Q.P. CODE: 79693

MATHEMATICS SOLUTION

(DEC-2019 SEM-4 COMPS)

Q1] A) Find all the basics solutions to the following problem: (5)

Maximise : $\mathbf{z = x_1 + 3x_2 + 3x_3}$

Subject to : $\mathbf{x_1 + 2x_2 + 3x_3 = 4}$

$\mathbf{2x_1 + 3x_2 + 5x_3 = 7}$

$\mathbf{x_1, x_2, x_3 \geq 0}$

SOLUTION :-

Maximise : $z = x_1 + 3x_2 + 3x_3$

Subject to : $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$

$x_1, x_2, x_3 \geq 0$

No of basic solution	Non basic variable	Basic variable	Equation and values of basic variables	Is solution feasible?	Is solution degenerate	Value of Z	Is solution optimal
1	$x_3 = 0$	x_1, x_2	$x_1 + 2x_2 = 4$ $2x_1 + 3x_2 = 7$ $x_1 = 2, x_2 = 1$	Yes	No	5	Yes
2	$x_2 = 0$	x_1, x_3	$x_1 + 3x_3 = 4$ $2x_1 + 5x_3 = 7$ $x_1 = 1, x_3 = 1$	Yes	No	4	No
3	$x_1 = 0$	x_2, x_3	$2x_2 + 3x_3 = 4$ $3x_2 + 5x_3 = 7$ $x_2 = -1, x_3 = 2$	no	no	-	-

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Q1] B) Evaluate $\int_c^1 (z - z^2)dz$, where c is upper half of the circle |z| = 1 (5)

SOLUTION:-

Evaluate $\int_c^1 (z - z^2)dz \qquad |z| = 1$

$z = e^{i\theta} \qquad \therefore dz = e^{i\theta}d\theta$ and θ varies from 0 to π

$$\int_c^1 (z - z^2)dz = \int_0^\pi (e^{i\theta} - e^{2i\theta})e^{i\theta}.id\theta = i\int_0^\pi (e^{2i\theta} - e^{3i\theta})d\theta = i\left[\frac{e}{2i} - \frac{e}{3i}\right]_0^\pi$$

$$= \left[\frac{e^{2i}}{2} - \frac{e^{3i\pi}}{2} - \frac{1}{2} + \frac{1}{3}\right] = \left[\frac{1}{2} + \frac{1}{3} - \frac{1}{2} + \frac{1}{3}\right] = \frac{2}{3}$$

The value of the integral for the lower half of the same circle in the same positive direction i.e when θ varies from π to 2π.

$$\int_c^1 (z - z^2)dz = i\left[\frac{e^{2i\theta}}{2i} - \frac{e^{3i\theta}}{3i}\right]_\pi^{2\pi} = i\left[\frac{e^{4i\pi}}{2i} - \frac{e^{6i\pi}}{3i} - \frac{e^{2i\pi}}{2i} + \frac{e^{3i\pi}}{3i}\right]$$

$$= \left[\frac{\cos 4\pi + i\sin 4\pi}{2} - \frac{\cos 6\pi + i\sin 6\pi}{3} - \frac{\cos 2\pi + i\sin 2\pi}{2} + \frac{\cos 3\pi + i\sin 3\pi}{2}\right] = \left[\frac{1}{2} - \frac{1}{3} - \frac{1}{2} - \frac{1}{3}\right] = -\frac{2}{3}$$

Q1] C) Ten individuals are chosen at random from a population and heights are found to be 63,63,64,65,66,69,69,70,70,71 inches. Discuss the suggestion that the height of universe is 65 inches. (5)

SOLUTION:-

N = 10(< 30, so it is small sample)

Null hypothesis(H_0): $\mu = 65$

Alternate hypothesis(H_a): $\mu \neq 65$

LOS = 5%

Degree of freedom = n-1=10-1 =9

Critical value(t_x) = 2.2622

Values (x_i)	$D_i = x_i - 67$	${D_i}^2$
63	-4	16
63	-4	16
64	-3	9
65	-2	4
66	-1	1

