Q.P Code: 67635

MATHEMATICS SOLUTION

SEM 4 (CBCGS – MAY 2019)

BRANCH – COMPUTER ENGINEERING

Q1] A) Find the basic, feasible and degenerate solutions for the following equations: **(5)**

$$\mathbf{2x_1 + 6x_2 + 2x_3 + x_4 = 3; \quad 6x_1 + 4x_2 + 4x_3 + 6x_4 = 2}$$

SOLUTION :-

$2x_1 + 6x_2 + 2x_3 + x_4 = 3$

$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$

Since there are four variables and 2 constraints there are $4C_2 = 6$ basic solutions

No of basic solution	No of basic variables	Basic variables	Equation and the values of basic variables	Is the solution feasible	degenerate
1	$x_3 = 0$ $x_4 = 0$	x_1, x_2	$2x_1 + 6x_2 = 3$ $6x_1 + 4x_2 = 2$ $x_1 = 0, x_2 = \frac{1}{2}$	Yes	Yes
2	$x_2 = 0$ $x_4 = 0$	x_1, x_3	$2x_1 + 3x_3 = 3$ $6x_1 + 4x_3 = 2$ $x_1 = -2, x_3 = \frac{7}{2}$	No	No
3	$x_1 = 0$ $x_4 = 0$	x_2, x_3	$6x_2 + 2x_3 = 3$ $4x_2 + 4x_3 = 2$ $x_2 = \frac{1}{2}, x_3 = 0$	Yes	Yes
4	$x_1 = 0$ $x_3 = 0$	x_2, x_4	$6x_2 + x_4 = 3$ $4x_2 + 6x_4 = 2$ $x_2 = \frac{1}{2}, x_4 = 0$	Yes	Yes
5	$x_2 = 0$ $x_3 = 0$	x_1, x_4	$2x_1 + x_4 = 3$ $6x_1 + 4x_4 = 2$ $x_1 = \frac{8}{3}, x_2 = \frac{-7}{3}$	No	No
6	$x_1 = 0$ $x_2 = 0$	x_3, x_4	$2x_3 + x_4 = 3$ $4x_3 + 6x_4 = 2$ $x_3 = 2, x_4 = -1$	No	No

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Q1] B)Integrate the function $f(z) = x^2 + ixy$ from A = (1,1) to B(2,4) along the curve x = t and $y = t^2$ (5)

SOLUTION:-

$f(z) = x^2 + ixy$ from A = (1,1) to B(2,4) along the wave x = t and $y = t^2$

Putting $x = t$ and $y = t^2$ in $f(z)$

We get $f(z) = t^2 + it^3$

$d(z) = dx + idy = dt + 2itdt = dt(1 + 2it)$

$\int_A^B f(z)dz = \int_1^2 (t^2 + it^3)(1 + 2it)dt = \int_1^2 t^2 + 2it^3 + it^3 - 2t^4\, dt = \int_1^2 (t^2 - 2t^4 + 3it^3)dt$

$= \left[\frac{t^3}{3} - \frac{2t^5}{5} + \frac{3it^4}{4}\right]_1^2 = \frac{8}{3} - \frac{2\times 32}{5} + \frac{(3i)(16)}{4} - \left[\frac{1}{3} - \frac{2}{5} + \frac{3i}{4}\right] = \frac{8}{3} - \frac{64}{5} + 12i - \frac{1}{3} + \frac{2}{5} - \frac{3i}{4}$

$= \frac{7}{3} - \frac{62}{5} + \frac{45i}{4} = -\frac{151}{15} + \frac{i45}{4}$

Answer:- $-\frac{151}{15} + \frac{i45}{4}$

Q1] C) A machine is set to produce metal plates of thickness 1.5cms with S.D. of 0.2cms. a sample of 100 plates produced by the machine gave an average thickness of 1.52 cms. Is the machine fulfilling the purpose? Test at 1% level of significance. (5)

SOLUTION :-

We have $\overline{X} = 1.52$; $\mu = 1.5$

$\sigma = 0.2$

sample size(n) = 100

$Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{1.52 - 1.5}{0.2/\sqrt{100}} = \frac{0.2}{2.0} = 0.1$

$Z = 0.1$

But at $\alpha = 1\%$ we have $Z = 2.576$

$Z_{calc} < Z_{obs}$

Yes the machine is fulfilling its purpose.

