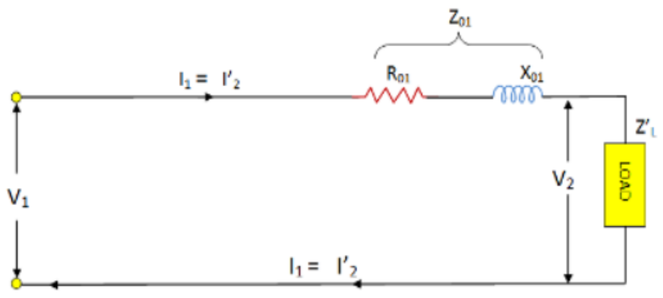
(fig-3)

And It can be simplified the terminals shown in fig – 4 & further simplify the equivalent circuit is shown in fig- 5

At last, the circuit is simplified by omitting I_0 altogether as shown in fig- 5 .

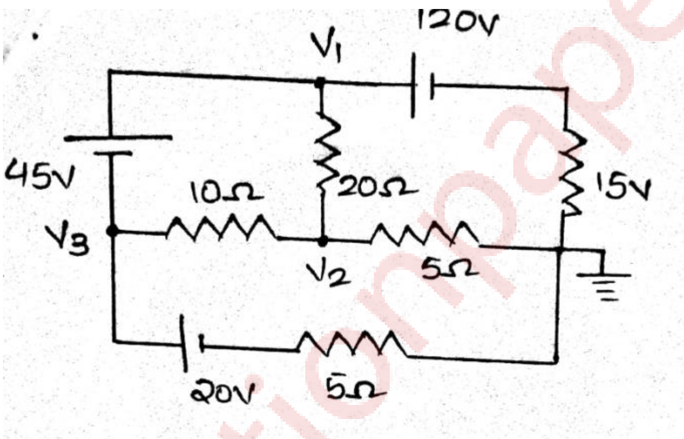


(fig-4)



(fig-5)

Q4] a) Obtain current through 15Ω resistance by nodal analysis. Take reference node as marked. (10)



Solution:-

Applying KCL at node 1

$$\frac{V_1-V_2}{20}+\frac{V_1-120}{15}+V_1-45-V_3=0$$

$$\frac{V_1}{20}-\frac{V_2}{20}+\frac{V_1}{15}-\frac{120}{15}+V_1-V_3=45$$

$$V_1\left(\frac{1}{20}+\frac{1}{15}+1\right)-V_2\left(\frac{1}{20}\right)-V_3=45+\frac{120}{15}$$

$$1.116V_1-0.05V_2-V_3=53 \quad \text{.......................(1)}$$

Applying KCL at node 2

$$\left(\frac{V_2-V_1}{20}\right)+\frac{V_2}{5}+\frac{V_2-V_3}{10}=0$$

$V_1\left(-\frac{1}{20}\right) - V_2\left(\frac{1}{20}+\frac{1}{5}+\frac{1}{10}\right) - V_3(-\frac{1}{10}) = 0$

$-0.05V_1 + 0.35V_2 - 0.1V_3 = 0$ …………..(2)

Applying KCL at node 3

$\left(\frac{V_3-V_2}{10}\right) + \left(\frac{V_3-20}{5}\right) + (V_3 - (-45) - V_1) = 0$

-$V_1 + V_2\left(\frac{-1}{10}\right) + V_3\left(\frac{1}{10}+\frac{1}{5}+1\right) - \frac{20}{5} + 45 = 0$

$-V_1 - 0.1V_2 + 1.3V_3 = -41$ …………….(3)

From (1) , (2) and (3) we get,

$\boldsymbol{V_1 = 67.70}$**V** $\boldsymbol{V_2 = 15.89}$**V** $\boldsymbol{V_3 = 21.76}$**V**

Current through 15Ω resistor :-

Applying KCL :-

$I = \frac{V_1-120}{15} = \frac{67.70-120}{15} = -3.48$ A

$\boldsymbol{I = 3.48}$**(←)A**

Q4] b) In a balanced 3 phase, star connected system, a wattmeter is connected with its current coil in series with Y line pressure coil between Y and R lines. Draw a neat circuit diagram showing the above wattmeter connection. Assuming a lagging power factor, draw the corresponding phasor diagram and derive the wattmeter reading in terms of line voltage, phase current and power factor. (10)

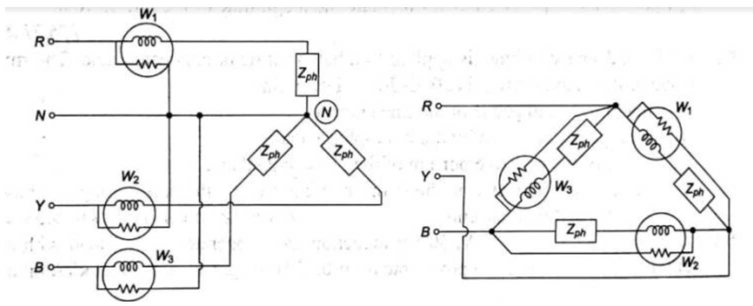
Solution:-

three wattmeter are inserted in each of the three phase of the load whether star connected or delta connected. Each wattmeter will measure the power consumed in each phase.

