QP CODE: 40558

MATHEMATICS SOLUTION

(CBCGS SEM – 4 NOV 2018)

BRANCH – COMPUTER ENGINEERING

Q1] A) Find all the basic solutions to the following problem: (5)

Maximise : $\mathbf{z = x_1 + x_2 + 3x_3}$

Subject to : $\mathbf{x_1 + 2x_2 + 3x_3 = 9}$

$$\mathbf{3x_1 + 2x_2 + 2x_3 = 15}$$

$$\mathbf{x_1, x_2, x_3 \geq 0}$$

SOLUTION:-

Maximise : $z = x_1 + x_2 + 3x_3$

Subject to : $x_1 + 2x_2 + 3x_3 = 9$

$$3x_1 + 2x_2 + 2x_3 = 15$$

$$x_1, x_2, x_3 \geq 0$$

No of basic solution	Non basic variables	Basic variable	Equation and the value of basic variables	Is the solution feasible	Is the solution degenerate	Value of Z
1	$x_3 = 0$	x_1, x_2	$x_1 + 2x_2 = 9$ $3x_1 + 2x_2 = 15$ $x_1 = 3$ $x_2 = 3$	yes	no	6
2	$x_2 = 0$	x_1, x_3	$x_1 + 3x_3 = 9$ $3x_1 + 2x_3 = 15$ $x_1 = 3.86$ $x_3 = 1.71$	yes	no	8.99
3	$x_1 = 0$	x_2, x_3	$2x_2 + 3x_3 = 9$ $2x_2 + 2x_3 = 15$ $x_2 = 13.5$ $x_3 = -6$	no	no	-

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Q1] B) Evaluate $\oint \mathbf{zdz}$ from $\mathbf{z = 0}$ to $\mathbf{z = 1 + i}$ along the curve $\mathbf{z = t^2 + it}$

(5)

SOLUTION :-

When z = 0, t =0

When $z = 1 + i; \ \ t = 1$

$z = t^2 + it$

$dz = (2t + i)dt$

$\int_0^{1+i} zdz = \int_0^1 (t^2 + it)(2t + i)dt = \int_0^1 (2t^3 + it^2 + 2it^2 - t)dt = \int_0^1 2t^3 - t + 3it^2)dt$

$= \left[\frac{t^4}{2} - \frac{t^2}{2} + it^3\right]_0^1 = i$

$\oint \mathbf{zdz = i} \quad \mathbf{where\ z = t^2 + it\ \ along\ z = 0\ to\ i + 1}$

Q1] C) A sample of 100 students is taken from a large population. The mean height of the students in this sample is 160cm. Can it be reasonably regarded that in the population, the mean height is 165cm, and the standard deviation is 10cm? (5)

SOLUTION :-

Null hypothesis : $\mu = 160$

Alternate hypothesis : $\mu \neq 160$

$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{165 - 160}{10/\sqrt{100}} = 5$

$|z| = 5$

At 5% level of significance, z is 1.96.

$z_{cal} > z_{critical}$

Null hypothesis is rejected.

Therefore, No it wouldn't be reasonable to suppose the assumption.

Q1] D) The sum of the Eigen values of a 3 X 3 matrix is 6 and the product of the Eigen value id also 6. If one of the Eigen value is one, find the other two Eigen values. (5)

SOLUTION:-

Let the Eigen values be λ_1, λ_2 and λ_3

$\lambda_1 + \lambda_2 + \lambda_3 = 6$

And $\lambda_1 . \lambda_2 . \lambda_3 = 6$

Given : one Eigen value = 1

Let $\lambda_1 = 1$

$1 + \lambda_2 + \lambda_3 = 6$

$\lambda_2 + \lambda_3 = 5$

$\lambda_3 = (5 - \lambda_2)$

And $(\lambda_2 - \lambda_3) = 6$

$\lambda_2(5 - \lambda_2) = 6$

$5\lambda_2 - {\lambda_2}^2 = 6$

${\lambda_2}^2 - 5\lambda_2 + 6 = 0$

${\lambda_2}^2 - 3\lambda_2 - 2\lambda_2 + 6 = 0$

$\lambda_2(\lambda_2 - 3) - 2(\lambda_2 - 3) = 0$

$\lambda_2 = 2$ or $\lambda_2 = 3$

If $\lambda_2 = 2$ and $\lambda_3 = 3$ and if $\lambda_2 = 3$ and $\lambda_3 = 2$

The Eigen values are 1,2,3.