Q1] D) The sum of the Eigen values of a 3 x 3 matrix is 6 and the product of the Eigen values is also 6. If one of the Eigen value is one, find the other two Eigen values. (5)

SOLUTION :-

Let the eigen values be λ_1, λ_2 and λ_3

$\lambda_1 + \lambda_2 + \lambda_3 = 6$

And $\lambda_1 . \lambda_2 . \lambda_3 = 6$

Given : one eigen value = 1

Let $\lambda_1 = 1$

$1 + \lambda_2 + \lambda_3 = 6$

$\lambda_2 + \lambda_3 = 5$

$\lambda_3 = (5 - \lambda_2)$

And $(\lambda_2 - \lambda_3) = 6$

$\lambda_2(5 - \lambda_2) = 6$

$5\lambda_2 - {\lambda_2}^2 = 6$

${\lambda_2}^2 - 5\lambda_2 + 6 = 0$

${\lambda_2}^2 - 3\lambda_2 - 2\lambda_2 + 6 = 0$

$\lambda_2(\lambda_2 - 3) - 2(\lambda_2 - 3) = 0$

$\lambda_2 = 2 \quad \text{or} \quad \lambda_2 = 3$

If $\lambda_2 = 2$ and $\lambda_3 = 3$ and if $\lambda_2 = 3$ and $\lambda_3 = 2$

The eigen values are 1,2,3.

Q2] A) Evaluate $\oint \frac{\sin^6 z}{(z-\frac{\pi}{6})^n} dz$ where c is the circle |z| = 1 for n =1, n=3 (6)

SOLUTION:-

$\oint \frac{\sin^6 z}{(z-\frac{\pi}{6})^n} dz$ where c is a is a circle $|z| = 1$

For n = 1 we have $\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^n} dz$

$Z_0 - \frac{\pi}{6} = 0$

$Z_0 = \frac{\pi}{6}$ pt lies inside the circle ; |z| = 1

$\int \frac{f(z)}{z-z_0} dz = 2\pi i f(z_0)$ { By Cauchy's Integra formula}

$$\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^n} dz = 2\pi i \sin^2\left(\frac{\pi}{6}\right) = 2\pi i \left(\frac{1}{2}\right)^6 = \frac{2\pi i}{64} = \frac{\pi i}{32}$$

For n =3; we have $\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^n} dz$

$Z = \frac{\pi}{6}$ lies inside the circle , $|Z| = 1$

Order of pole = 3

$\int \frac{f(z)}{(Z-Z_0)^n} = \frac{2\pi i}{(n-1)!} f^{n-1}(z_0)$

$f(z) = \sin^6 z$

$f'(z) = 6\sin^5 z.\cos z$

$f''(z) = 6[5\sin^4 z\cos^2 z - \sin^5 z.\sin z] = 6[5\sin^4 z\cos^2 z - \sin^6 z]$

$f''(z) = 6\left[5\sin^4\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{6}\right) - \sin^6\left(\frac{\pi}{6}\right)\right] = \frac{21}{16}$

$$\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz = \frac{2\pi i}{2!} . \frac{21}{16} = \frac{21\pi i}{16}$$

Q2] B) Solve the following LPP using Simplex method **(6)**

Maximize $\mathbf{z = 3x_1 + 5x_2;}$

Subject to $\mathbf{3x_1 + 2x_2 \leq 18}$

$\mathbf{x_1 \leq 4;}$

$\mathbf{x_2 \geq 6}$

$\mathbf{x_1, x_2 \geq 0}$

SOLUTION :-

$z = 3x_1 + 5x_2; \quad 3x_1 + 2x_2 \leq 18$

$x_1 \leq 4; \quad x_2 \geq 6$

We first express the problem in standard form;