69	2	4
69	2	4
70	3	9
70	3	9
71	4	16
Total	0	88

$\bar{d} = \frac{\sum d_i}{n} = \frac{0}{10} = 0$

$\bar{x} = a + \bar{d} = 67 + 0 = 67$

Since sample is small,

$s = \sqrt{\frac{\sum {d_i}^2}{n} - \left(\frac{\sum d_i}{n}\right)^2} = \sqrt{\frac{88}{10} - \left(\frac{0}{10}\right)^2} = 2.9965$

S.E = $\frac{s}{\sqrt{n-1}} = \frac{2.9965}{\sqrt{9}} = 0.9888$

Test statistics

$t_{cal} = \frac{\bar{x}-\mu}{S.E} = \frac{67-65}{0.988} = 2.0227$

Decision

Since $|t_{cal}| < t_x$, H_0 is accepted.

The man height of the universe is 65 inches

Q1] D) If $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$ then find A^{100} **(5)**

SOLUTION:-

$A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$

The characteristics equation is

$\begin{bmatrix} 2-\lambda & 3 \\ -3 & -4-\lambda \end{bmatrix} = 0$

$(2-\lambda)(-4-\lambda)-(-3\times 3)=0$

$-8-2\lambda+4\lambda+\lambda^2+9=0$

$(\lambda+1)^2=0$

$\lambda=\ -1$

Since matrix is of order 2 we consider

$\varphi(A)=\ A^{100}=\ \alpha_1 A+\alpha_0 I$

λ satisfies this equation $\lambda^{100}=\ \alpha_1\lambda+\alpha_0 I$

Putting $\lambda=\ -1$

$(-1)^{100}=\ \alpha_1(-1)+\alpha_0 I$

$1=\ -\alpha_1+\alpha_0 I$

$100\lambda^{99}=\alpha_1+0$

$\alpha_1\ =\ -100$

$1=\ -\alpha+\alpha_0$

$\alpha_0\ =\ -99$

$A^{100}=-100A-99I$

$A^{100}=\ -100\begin{bmatrix}2 & 3\\ -3 & -4\end{bmatrix}-99\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}-200-99 & -300-0\\ 300-0 & 400-99\end{bmatrix}=\begin{bmatrix}-299 & -300\\ 300 & 301\end{bmatrix}$

Q2] A) Evaluate $\int_c^1 \frac{z+2}{(z-3)(z-4)}$ dz, where c is the circle |z| = 1 **(6)**

SOLUTION:-

$\int_c^1 \frac{z+2}{(z-3)(z-4)}dz$ $|z|=1$

$|z|=1$ is a circle with center at the origin and radius 1 hence both points z = 3 and z = 4 lie outside the circle C and f(z) is analytic in C

By Cauchy's theorem $\int_c^1 \frac{z+2}{(z-3)(z-4)}dz=0$

Q2] B) An I.Q test was administered to 5 persons and after they were trained. The results are given below. (6)

Test whether there is change in I.Q after the training program use 1% LOS

	I	**II**	**III**	**IV**	**V**
I.Q before training	**110**	**120**	**123**	**132**	**125**
I.Q after training	**120**	**118**	**125**	**136**	**121**

SOLUTION:-

	I	II	III	IV	V
I.Q before training	110	120	123	132	125
I.Q after training	120	118	125	136	121

deviation in each case is 10 -2 2 4 -4
sum of d=10
d^2=100+4+4+16+16=140

$$\mathrm{d} = \text{sum of}\frac{\mathrm{d}}{\mathrm{n}} = \frac{10}{2} = 5 \quad \text{and } \mathrm{s} = \sqrt{\frac{\mathrm{d}^2 - \mathrm{nd}}{\mathrm{n}-1}} = \sqrt{\frac{140- \ (4)}{5-1}} = 5.673$$

Hence there is no change of IQ after training since given
t0.01(4) = 4.6

Q2] C) Solve the following LPP using Simplex Method (8)

Maximize $z = 4x_1 + 10x_2$

Subject to $2x_1 + x_2 \leq 10$

$2x_1 + 5x_2 \leq 20$

$2x_1 + 3x_2 \leq 18$

$x_1, \ x_2 \geq 0$

Solution:-

We first express the given problem in standard form.