Q2] A) Evaluate $\oint \frac{\sin^6 z}{(z - \frac{\pi}{6})^n} dz$ where c is the circle |z| = 1 for n = 1, n = 3 (6)

SOLUTION:-

$\oint \frac{\sin^6 z}{(z - \frac{\pi}{6})^n} dz$ where c is a is a circle $|z| = 1$

For n = 1 we have $\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^n} dz$

$Z_0 - \frac{\pi}{6} = 0$

$Z_0 = \frac{\pi}{6}$ pt lies inside the circle ; |z| = 1

$\int \frac{f(z)}{z-z_0} dz = 2\pi i f(z_0)$ { By Cauchy's Integra formula}

$$\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^n} dz = 2\pi i \sin^2\left(\frac{\pi}{6}\right) = 2\pi i \left(\frac{1}{2}\right)^6 = \frac{2\pi i}{64} = \frac{\pi i}{32}$$

For n =3; we have $\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^n} dz$

$Z = \frac{\pi}{6}$ lies inside the circle , $|Z| = 1$

Order of pole = 3

$\int \frac{f(z)}{(Z-Z_0)^n} = \frac{2\pi i}{(n-1)!} f^{n-1}(z_0)$

$f(z) = \sin^6 z$

$f'(z) = 6\sin^5 z.\cos z$

$f''(z) = 6[5\sin^4 z\cos^2 z - \sin^5 z.\sin z] = 6[5\sin^4 z\cos^2 z - \sin^6 z]$

$f''(z) = 6\left[5\sin^4\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{6}\right) - \sin^6\left(\frac{\pi}{6}\right)\right] = \frac{21}{16}$

$$\oint \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz = \frac{2\pi i}{2!} \cdot \frac{21}{16} = \frac{21\pi i}{16}$$

Q2] B) The following data is collected on two characters. Based on this, can you say that there is no relation between smoking and literacy? Use Chi-square test at 5% Level of significance (6)

	Smokers	**Non smokers**
Literates	**83**	**57**
Illiterates	**45**	**68**

Solution:-

	Smokers	Non smokers	Total
Literates	83	57	140
Illiterates	45	68	113
Total	128	125	253

Expected frequency (literate and smokers) = $\frac{140\times128}{253} = 70.83$

Expected frequency (literate and non - smokers) = $\frac{140\times125}{253} = 69.17$

Expected frequency (illiterate and smokers) = $\frac{113\times128}{253} = 57.17$

Expected frequency (illiterate and non - smokers) = $\frac{113\times125}{253} = 55.83$

O	E	$(0-\mathrm{E})^2$	$\frac{(\mathrm{O}-\mathrm{E})^2}{\mathrm{E}}$
83	70.83	148.11	2.0911
57	69.17	148.11	2.1412
45	57.17	148.11	2.5907
68	55.83	148.11	2.6529
			$\mathrm{X}^2 = 9.4759$

Level of significance = (0.05)

Degree of freedom = (r-1)(c-1) = (2-1)(2-1) = 1

Critical value at 1 degree of freedom at 5% level of significance is 3.84.

$\mathbf{X^2}_{\mathbf{table}} < \mathbf{X^2}_{\mathbf{calc}}$

There is no association.