$z - 3x_1 - 5x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

$3x_1 + 2x_2 + 1s_1 + 0s_2 + 0s_3 = 0$

$x_1 + 0s_1 + s_2 + 0s_3 = 0$

$0x_1 + x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

Simple table

Iteration no	Basic variables	Coefficient of					RHS soln	Ratio
		x_1	x_2	s_1	s_2	s_3		
0	Z	-3	-5	0	0	0	0	
s_3 leaves	s_1	3	2	1	0	0	18	9
x_2 enters	s_2	1	0	0	1	0	4	-
	s_3	0	1	0	0	1	6	6
1	Z	-3	0	0	0	5	30	
s_1 leaves	s_1	3	0	1	0	-2	6	2
x_1 enters	s_2	1	0	0	1	0	4	4
	x_2	0	1	0	0	1	6	-
2	Z	0	0	1	0	3	36	
	x_1	1	0	1/3	0	-2/3	2	
	s_2	0	0	-1/3	1	2/3	2	

	x_2	0	1	0	0	1	6	

$x_1 = 2;\ \ x_2 = 6$

$$\mathbf{Z_{max} = 3(x_1) + 5(x_1) = 3(2) + 5(6) = 6 + 30 = 36}$$

Q2] C) The following data is collected on two characters. Based on this, can you say that there is no relation between smoking and literacy? Use chi-square test at 5% Level of significance. (8)

	Smokers	**Non smokers**
Literates	**40**	**35**
Illiterates	**35**	**85**

SOLUTION :-

	Smokers	Non smokers	Total
Literates	40	35	75
Illiterates	35	85	120
Total	75	120	195

Expected frequency (literate and smokers) = $\frac{75\times75}{195} = 28.846$

Expected frequency (literate and non - smokers) = $\frac{75\times120}{195} = 46.15$

Expected frequency (illiterate and smokers) = $\frac{120\times75}{195} = 46.15$

Expected frequency (illiterate and non - smokers) = $\frac{120\times120}{195} = 73.85$

Null hypothesis, H_0 : no association

Alternate hypothesis , H_a : there is association

O	E	$(0-E)^2$	$\frac{(O-E)^2}{E}$
40	28.85	124.32	4.3093
35	46.15	124.32	2.6938
35	46.15	124.32	2.6938
85	73.85	124.32	1.6834
			$X^2 = 11.3803$

Level of significance = (0.05)

Degree of freedom = (r-1)(c-1) = (2-1)(2-1) = 1

Critical value at 1 df for 5% level of significance is 3.84.

$\mathbf{X^2}_{\mathbf{table}} < \mathbf{X^2}_{\mathbf{calc}}$

There is no association.

Q3] A) Find the Eigen values and Eigen vectors of the following matrix. (6)

$$\mathbf{A} = \begin{bmatrix} \mathbf{3} & \mathbf{10} & \mathbf{5} \\ \mathbf{-2} & \mathbf{-3} & \mathbf{-4} \\ \mathbf{3} & \mathbf{5} & \mathbf{7} \end{bmatrix}$$

SOLUTION :-

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$$

The characteristics equation is given by

$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$

$\lambda = 3,2,2$

For $\lambda = 3$

$[A - 3I]X = 0$

$$\begin{bmatrix} 0 & 10 & 5 \\ -2 & -6 & -4 \\ 3 & 5 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0x_1 + 10x_2 + 5x_3 = 0$

$-2x_1 - 6x_2 - 4x_3 = 0$

$3x_1 + 5x_2 + 4x_3 = 0$

Consider last two equations,

$$\frac{x_1}{\begin{vmatrix} -6 & -4 \\ 5 & 4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -2 & -4 \\ 3 & 4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & -6 \\ 3 & 5 \end{vmatrix}} = t$$

$\frac{x_1}{-4} = -\frac{x_2}{4} = \frac{x_3}{8} = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