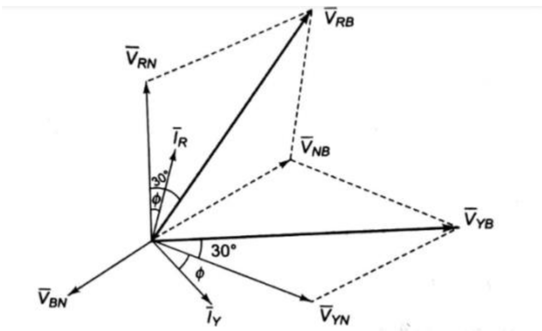
For balanced load, $W_1 = W_2 = W_3$

For unbalanced load, $W_1 \neq W_2 \neq W_3$

Total power P = , $W_1 + W_2 + W_3$



Let V_{RN}, V_{YN} and V_{BN} be the three phase voltages, I_R, I_Y and I_B be the phase currents. The phase currents will lag behind their respective phase voltage by angle



Current through current coil of $W_1 = I_R$

Voltages across voltage coil of $W_1 = V_{RB} = V_{RN} + V_{NB} = V_{RN} - V_{BN}$

$W_1 = V_{RB} I_R \cos(30 - \varphi)$

Current through current coil of $W_2 = I_Y$

Voltage across voltage coil of $W_2 = V_{YB} = V_{YN} + V_{NB} = V_{YN} - V_{BN}$

$W_2 = V_{YB} I_Y \cos(30 + \varphi)$

$I_R = I_Y = I_B$

$W_1 = V_{RB} I_R \cos(30 - \varphi)$

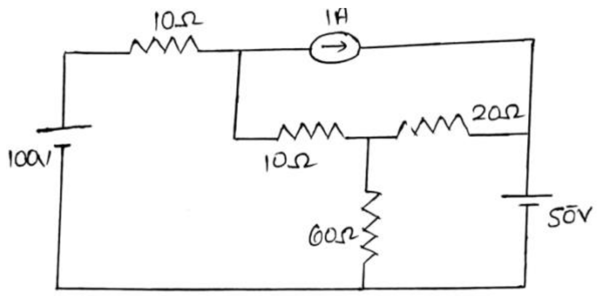
$W_2 = V_{YB} I_Y \cos(30 + \varphi)$

$W_1 + W_2 = V_L I_L [\cos(30 + \varphi) + \cos(30 - \varphi)] = \sqrt{3} V_L I_L cos\varphi$

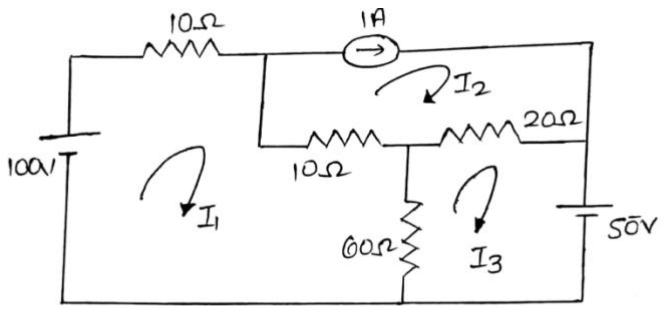
In terms of phase current ,

P = $3 V_{ph} I_{ph} cos\varphi$

Q5] a) Obtain current through 60Ω resistance by Mesh analysis (6)



Solution:-



Current through mesh 2 is $\mathrm{I} = 1$A.

$I_2 = 1A$

Applying mesh analysis:-

Mesh 1

$-100 + 10I_1 + 10(I_1 - 1) + 60(I_1 - I_3) = 0$

$10I_1 + 10I_1 + 60I_1 - 60I_3 = 110$

$80I_1 - 60I_3 = 110$ ....(1)

Mesh 3

$-60(I_3 - I_1) - 20(I_3 - 1) + 50 = 0$

$-60I_3 + 60I_1 - 20I_3 + 20 + 50 = 0$

$-60I_1 + 80I_3 = 70$ ....(2)