Q2] C) Solve the following LPP using Simple Method **(8)**

Maximise $\mathbf{z = 3x_1 + 5x_2}$

Subject to $\mathbf{3x_1 + 2x_2 \leq 18,\ \ x_1 \leq 4,\ x_2 \geq 6,\ \ x_1, x_2 \geq 0}$

SOLUTION :-

$z = 3x_1 + 5x_2$

$3x_1 + 2x_2 \leq 18$

$x_1 \leq 4$

$x_2 \geq 6$

We first express the given problem in standard form;

$z - 3x_1 - 5x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

$3x_1 + 2x_2 + 1s_1 + 0s_2 + 0s_3 = 0$

$x_1 + 0s_1 + 0s_2 + 0s_3 = 0$

$0x_1 + x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

Simple table

Iteration number	Basic variables	Coefficient of					RHS soln	ratio
		x_1	x_2	s_1	s_2	s_3		
0	Z	-3	-5	0	0	0	0	
s_3 leaves	s_1	3	2	1	0	0	18	9
x_2 enters	s_2	1	0	0	1	0	4	-
	s_3	0	1	0	0	1	6	6
1	Z	-3	0	0	0	5	30	
s_1 leaves	s_1	3	0	1	0	-2	6	2
x_1 enters	s_2	1	0	0	1	0	4	4
	x_2	0	1	0	0	1	6	-
2	Z	0	0	1	0	3	36	
	x_1	1	0	1/3	0	-2/3	2	
	s_2	0	0	-1/3	1	2/3	2	
	x_2	0	1	0	0	1	6	

$x_1 = 2; \quad x_2 = 6;$

$$\mathbf{Z_{max} = 3(x_1) + 5(x_1) = 3(2) + 5(6) = 6 + 30 = 36}$$

Q3] A) Find the Eigen values and Eigen vectors of the following matrix. (6)

$$\mathbf{A} = \begin{bmatrix} \mathbf{4} & \mathbf{6} & \mathbf{6} \\ \mathbf{1} & \mathbf{3} & \mathbf{2} \\ \mathbf{-1} & \mathbf{-4} & \mathbf{-3} \end{bmatrix}$$

SOLUTION :-

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

The characteristics equation is given by

$$\lambda^3 - 4\lambda^2 + (-1 - 6 + 6)\lambda - 4 = 0$$

$$\lambda^3 - 4\lambda^2 - \lambda - 4 = 0$$

$$\lambda^2(\lambda - 4) - 1(\lambda - 4) = 0$$

$$(\lambda - 4)(\lambda^2 - 1) = 0$$

$$\lambda = 4 \quad \text{and} \quad \lambda = +1, -1$$

For $\lambda = 4$

$$\begin{bmatrix} 0 & 6 & 6 \\ 1 & -1 & 2 \\ -1 & -4 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0x_1 + 6x_2 + 6x_3 = 0$$

$$1x_1 - 1x_2 + 2x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 6 & 6 \\ -1 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 6 \\ 1 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix}} = t$$

$$\frac{x_1}{18} = \frac{x_2}{6} = \frac{x_3}{6} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3t \\ t \\ t \end{bmatrix}$$

For $\lambda = 1$

$$\begin{bmatrix} 3 & 6 & 6 \\ 1 & 2 & 2 \\ -1 & -4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 + x_3 = 0$$

$$-x_1 - 4x_2 - 4x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 2 & 1 \\ -4 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 5 \\ -1 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 2 \\ -1 & -4 \end{vmatrix}} = t$$

$$\frac{x_1}{-4} = \frac{x_2}{3} = \frac{x_3}{-2} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4t \\ 3t \\ -2t \end{bmatrix}$$

For $\lambda = -1$

$$\begin{bmatrix} 5 & 6 & 6 \\ 1 & 4 & 2 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$5x_1 + 6x_2 + 6x_3 = 0$$

$$-x_1 - 4x_2 - 2x_3 = 0$$

$$\frac{x_1}{\begin{vmatrix} 6 & 6 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & 6 \\ -1 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & 6 \\ -1 & -4 \end{vmatrix}} = t$$

$$\frac{x_1}{12} = \frac{x_2}{4} = \frac{x_3}{-14} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6t \\ 2t \\ -7t \end{bmatrix}$$

Eigen values = 4,1,-1

Eigen vector = [3t t t], [-4t 3t -2t], [6t 2t -7t]

Q3] B) The income of a group of 10,000 persons were found to be normally distributed with mean of Rs 750 and standard deviation of Rs 50. What is the lowest income of richest 250? (6)

SOLUTION :-

Standard normal variate; $Z = \frac{X-m}{\sigma} = \frac{X-750}{50}$

If we have to consider the richest 250 persons, then probability that a person selected at random will be one of them is $\frac{250}{10000} = 0.025$

Area from $(z = 0 \text{ to } z = \text{this value}) = 0.5 - 0.025 = 0.475$

From the table, we find that the area from $z = 0$ to $z = 1.96$ is 0.475

The required $z = 1.96$

When $z = 1.96$, $1.96 = \frac{X-750}{50}$

$X - 750 = 1.96 \times 50 = 848$

Lowest income of richest 250 persons= Rs 848.