For $\lambda = 2$

$[A - 2I]X = 0$

$$\begin{bmatrix} 1 & 10 & 5 \\ -2 & -5 & -4 \\ 3 & 5 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \ \to R_2 + 2R_1$ and $R_3 \to \ R_3 - 3R_1$

$$\begin{bmatrix} 1 & 10 & 5 \\ 0 & 15 & 16 \\ 0 & -25 & -25 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

rank of matrix = 3 and number of variables = 3

$x_1 + 10x_2 + 5x_3 = 0$

$-2x_1 - 5x_2 - 4x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 10 & 5 \\ -5 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 5 \\ -2 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 10 \\ -2 & -5 \end{vmatrix}} = t$$

$$\frac{x_1}{-15} = -\frac{x_2}{6} = \frac{x_3}{15} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -15 \\ -6 \\ 15 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ -5 \end{bmatrix}$$

Eigen values = 3,2

Eigen vector = [1 1 -2], [5 -2 -5]

Q3] B) The incomes of a group of 10,000 person's were found to be normally distributed with mean of Rs 750 and standard deviation of Rs. 50 . what is the lowest income of richest 250? (6)

SOLUTION :-

Standard normal variate; $Z = \frac{(X-m)}{\sigma} = \frac{X-750}{50}$

If we have to consider the richest 250 persons, then probability that a person selected at random will be one of them is $\frac{250}{10000} = 0.025$

Area from (z=0 to z = this value) = 0.5-0.025 = 0.475

From the table, we find that the area from z = 0 to z = 1.96 is 0.476 the required z = 1.96;

$1.96 = \frac{X-750}{50}$

$X - 750 = 1.96 \times 50 = 845$

Lowest income of richest 250 persons = Rs 848

Q3] C) Obtain Taylor's and Laurent's expansion of $f(z) = \frac{z-1}{z^2-2z-3}$ indicating region of convergence (8)

SOLUTION:-

$f(z) = \frac{z-1}{z^2-2z-3} = \frac{z-1}{(z+1)(z-3)}$

Applying partial fractions;

$\frac{z-1}{z^2-2z-3} = \frac{A}{z+1} + \frac{B}{z-3}$

$(z-1) = A(z-3) + B(z+1)$

Put z = 3

3-1 = B(4)

2 = 4B

B = $\frac{1}{2}$

Put z = -1

-2 = A(-4)

A = $\frac{1}{2}$

$f(z) = \frac{1}{2(z+1)} + \frac{1}{2(z-3)} = \frac{1}{2}[\frac{1}{z+1} - \frac{1}{z-3}]$

(1) for $|z| < 1$

$f(z) = \frac{1}{2}\left[(1+z)^{-1} - \frac{1}{3}\left(\frac{1}{1-\frac{z}{3}}\right)\right]$

$$= \frac{1}{2}\left[\left(1 - z + \frac{z^2}{2} + \cdots\right) - \frac{1}{3}\left(1 - \frac{z}{3}\right)^{-1}\right] = \frac{1}{2}\left[\left(1 - z + \frac{z^2}{2} + \cdots\right) - \frac{1}{3}\left(1 + \frac{z}{3} + \frac{z^2}{9} + \cdots\right)\right]$$
$$= \frac{1}{2}\left[\frac{2}{3} - \frac{10z}{9} + \frac{29z^2}{27} - \cdots\right]$$

(2) For 1 < |z| < 3

$$f(z) = \frac{1}{2}\left[\frac{1}{z}(1+z)^{-1} - \frac{1}{3}\left(\frac{1}{1-\frac{z}{3}}\right)^{-1}\right] = \frac{1}{2}\left[\frac{1}{z}\left(1 - \frac{1}{z} + \frac{1}{z^2}\ldots\right) - \frac{1}{3}\left(1 - \frac{z}{3} + \frac{z^2}{9} + \cdots\right)\right]$$