Maximize $z = 4x_1 + 10x_2 + 0s_1 + 0s_2 + 0s_3$

i.e. $z - 4x_1 - 10x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

Subject to $2x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 10$

$2x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 = 20$

$2x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 = 18$

We put this information in tabular form as follows.

Iteration no	Basic variable	Coefficients of					RHS soln	ration
		x_1	x_2	s_1	s_2	s_3		
0	Z	-4	-10	0s	0	0	0	
s_2 leaves	s_1	2	1	1	0	0	10	10
x_2 enters	s_2	2	5	0	1	0	20	4
	s_3	2	3	0	0	1	18	6
1	z	0	0	0	2	0	40	
s_1 leaves	s_1	8/5	0	1	-1/5	0	6	15/4
x_1 enters	x_2	2/5	1	0	1/5	0	4	10
	s_3	4/5	0	0	-3/5	1	6	15/2
2	Z	0	0	0	-1/5	0	40	
	x_1	1	0	5/8	-1/8	0	15/4	
	x_2	0	1	-1/4	1/5	0	5/2	
	s_3	0	0	-1/2	-1/2	1	3	

$x_1 = \frac{15}{4}$; $x_2 = \frac{5}{2}$ and $z_{max} = 40$

This is an alternative solution. But this does not improve the above optimal solution.

Thus we have two solutions, $x_1 = 0$; $x_2 = 4$ and $z_{max} = 40$

And $x_1 = \frac{15}{4}$; $x_2 = \frac{5}{2}$ and $z_{max} = 40$

If there are two solutions to a problem then there are infinite number solutions.

Let $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $X_1 = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$, $X_2 = \begin{bmatrix} \frac{15}{4} \\ \frac{5}{2} \end{bmatrix}$, $then\ X = \lambda\ \ X_1 + (1-\lambda)X_2\ \ for\ 0 \le\ \lambda \le 1$

i.e. $X = \begin{bmatrix} \frac{15}{4}(1-\lambda) \\ 4 + \frac{5}{2}(1-\lambda) \end{bmatrix}$

Gives infinite number of feasible solutions, all giving $z_{max} = 40$

Thus we get two points A(0, 4) and G(15/4, 5/2) giving the same maximum value of z(=40).

Q3] A) Find the Eigen values and Eigen vectors of the following matrix. (6)

A = $\begin{bmatrix} \mathbf{2} & \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{3} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{2} \end{bmatrix}$

SOLUTION:-

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

The characteristics equation is

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} = 0$$

After simplification we get,

$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$(\lambda - 1)(\lambda - 1)(\lambda + 5) = 0$

$\lambda = 1,1,5$

Hence 1,1,5 are the Eigen values.

(1) For $\lambda = 1$ $[A - \lambda_1 I]X = 0$ gives

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By $R_2 - R_1,\ \ R_3 - R_1$

$$\begin{bmatrix}1 & 2 & 1\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$x_1 + 2x_2 + x_3 = 0$$

We see that the rank of the matrix is 1 and number of variables is 3. Hence there are 3-1 = 2 linearly independent solutions i.e there are two parameters we shall denote these parameters by s and t.

Putting $x_2 = -5,\ \ x_3 = -t,$ we get $x_1 = -2x_2 - x_3 = 2s + t$

$$X = \begin{bmatrix}2s+t\\-s+0\\0-t\end{bmatrix} = s\begin{bmatrix}2\\-1\\0\end{bmatrix} + t\begin{bmatrix}1\\0\\-1\end{bmatrix}$$

$$x_1 = \begin{bmatrix}2\\-1\\0\end{bmatrix},\ x_2 = \begin{bmatrix}1\\0\\-1\end{bmatrix}$$

The vectors $\begin{bmatrix}2\\-1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}$ are linearly independent.