Q3] C) Expand $\frac{z^2-1}{z^2+5z+6}$ around z = 0 **(8)**

SOLUTION :-

Degree of numerator = degree of denominator

Dividing numerator by denominator

$$f(z) = 1 - \frac{8}{z+3} + \frac{3}{z+2}$$

case (i); when |z| < 2

$$f(z) = 1 - \frac{8}{3\left(1+\frac{z}{3}\right)} + \frac{3}{2\left(1+\frac{z}{2}\right)} = 1 - \frac{8}{3}\left(1+\frac{z}{3}\right)^{-1} + \frac{3}{2}\left(1+\frac{z}{2}\right)^{-1} = 1 - \frac{8}{3}\left(1 - \frac{z}{3} + \frac{z^2}{9} + \cdots\right) + \frac{3}{2}\left(1 - \frac{z}{2} + \frac{z^2}{4} + \cdots\right)$$

case (ii); when 2 < |z| < 3

$$f(z) = 1 - \frac{8}{3\left(1+\frac{z}{3}\right)} + \frac{2}{z\left(1+\frac{z}{2}\right)} = 1 - \frac{8}{3}\left(1+\frac{z}{3}\right)^{-1} + \frac{2}{z}\left(1+\frac{z}{2}\right)^{-1}$$

$$= 1 - \frac{8}{3}\left(1 - \frac{z}{3} + \frac{z^2}{9} + \cdots\right) + \frac{2}{z}\left(1 - \frac{2}{z} + \frac{4}{z^2} + \cdots\right)$$

Case (iii), when |z| > 3

$$f(z) = 1 - \frac{8}{z\left(1+\frac{3}{z}\right)} + \frac{2}{z\left(1+\frac{2}{z}\right)} = 1 - \frac{8}{z}\left(1+\frac{3}{z}\right)^{-1} + \frac{2}{z}\left(1+\frac{2}{z}\right)^{-1}$$

$$\mathbf{f(z) = 1 - \frac{8}{z}\left(1 - \frac{3}{z} + \frac{9}{z^2} + \cdots\right) + \frac{2}{z}\left(1 - \frac{2}{z} + \frac{4}{z^2} + \cdots\right)}$$

Q4] A)The mean breaking strength of cables supplied by a manufacturer is 1800 with S.D. 100. By a new technique in the manufacturing process it is claimed that the breaking strength of the cable has increased. In order to test the claim a sample of 50 cables are tested. It is found that the mean breaking strength is 1850. Can we support the claim at 1% LOS. (6)

SOLUTION :-

Null hypothesis : $\mu = 1800$

alternate hypothesis : $\mu \neq 1800$

$|Z| = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1850-180}{\frac{100}{\sqrt{50}}} = 3.53$

The value of Z at 1% level of significance = 2.576

$Z_{calc} > Z_{critical}$

Null hypothesis is rejected

Therefore the claim is not supported.

Q4] B) Using the Residue theorem, Evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$ (6)

SOLUTION :-

$\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$

Let $z = e^{i\theta} d\theta$

$dz = ie^{i\theta} d\theta$

$d\theta = \frac{dz}{ie^{i\theta}}$

$d\theta = \frac{dz}{iz}$ where C is unit circle $|z| = 1$

$$\int_0^{2\pi} \frac{1}{5-3\left(\frac{z^2+1}{2z}\right)} \left(\frac{dz}{iz}\right) = \int_0^{2\pi} \frac{-2dz}{(10z-3z^2-3)i} = \int_0^{2\pi} \frac{-2dz}{(-10z+\ ^2+3)i} = \int \frac{-2}{(3z-1)(z-3)i}$$

Z-3 lies outside the circle.