(3) For |z| > 3

$$f(z) = \frac{1}{2}\left[\frac{1}{z+1} + \frac{1}{z-3}\right] = \frac{1}{2}\left[\frac{1}{z}\left(1 + \frac{1}{z}\right)^{-1} + \frac{1}{z}\left(1 - \frac{3}{z}\right)^{-1}\right]$$
$$= \frac{1}{2}\left[\frac{1}{z}\left[1 - \frac{1}{z} + \frac{1}{z^2} - \cdots\right] + \frac{1}{z}\left(1 + \frac{3}{z} + \frac{z^2}{9} + \cdots.\right)^{-1}\right]$$
$$= \frac{1}{z}\left[1 + \frac{1}{z} + \frac{5}{z^2} + \cdots\right] \quad = \frac{1}{z} + \frac{1}{z^2} + \frac{5}{z^3} + \cdots$$

(1) Gives taylor series (2) and (3) gives Laurent series.

Q4] A) A man buys 100 electric bulbs of each of two well-known makes taken at random from stock for testing purpose. He finds that 'make A' has a mean life of 1300 hrs with a S.D. of 82 hours and 'make B' has a mean life of 1248 hours with S.D. of 93 hours. Discuss the significance of these results. (6)

SOLUTION :-

Null hypothesis : $\mu_1 = \mu_2$, alternate hypothesis : $\mu_1 \neq \mu_2$

a) We have $n_1 = n_2 = 100$

$\sigma_1 = 82$

$\sigma_2 = 93$

$$SE = \sqrt{\frac{\sigma_1{}^2}{n_1} + \frac{\sigma_2{}^2}{n_2}} = \sqrt{\frac{82^2+93^2}{100}} = 12.3988$$

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{SE} = \frac{1300-1248}{12.3988} = 4.1939$$

Level of significance = 5%

Critical value = 1.96

$Z_{calc} > Z_{obs}$

There is significant difference.

Q4] B) Using the Residue theorem, Evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$ (6)

SOLUTION :-

$\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$

Let $z = e^{i\theta} d\theta$

$dz = ie^{i\theta} d\theta$

$d\theta = \frac{dz}{ie^{i\theta}}$

$d\theta = \frac{dz}{iz}$ where C is unit circle $|z| = 1$

$$\int_0^{2\pi} \frac{1}{5-3\left(\frac{z^2+1}{2z}\right)}\left(\frac{dz}{iz}\right) = \int_0^{2\pi} \frac{-2dz}{(10z-3z^2-3)i} = \int_0^{2\pi} \frac{-2dz}{(-10z+3z^2+3)i} = \int \frac{-2dz}{(3z-1)(z-3)i}$$

Z-3 lies outside the circle.

Residue of f(z) at $z = \frac{1}{3} = \lim_{z \to \frac{1}{3}} \left(z - \frac{1}{3}\right) \frac{-2}{i(3z-1)(z-3)} = \lim_{z \to \frac{1}{3}} \left(\frac{3z-1}{3}\right) \frac{-2}{i(3z-1)(z-3)} = \frac{1}{3i}\left(-\frac{2}{\frac{1}{3}-3}\right) = \frac{1}{4i}$

$$\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta} = 2\pi i \left(\frac{1}{4i}\right) = \frac{\pi}{2}$$

$$\boldsymbol{\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta} = \textbf{real part of} \left(\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}\right) = \frac{\pi}{2}}$$

Q4] C) (1) Out of 1000 families with 4 children each, how many would you expect to have (a) at least one boy (b) at most 2 girls.

(2)Find the Moment Generating Function of Binomial Distribution and hence find its mean. (8)

SOLUTION :-

(1)

BBBB	BBBG	BBGB	BGBB
BBGG	BGGB	BGBG	BGGG
GGGG	GGGB	GGBG	GBGG
GGBB	GBBG	GBGB	GBBB

a) P(at last one boy) $= \frac{15}{16}$

Families having at least one boy = $N \times P = 1000 \times \frac{15}{16} = 37.5 = 938$

b) P(at most 2 girls) = P(2 girls) + P(3 girls) +P(4 girls) = $\frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{11}{16}$

Families having at most 2 girls = $N \times P = 1000 \times \frac{11}{16} = 687.5 = 688$

938 families have at least 1 boy
688 families have at most 2 girls.