Hence corresponding to λ=1 the Eigen vectors are $\begin{bmatrix}2\\-1\\0\end{bmatrix}$ and $\begin{bmatrix}1\\0\\-1\end{bmatrix}$

(1) For $\lambda = 5,\quad [A - \lambda_1 I]X = 0$ gives

$$\begin{bmatrix}-3 & 2 & 1\\1 & -2 & 1\\1 & 2 & -3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \text{ by } R_{13}\begin{bmatrix}1 & 2 & -3\\1 & -2 & 1\\-3 & 2 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

By $R_2 - R_1,\ \ R_3 + 3R_1$

$$\begin{bmatrix}1 & 2 & -3\\0 & -4 & 4\\0 & 8 & -8\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

By $R_3 + 2R_2$

$$\begin{bmatrix}1 & 2 & -3\\0 & -4 & 4\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$x_1 + 2x_2 - 3x_3 = 0$ and $-4x_2 + 4x_3 = 0$

Putting $x_3 = t$ we get $x_2 = t$ and $x_1 = -2x_2 + 3x_3 = -2t + 3t = t$

$X = \begin{bmatrix}t\\t\\t\end{bmatrix} = t\begin{bmatrix}1\\1\\1\end{bmatrix}$ hence corresponding to λ = 5, the Eigen vector is $\begin{bmatrix}1\\1\\1\end{bmatrix}$

Q3] B) If the height of 500 students is normally distributed with mean 68 inches and standard deviation 4 inches. Find the expected number of students having heights between 65 and 71 inches. (6)

SOLUTION:-

Evaluating no of students having height between 65 and 71 inches

When X = 65

$z = \frac{65-68}{4} = -\frac{3}{4} = -0.75$

When $x = 71$

$z = \frac{71-6}{4} = \frac{3}{4} = 0.75$

$P(-0.75 < Z < 0.75) = Area\ from\ (Z = 0\ to\ Z = -0.75) + Area\ from\ (Z = 0\ to\ 0.75)$

$\frac{1}{\sqrt{2\pi}}\int_{-0.75}^{0} C^{-\frac{1}{2}z^2dz} + \frac{1}{\sqrt{2\pi}}\int_{0}^{0.75} C^{-\frac{1}{2}z^2dz} = 0.2734 + 0.2734 = 0.5468$

Probability of students having height between 65 to 71 inches is 0.5468

No of students = $N \times P = 500 \times 0.5468 = 273$

No of students having height between 65 to 71 inches is 273

Q3] C) Obtain Taylors and Laurents expansion of $f(z) = \frac{z^2-1}{z^2+5z+6}$ around z = 0 (8)

SOLUTION:-

Since the degree of the numerator is equal to the degree of the denominator we first divide the numerator by the denominator.

$f(z) = \frac{z^2-1}{z^2+5z+6} = 1 - \frac{5z+7}{z^2+5z+6}$

Let $\frac{-5z-7}{z^2+5z+6} = \frac{a}{z+3} + \frac{b}{z+2}$ $\quad -5z - 7 = a(z+2) + b(z+3)$

When z = -2 b = 3 when z = -3 and a = -8

$f(z) = \frac{z^2-1}{z^2+5z+6} \;\; = 1 - \frac{8}{z+3} + \frac{3}{z+2}$ ………………. (1)

Case (1): when |z| < 2 we write

$$f(z) = 1 - \frac{8}{3\left[1+\left(\frac{z}{3}\right)\right]} + \frac{3}{2\left[1+\left(\frac{z}{2}\right)\right]}$$

When |z| < 2, clearly |z| < 3

$$f(z) = 1 - \frac{8}{3}\left[1+\left(\frac{z}{3}\right)\right]^{-1} + \frac{3}{2}\left[1+\left(\frac{z}{2}\right)\right]^{-1} = 1 - \frac{8}{3}\left[1-\left(\frac{z}{3}\right)+\left(\frac{z}{3}\right)^2 - \;\ldots\ldots.\right] + \frac{3}{2}\left[1-\left(\frac{z}{3}\right)+\left(\frac{z}{3}\right)^2 - \;\ldots\ldots\right]$$