$I_1 = 4.64$A $I_3 = 4.35$A

Current through 60Ω is = $I_1 - I_3$ = 4.64-4.35 = 0.29A

$\mathbf{I(60\Omega) = 0.29A}$

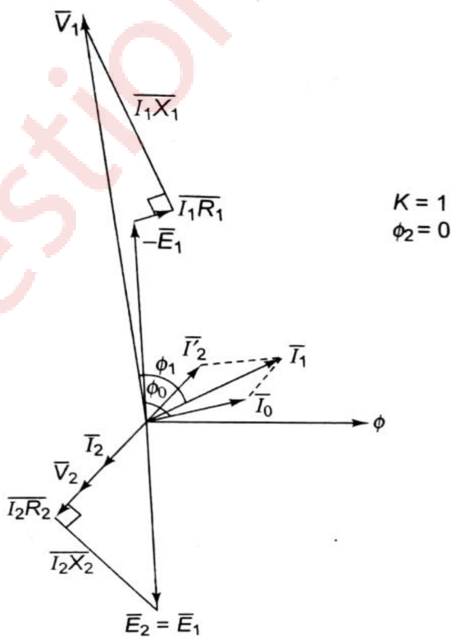
Q5] b) Develop the phasor diagram of a single transformer supplying to a resistive load . (8)

Solution:-

If the load is resistive or power factor is unity, the voltage V2 and I2 are in phase. Steps to draw the phasor diagram are,

1. Consider flux Φ as reference
2. E1 lags Φ by 90. Reverse E1 to get -E1.
3. E1 and E2 are inphase
4. Assume V2 in a particular direction
5. I2 is in phase with V2.
6. Add I2 R2 and I2 X2 to to get E2.
7. Reverse I2 to get I2'.
8. Add Io and I2' to get I1.
9. Add I1 R1 and to -E1 to get V1.

Angle between V1 and I1 is Φ1 and cosΦ1 is primary power factor. Remember that I1X1 leads I1 direction by 90 and I2 X2 leads I2 by 90 as current through inductance lags voltage across inductance by 90 . The phasor diagram is shown in the Fig. shown below



Q5] c) Derive the emf equation of a DC generator (6)

Solution:-

Let φ = flux per pole in Webbers .

Z = Total number of armature conductors.

N = Speed of the armature in revolution per minute(rpm)

P = Number of poles.

A = Number of parallel paths.

When the armature completes one revolution, each conductor cuts the magnetic flux. Therefore flux cut by one conductor in one revolution of the armature.

= flux per pole × number of poles

= φ p Webbers

Time taken to complete one revolution = $\frac{60}{N}$ seconds

Hence, average emf induced in one conductor = $\frac{flux\ cut}{time\ taken}$

= $\frac{\varphi P}{60/N}$ = $\frac{\varphi PN}{60}$ volts

Induced emf (E) = Resultant emf per parallel path

= Average emf per conductor × Number of conductors in series per parallel paths

Induced emf(E) = $\frac{\varphi PN}{60} \times \frac{Z}{A}$ = $\frac{\varphi ZN}{60} \frac{P}{A}$ volts

In case of a DC generator, this emf is called generator emf(Eg). In case of a DC motor , this emf apposes the applied emf and hence, it is called back emf (Eg)

Q6] a) A resistor and a pure reactance are connected in series across a 150V ac supply. When the frequency is 40Hz, the circuit draws 5A. When the frequency is increased to 50Hz , the circuit draws 6A. find the value of resistance and the element value of the reactance. Also find the power drawn in the second case. (10)

Solution:-

V = 150V

$f_1 = 40Hz$

$f_2 = 50Hz$

$I_1 = 5A$

$I_2 = 6A$

1. Values of R and C

For $f_1 = 40Hz$

$Z_1 = \frac{V}{I_1} = \frac{150}{5} = 30\Omega$

$Z_1 = \sqrt{R^2 + (\frac{1}{2\pi f_1 C})^2} = \sqrt{R^2 + (\frac{1}{80\pi C})^2}$

$R^2 + (\frac{1}{80\pi C})^2 = 900$ ……………….(1)

For, $f_2 = 50Hz$

$Z_2 = \frac{V}{I_2} = \frac{150}{6} = 25\Omega$

$Z_2 = \sqrt{R^2 + (\frac{1}{2\pi f_2 C})^2} = \sqrt{R^2 + (\frac{1}{100\pi C})^2}$

$R^2 + (\frac{1}{100\pi C})^2 = 625\Omega$ ……………………..(2)

Solving equation (1) and (2),

R = 19.96Ω

C = 69.4μF

2. Power drawn in the second case

$P_2 = I_2^2 R = (6)^2 \times 19.96$ = 718.56W

Q6] b) A single phase 10KVA, 500 V/250V, 50Hz transformer has the following constants.

Resistance :primary = 0.2 ohms, secondary= 0.5 ohms

Reactance :primary = 0.4 ohms, secondary = 0.2 ohms

Resistance of equivalent exciting circuit w.r.t. primary = 1500 ohms

Reactance of equivalent exciting circuit w.r.t primary = 750 ohms

What will be the reading of the instrument placed in primary side when the transformer is connected for OC and SC test?

Solution:-