(1) Moment generating function about origin:

$M_0(t) = E(e^{tx}) = \sum p_i e^{txi} = \sum nC_x p^x q^{n-x}.e^{tx} = \sum nC_x q^{n-x}(pe^t)^x$

$M_0(t) = (q + pe^t)^n$

Differentiating $M_0(t)$ and putting $t = 0$ to find mean.

$\frac{d}{dt}[M_0(t)] = n(q + pe^t)^n\ pe^t = np[e^t(q + pe^t)^{n-1}]$

$\frac{d}{dt}[M_0(t)] = np(q + p)^{n-1} = np \quad \ldots\ldots(p + q = 1)$

$\mathbf{mean = np}$

Q5] A) Check whether the following matrix is Derogatory or Non- Derogatory:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{-3} & \mathbf{3} \end{bmatrix}$$ **(6)**

SOLUTION :-

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$

Characteristics equation :

$\lambda^3 - 3\lambda + (3 - 0 - 0)\lambda - 1 = 0$

$\lambda^3 - 3\lambda + 3\lambda - 1 = 0$

$(\lambda - 1\,)^3 = 0$

$\lambda = 1,1,1$

Let us find minimal polynomial.

$(x - 1)(x - 1) = 0$

Assuming ; $x^2 - 2x + 1 = 0 \quad$ annihilates A

$A^2 - 2A + I$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -3 & 3 \\ 3 & -8 & 6 \end{bmatrix}$$

$$2A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 2 & -6 & 6 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A^2 - 2A + I \neq 0$

It is not a minimal polynomial

Minimal polynomial = (x-1)(x-1)(x1)

As a degree of minimal polynomial is equal to order the matrix is non derogatory

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Q5] B) The means of two random samples of sizes 9 and 7 are 196.42 and 198.82 respectively. The sum of the squares of the derivatives from the means are 26.94 and 18.73 respectively. Can the samples be regarded to have been drawn from the same normal population? (6)

SOLUTION:-

Null hypothesis H_0: $\mu_1 = \mu_2$

alternative hypothesis : $\mu_1 \neq \mu_2$

(ii) calculation of test statistic

$$S_p = \sqrt{\frac{\sum(X_i-\bar{X})^2+\sum(Y_i-\bar{Y})^2}{n_1+n_2-2}} = \sqrt{\frac{26.94+18.73}{9+7-2}} = \sqrt{\frac{14.67}{14}} = 1.87$$

Standard error of the difference between the means

$$SE = sp \times \sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 1.81\sqrt{\frac{1}{9}+\frac{1}{7}} = 0.91$$

$$t = \frac{\overline{X_1}-\overline{X_2}}{SE} = \frac{196.42-198.82}{0.91} = -2.64$$

$|t| = 2.64$

Level of significance = 5%

Critical value; $v = 9 + 7 - 2 = 14$ degrees of freedom is 2.145

Decision; $t_{calc} > t_{table}$

Thus; null hypothesis is rejected alternative hypothesis is accepted

The sample cannot be considered to have been drawn from same population.

5] C) Use the dual simplex method to solve the following L.P.P. (6)

Minimize : $\mathbf{z = x_1 + x_2}$

Subject to : $\mathbf{2x_1 + x_2 \geq 2}$

$\mathbf{-x_1 - x_2 \geq 1}$

$\mathbf{x_1, x_2 \geq 0}$

SOLUTION :-

Minimize : $z = x_1 + x_2$

Subject to : $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1, x_2 \geq 0$

We first express the given problem using $\leq$ in the given constraints.