Case (2): when 2 < |z| < 3 we write

$$f(z) = 1 - \frac{8}{3\left[1+\left(\frac{z}{3}\right)\right]} + \frac{3}{z\left[1+\left(\frac{z}{2}\right)\right]} \;=\; 1 - \frac{8}{3}\left(1+\frac{z}{3}\right)^{-1} + \frac{3}{z}\left(1+\frac{2}{z}\right)^{-1}$$

$$f(z) = 1 - \frac{8}{3}\left[1-\left(\frac{z}{3}\right)+\left(\frac{z}{3}\right)^2 - \;\ldots..\right] + \frac{3}{z}\left[1-\left(\frac{2}{z}\right)+\left(\frac{2}{z}\right)^2 - \;\ldots\ldots..\right]$$

Case (3): when |z| > 3 we write

$$f(z) = 1 - \frac{8}{z\left[1+\left(\frac{3}{z}\right)\right]} + \frac{3}{z\left[1+\left(\frac{2}{z}\right)\right]}$$

When |z| > 3 clearly |z| > 2

$$f(z) = 1 - \frac{8}{z}\left(1+\frac{3}{z}\right)^{-1} + \frac{3}{2}\left(1+\frac{2}{z}\right)^{-1} = 1 - \frac{8}{z}\left[1-\left(\frac{3}{z}\right)+\left(\frac{3}{z}\right)^2-\left(\frac{3}{z}\right)^3+\cdots\right] + \frac{3}{z}\left[1-\left(\frac{2}{z}\right)+\left(\frac{2}{z}\right)^2-\left(\frac{2}{z}\right)^3+\cdots\right]$$

Q4] A) A machine is claimed to produce nails of mean length 5cms and standard deviation of 0.45cm. A random sample of 100 nails gave 5.1 as their average length. Does the performance of the machine justify the claim? Mention the level of significance you apply. (6)

SOLUTION:-

$\overline{X} = 5\text{cm}$

SD = 0.45cm

$\mu = 5.1\text{cm}$

n = 100

we have,

$z = \left|\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}\right| = \left|\frac{5-5.1}{0.45/\sqrt{100}}\right| = 2.22$

Level of significance $\alpha = 0.05$

Critical value : the value of Z_α at 5% level of significance is 1.96

The performance of the machine increases.

Q4] B) Using the Residue theorem, Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$ (6)

SOLUTION:-

Let $e^{i\theta} = z$ $\quad\quad e^{i\theta}.\,id\theta = dz;$

$d\theta = \frac{dz}{iz}$ $\quad$ and $\quad \sin\theta = \frac{z^2-1}{2iz}$

$I = \int_c^{iz} \frac{1}{5+3\left(\frac{z^2-1}{2iz}\right)} \left(\frac{dz}{iz}\right) = \int_c^{1} \frac{2}{3z^2+10iz} dz = \int_c^{1} \frac{2}{(3z+i)(z+3i)} dz$

Where c is the circle |z|=1

Now the poles of f(z) are given by $(3z+i)(z+3i) = 0$, $z = -\left(\frac{i}{3}\right)$ and $z = -3i$ are simple poles. But $z = -\left(\frac{i}{3}\right)$ lies inside and z = -3i lies outside the circle |z| = 1

Resides $\left(\text{at } z = -\frac{i}{3}\right) = \lim_{z\to -\frac{i}{3}} \left[z + \frac{i}{3}\right] . \frac{2}{(3z+i)(z+3i)} = \lim_{z\to -\frac{i}{3}} \frac{2}{3(z+3i)} = \frac{1}{4i}$

$I = 2\pi i \left(\frac{1}{4i}\right) = \frac{\pi}{2}$

Q4] C) (1) In a certain manufacturing process 5% of the tools produced turnout to be defective. Find the probability that in a sample of 40 tools at most 2 will be defective.