Minimize; $z = x_1 + x_2$

$-2x_1 - x_2 \leq -2$

$x_1 + x_2 \le 1$

Introducing stack variables s_1, s_2, we have

$z = x_1 + x_2 - 0s_1 - 0s_2$

$z - x_1 - x_2 - 0s_1 - 0s_2 = 0$

Subject ; $-2x_1 - x_2 + s_1 - 0s_2 = -2$

$x_1 + x_2 + 0s_1 + s_2 = -1$

Simple table

Iteration number	Basic variables	Coefficient of				RHS soln
		x_1	x_2	s_1	s_2	
0	Z	-1	-1	0	0	0
s_2 leaves	s_1	-2	-1	1	0	-2
x_1 enters	s_2	1	1	0	1	-1
Ratio		½	1			
1	Z	0	-½	-½	0	1
	x_1	1	½	-½	0	1
	s_2	0	½	½	1	-2
Ratio		-	-1	-1	-	

Since s_2 row is –ve, s_2 leaves; but since all ratios are –ve the LPP has no feasible solution.

Q6] A) Show that the matrix A satisfies Cayley-Hamilton theorem and hence find A^{-1} (6)

Where $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

SOLUTION :-

$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

Characteristics equation,

$\lambda^3 - 6\lambda^2 + (4 + 3 + 4)\lambda - 6 = 0$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$ .... (1)

λ = 1,3,2

By cayley Hamilton theorem;

Matrix A satisfies equation (1)

$A^3 - 6A^2 + 11A - 6I = 0$

$$A^3 = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 0 & -13 \\ 0 & 8 & 0 \\ -13 & 0 & 14 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 5 \end{bmatrix}$$

$A^3 - 6A^2 + 11A - 6I = 0$

$$= \begin{bmatrix} 14 & 0 & -13 \\ 0 & 8 & 0 \\ -13 & 0 & 14 \end{bmatrix} - \begin{bmatrix} 30 & 0 & -24 \\ 0 & 24 & 0 \\ -24 & 0 & 30 \end{bmatrix} + \begin{bmatrix} 22 & 0 & -11 \\ 0 & 22 & 0 \\ -11 & 0 & 22 \end{bmatrix} - \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Cayley Hamilton theorem is satisfied.

Multiplying (i) by A^{-1}

$A^3 - 6A^2 + 11A - 6A^{-1} = 0$

$$6A^{-1} = \begin{bmatrix} 5 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 12 & 0 & -6 \\ 0 & 12 & 0 \\ -6 & 0 & 12 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$\mathbf{A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 4 \end{bmatrix}}$$

Q6] B) A random variable X has the probability distribution (6)

$\mathbf{P(X = x) = \frac{1}{8}\,.3C_x;\quad x = 0, 1, 2, 3}$ **. find the mean and variance.**

SOLUTION :-

$P(X = x) = \frac{1}{8} \cdot 3C_x; \quad x = 0,1,2,3$

X	0	1	2	3
P(X= x)	1/8	3/8	3/8	1/8

Mean = $\sum p_i x_i \;=\; 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} = 0 + \frac{3}{8} + \frac{6}{8} + \frac{1}{8} = \frac{10}{8}$

$E(X^2) = \sum p_i x_i^2 = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 4 \times \frac{3}{8} + 9 \times \frac{3}{8} = 0 + \frac{3}{8} + \frac{12}{8} + \frac{27}{8} = \frac{42}{8}$

$\text{variance} = E(X^2) - [E(X)]^2 = \frac{42}{8} - \left[\frac{10}{8}\right]^2 \;= 3.6875$

Mean = 1.25; $\mathbf{variance = 3.6875}$

Q6] C) Using Kuhn-Tucker conditions, solve the following NLPP (8)