(2)A random variable x has the probability distribution $\mathbf{P(X=x),=\frac{1}{8}\,.3C_x, x=0,1,2,3.}$ $\mathbf{find\ the\ moment\ generating\ function\ of\ x}$ (8)

SOLUTION:-

(1) n = 40 p = 0.05 $f(x)=\frac{e^{-2}.2^x}{x}$

n = np = $40\times0.05=2$ use poisson p(at most 2) = $p(x\le2)=\frac{e^{-2}.2^0}{0}+\frac{e^{-2}.2^1}{1!}+\frac{e^{-2}.2^2}{2!}=$ 0.675

(2) $P(X=x),=\frac{1}{8}\,.3C_x, x=0,1,2,3$

$P(X=0)=\frac{1}{8}.3C_0=\frac{1}{8}$

$P(X=1)=\frac{1}{8}.3C_1=\frac{3}{8}$

$P(X=2)=\frac{1}{8}.3C_2=\frac{3}{8}$

$P(X=3)=\frac{1}{8}.3C_3=\frac{1}{8}$

Moment generating function, $M_0(t)=E(e^{txi})=\sum p_i\,e^{txi}$

From above values

$M_0(t)=\frac{1}{8}e^{t(0)}+\frac{3}{8}e^{t(1)}+\frac{3}{8}e^{t(2)}+\frac{1}{8}e^{t(3)}=\frac{1}{8}\left[1+3e^{t(0)}+3e^{t(2)}+e^{t(3)}\right]=\frac{1}{8}(1+e^t)^3$

$M_0(t)=\frac{1}{8}\frac{1}{8}(1+e^t)^3$

Q5] A) Check whether the following matrix is Derogatory or Non- Derogatory

$$\mathbf{A=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}}$$ **(6)**

SOLUTION:-

$$A=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$$

Characteristics equation :

$\lambda^3 - 12\lambda^2 + (8 + 14 + 14)\lambda - 32 = 0$

Λ = 8,2,2

Let us assume $(x - 8)(x - 2) = x^2 - 10x + 16$ annihilates A

Now $A^2 - 10A + 16I$

$$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - 10 \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + 16 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & 0 & 0 \\ 0 & -16 & 0 \\ 0 & 0 & -16 \end{bmatrix} + \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x^2 - 10x + 16$ annihilates A thus f(x) is the monic polynomial of lowest degree

$\text{minimal polynomial} = x^2 - 10x + 16$

Q5] B) In an industry 200 workers employed for a specific job were classified according to their performance and training received to test independence of training received and performance. The data are summarized as follows: (6)

Performance	**Good**	**Not good**	**Total**
Trained	**100**	**50**	**150**
Untrained	**20**	**30**	**50**
Total	**120**	**60**	**200**

Use χ^2 − test for independence at 5% level of significance and write your conclusion.

SOLUTION:-

Null Hypothesis, H_o there is no independence

Alternative Hypothesis, H_a there is an independence

Calculate of test statistics;

Trained and good = $\frac{120\times150}{200} = 90$

Untrained and good = $\frac{120\times50}{200} = 30$

Trained and not good = $\frac{80\times150}{200} = 60$

untrained and not good = $\frac{80\times50}{200} = 20$

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
90	100	100	1
30	20	100	5
60	50	100	2
20	30	100	3.33 X = 11.33

$\alpha = 0.05$

Degree of freedom = $(r-1)(c-1) \;=\; (2-1)(2-1) \;=\; 1$

Critical value = 3.841

$\chi^2\text{cal} \;>\; \chi^2\text{table}$

Thus Null hypothesis is rejected

There is an independence relationship.

Q5] C) Use the dual simplex method to solve the following L.P.P (8)

Minimize $z = 2x_1 + x_2$

Subject to $3x_1 + x_2 \geq 3$

$\mathbf{4x_1 + 3x_2 \geq 6}$

$\mathbf{x_1 + 2x_2 \leq 3}$

$\mathbf{x_1, x_2 \geq 0}$

SOLUTION :-

Minimize $z = 2x_1 + x_2$

Subject to $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq \; 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