Maximize : $\mathbf{Z = 10x_1 + 10x_2 - x_1{}^2 - x_2{}^2}$

Subject to ; $\mathbf{x_1 + x_2 \leq 8}$

$\mathbf{-x_{1+}x_2 \leq 5}$

$\mathbf{x_1, x_1 \geq 0}$

SOLUTION :-

Maximize : $Z = 10x_1 + 10x_2 - x_1{}^2 - x_2{}^2$

Subject to ; $x_1 + x_2 \leq 8$

$-x_{1+}x_2 \leq 5$

$x_1, x_1 \geq 0$

We write the problem as

$f(x_1, x_2) = 10x_1 + 10x_2 - x_1{}^2 - x_2{}^2$

$h_1(x_1, x_2) = \; x_1 + x_2 - 8$

$h_2(x_1, x_2) = \; -x_1 + x_2 - 5$

The kuhn jucker conditions are:

$\frac{\partial f}{\partial x_1} - \lambda \frac{\partial h_1}{\partial x_1} - \lambda_2 \frac{\partial h_2}{\partial x_2} = 0$

$10 - 2x_1 - \lambda_1(1) - \; \lambda_2(1) = 0$

$10 - 2x_2 - \lambda_1 - \lambda_2 = 0$ .... (1)

Also, $\frac{\partial f}{\partial x_2} - \lambda \frac{\partial h_1}{\partial x_2} - \lambda_2 \frac{\partial h_2}{\partial x_2} = 0$

$10 - 2x_2 - \lambda_1(1) - \lambda_2(1) = 0$ .... (2)

$\lambda_1(-x_1 + x_2 - 5) = 0$ .... (3)

$\lambda_2(-x_1 + x_2 - 5) = 0$ .... (4)

$x_1 + x_2 - 8 \leq 0$ .... (5)

-$x_1 + x_2 - 5 \leq 0$ .... (6)

$x_1, x_2 > 0$ .... (7)

For minima; $\lambda_1, \lambda_2 \geq 0$

Case 1: $\lambda_1 = \lambda_2 = 0$

From 1 and 2 we get,

$10 - 2x_1 = 0 \quad \text{and} \quad 10 - 2x_2 = 0$

$x_1 = x_2 = 5$

These values do not satisfy all the equation

Case 2: $\lambda_1 = 0; \quad \lambda_2 \neq 0$

To find x_1 and x_2, we eliminate λ_2 from

$10 - 2x_1 + \lambda_2 = 0 \quad \text{and} \quad 10 - 2x_2 - \lambda_2 = 0$

Adding these equation,

We get $20 - 2x_1 - 2x_2 = 0$

$x_1 + x_2 = 10$

$\lambda_2 \neq 0$ we get from (4); $-x_1 + x_2 = 5$

Adding the two we get $2x_2 = 15$

$x_2 = 7.5$

$x_1 = 10 - 7.5 = 2.5$

But equation 5 is not satisfied reject this pair.

Case 3: $\lambda_1 \neq 0$; $\lambda_2 = 0$

Eliminating λ_1 from,

$16 - 2x_1 - \lambda_1 = 0$ and $10 - 2x_2 - \lambda_1 = 0$

By subtraction;

$-2x_1 + 2x_2 = 0$

$x_1 = x_2$

$\lambda_1 \neq 0$ from $3, x_1 + x_2 = 8$

$2x_1 = 8$

$x_1 = 4$ and $x_2 = 4$

From 10-2$x_1 - \lambda_1 = 0$

We get, $\lambda_1 = 10 - 2x_1 = 2$

These values satisfy the condition,

$z_{max} = 10(4) + 10(4) - 16 - 16 = 48$

Case 4: $\lambda_1 \neq 0, \lambda_2 \neq 0$

From 3 and 4,

$x_1 + x_2 = 8$

And -$x_1 + x_2 = 5$

Adding the two , we get $2x_2 = 13$

$x_2 = 6.5$

$x_1 = 8 - 6.5 = 1.5$

For these values of x_1 and x_2

$-\lambda_1 + \lambda_2 = -10 + 2x_1 = -7$

$-\lambda_1 - \lambda_2 = -10 + 2x_2 = 3$

Adding the two we get, $-2\lambda_1 = -4$

And $\lambda_2 = -\lambda_1 - 3 = -5$

We reject this pair, $\mathbf{z_{max} = 48}$