-$3x_1 - x_2 \leq -3$

-$4x_1 - 3x_2 \leq - \; 6$

$x_1 + 2x_2 \leq 3$

Introducing stack variables

$Z' - 2x_1 - x_2 - 0s_1 - 0s_2 - 0s_3$

$-3x_1 - x_2 + s_1 - 0s_2 - 0s_3 = \; -6$

$x_1 + 2x_2 + 0s_1 + 0s_2 + s_3 = 3$

Iterations no	Basics variables	Coefficients of					RHS Solution
		x_1	x_2	s_1	s_2	s_3	
0	Z’	-2	-1	0	0	0	0
s_2 leaves	s_1	-3	-1	1	0	0	-3
x_2 enters	s_2	-4	-3	0	1	0	-6
	s_3	1	2	0	0	1	3
Ratio:		-1/2	-1/3				
1	Z’	2	2	0	-1	0	6
	s_1	-5/3	0	0	1/3	0	-1
	x_2	4/3	1	0	1/3	0	2
	s_3	-5/3	0	0	-2/3	1	-1
Ratio:		-6/5			3/2	0	
2		11/3	2	0	-1/3	0	7
s_3 leaves	s_1	0	0	0	1	-1	0
x_1 enters	x_2	0	1	0	3	-4/3	6/5
	x_1	1	0	0	2/5	-3/5	3/5

$x_1 = \frac{3}{5}$; $x_2 = \frac{6}{5}$;

$Z_{min} = 2\left(\frac{3}{5}\right) + \frac{6}{5} = \frac{12}{5}$

$Z_{min} = \frac{12}{5}$

Q6] A) Show that the matrix A satisfies Cayley- Hamilton theorem and hence find A^{-1} **(6)**

Where A = $\begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{7} \\ \mathbf{4} & \mathbf{2} & \mathbf{3} \\ \mathbf{1} & \mathbf{2} & \mathbf{1} \end{bmatrix}$

We know that,

$|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1-\lambda & 3 & 7 \\ 4 & 2-\lambda & 3 \\ 1 & 2 & 1-\lambda \end{bmatrix} \right| = 0$$

$(1-\lambda)[(2-\lambda)(1-\lambda) - 6] - 3[4(1-\lambda) - 7] + 7[8 - (2-\lambda)] = 0$

$(1-\lambda)[2 - 2\lambda - \lambda + \lambda^2 - 6] - 3[4 - 4\lambda - 7] + 7[\lambda + 6] = 0$

$\lambda^2 - 3\lambda - 4 + \lambda^3 + 3\lambda^2 + 4\lambda + 12\lambda + 9 + 7\lambda + 42 = 0$

$\lambda^3 + 4\lambda^2 + 10\lambda + 47 = 0$

The characteristics equation of A is

$P(\lambda) = \lambda^3 + 4\lambda^2 + 10\lambda + 47 = 0$

By cayley-Hamilton theorem, A is satisfies its characteristics equation

So that replace λ with A

$P(A) = A^3 + 4A^2 + 10A + 47 = 0$ ………………… (1)

Since |A| = 35 $\neq 0$ by A^{-1} is exists.

Multiply equation (1) by A^{-1} we get,

$$A^{-1}(A^3 + 4A^2 + 10A + 47) = 0$$

$$A^2 = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 23 & 23 \\ 15 & 22 & 37 \\ 10 & 11 & 14 \end{bmatrix}$$

$$A^2 + 4A + 10I + 47A^{-1} = 0$$

$$A^2 + 4A + 10I = -47A^{-1}$$

$$A^{-1} = \frac{1}{-47}[A^2 + 4A + 10I]$$

$$A^{-1} = \frac{1}{-47}\left[\begin{bmatrix} 20 & 23 & 23 \\ 15 & 22 & 37 \\ 10 & 11 & 14 \end{bmatrix} + \begin{bmatrix} 4 & 12 & 28 \\ 16 & 8 & 12 \\ 4 & 8 & 4 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}\right] = -\frac{1}{47}\begin{bmatrix} 34 & 35 & 51 \\ 31 & 40 & 49 \\ 14 & 19 & 28 \end{bmatrix}$$

Q6] B) A discrete random variable has the probability density function given below **(6)**

$X = x_i$	**-2**	**-1**	**0**	**1**	**2**	**3**
$P(x_i)$	**0.2**	**k**	**0.1**	**2k**	**0.1**	**2k**

Find K, Mean, Variance.

SOLUTION:-

$X = x_i$	-2	-1	0	1	2	3
$P(x_i)$	0.2	k	0.1	2k	0.1	2k

$$\sum P(x_i) = 1$$

$$0.2 + k + 0.1 + 2k + 0.1 + 2k = 1$$

$$0.4 + 5k = 1$$

$$5k = 1 - 0.4 = 0.6$$

$$k = \frac{0.6}{5} = 0.12$$

$X = x_i$	-2	-1	0	1	2	3
$P(x_i)$	0.2	0.12	0.1	0.24	0.1	0.24

Mean =
$\sum x_i P(x_i) = (-2 \times 0.2) + (-1 \times 0.12) + (0 \times 0.1) + (1 \times 0.24) + (2 \times 0.1) + (3 \times 0.24)$

Mean = $-0.4 - 0.12 + 0 + 0.24 + 0.2 + 0.72 = 0.64$

Mean = E(x) = 0.64

$$E(X^2) = [4 \times 0.2] + [1 \times 0.12] + [0] + [1 \times 0.24] + [4 \times 0.1] + [3 \times 0.24]$$

$E(X^2) = 2.28$

Variance = $E(X^2) - [E(X)]^2 = 2.28 - 0.64^2 = 2.28 - 0.4096 = 1.8704$

Q6] C)Using Kuhn- Tucker conditions solve the following NLPP (8)

Maximize : $\mathbf{Z = 2x_1 - 7x_2 + 12x_1x_2}$

Subject to : $\mathbf{2x_1 + 5x_2 \leq 98}$

$\mathbf{x_1, x_2 \geq 0}$

SOLUTION:-

Maximize : $Z = 2x_1 - 7x_2 + 12x_1x_2$

Subject to : $2x_1 + 5x_2 \leq 98$

$x_1, x_2 \geq 0$

We rewrite the given problem as:

f(x) = $2x_1^2 - 7x_2^2 + 12x_1x_2$

$h(x_1x_2) = 2x_1 + 5x_2 - 98$

Kuhn tucker conditions are:

$\frac{\partial f}{\partial x_1} - \frac{\lambda \partial h}{\partial x_1} = 0; \frac{\partial f}{\partial x_2} - \frac{\lambda \partial h}{\partial x_2} = 0$

$\lambda h(x_1, x_2) = 0;\ h(x_1, x_2) \leq 0,\ \lambda \geq 0$

We get,

$4x_1 + 12x_2 - \lambda(2) = 0$ .... (1)

$12x_1 - 14x_2 - \lambda(5) = 0$ ....(2)

$\lambda(2x_1 + 5x_2 - 98) = 0$ ………………… (3)

$2x_1 + 5x_2 - 98 \le 0$ ……………… (4)

$\lambda \ge 0$ ………… (5)

From (3) we get either λ = 0 or $(2x_1 + 5x_2 - 98) = 0$

Case 1: λ = 0 and $(2x_1 + 5x_2 - 98) \neq 0$

From 1 and 2,

$4x_1 + 12x_2 = 0$

$12x_1 - 14x_2 = 0$

On solving simultaneously we get $x_1 = x_2 = 0$

Case 2:

$\lambda \neq 0$ and $2x_1 + 5x_2 - 98 = 0$

$4x_1 + 12x_2 - \lambda(2) = 0$

$12x_1 - 14x_2 - \lambda(5) = 0$

$\lambda = \frac{12\ \ _1 - 14x_2}{5}$

Equation I : $4x_1 + 12x_2 - 2\left[\frac{12x_1 - 14x_2}{5}\right] = 0$

$20x_1 + 60x_2 - 24x_1 + 28x_2 = 0$

$-4x_1 + 88x_2 = 0$ (divide through by 4)

$-x_1 + 22x_2 = 0$

Put $x_1 = 22x_2$ in 4

$2(22x_2) + 5x_2 = 98$

$44x_2 + 5x_2 = 98$

$49x_2 = 98$

$x_2 = 2$

$2x_1 + 10 = 98$

$2x_1 = 88$

$x_1 = 44$

These values satisfy all conditions,

$Z_{max} = 2(1936) - 7(4) + 12(44)(2) = 4900$

$Z_{max} = 4900$