Residue of f(z) at $z = \frac{1}{3} = \lim_{z\to\frac{1}{3}} \left(z - \frac{1}{3}\right) \frac{-2}{i(3z-1)(z-3)} = \lim_{z\to\frac{1}{3}} \left(\frac{3z-1}{3}\right) \frac{-2}{i(3z-1)(z-3)} =$

$\frac{1}{3i}\left(-\frac{2}{\frac{1}{3}-3}\right) = \frac{1}{4i}$

$\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta} = 2\pi i\left(\frac{1}{4i}\right) = \frac{\pi}{2}$

$$\boldsymbol{\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta} = \text{real part of } \left(\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}\right) = \frac{\pi}{2}}$$

Q4] C) (1) Out of 1000 families with 4 children each, how many would you expect to have (a) at least one boy (b) at most 2 girls.

(2)Find the Moment Generating Function of POISSON Distribution and hence find its mean. (8)

SOLUTION :-

(1)

BBBB	BBBG	BBGB	BGBB
BBGG	BGGB	BGBG	BGGG
GGGG	GGGB	GGBG	GBGG
GGBB	GBBG	GBGB	GBBB

a) P(at last one boy) $= \frac{15}{16}$

Families having at least one boy = $N \times P = 1000 \times \frac{15}{16} = 37.5 = 938$

b) P(at most 2 girls) = P(2 girls) + P(3 girls) +P(4 girls) = $\frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{11}{16}$

Families having at most 2 girls = $N \times P = 1000 \times \frac{11}{16} = 687.5 = 688$

938 families have at least 1 boy

688 families have at most 2 girls.

(1) Moment generating function about origin:

$M_0(t) = E(e^{tx}) = \sum p_i e^{txi} = \sum nC_x p^x q^{n-x}. e^{tx} = \sum nC_x q^{n-x}(pe^t)^x$

$M_0(t) = (q + pe^t)^n$

Differentiating $M_0(t)$ and putting $t = 0$ to find mean.

$\frac{d}{dt}[M_0(t)] = n(q + pe^t)^n\ pe^t = np[e^t(q + pe^t)^{n-1}]$

$$\frac{d}{dt}[M_0(t)] = np(q+p)^{n-1} = np \qquad \ldots\ldots(p+q=1)$$

$$\mathbf{mean = np}$$

Q5] A) Check whether the following matrix is Derogatory or Non- Derogatory:

$$\mathbf{A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}}$$ **(6)**

SOLUTION :-

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$

Characteristics equation,

$\lambda^3 - 3\lambda^2 + (3+0-0)\lambda - 1 = 0$

$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$ .... (1)

$(\lambda - 1)^3 = 0$

λ = 1,1,1

let us find minimal polynomial;

$(x-1)(x-1) = 0$

Assuming; $x^2 - 2x + 1 = 0$ annihilates A

$A^2 - 2A + I = 0$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -3 & 3 \\ 3 & -8 & 6 \end{bmatrix}$$

$$2A = 2\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 2 & -6 & 6 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A^2 - 2A + I \neq 0$

It is not a minimal polynomial.

Minimal polynomial = $(x-1)(x-1)(x-1)$

As degree of freedom of minimal polynomial is equal to order

The matrix is non derogatory

Q5] B) The means of two random samples of sizes 9 and 7 are 196 and 199 respectively. The sum of the squares of the deviations from the mean is 27 and 19 respectively. Can the samples be regarded to have been drawn from the same normal population? (6)

SOLUTION :-

$n_1 = 9 \qquad n_2 = 7 \qquad \overline{X_1} = 196 \qquad X_2 = 199$

Null hypothesis H_0: $\mu_1 = \mu_2$

alternative hypothesis : $\mu_1 \neq \mu_2$

$$S_p = \sqrt{\frac{\sum(X_i-\bar{X})^2+\sum(Y_i-\bar{Y})^2}{n_1+n_2-2}} = \sqrt{\frac{27+19}{14}} = 1.8126$$

Standard error of the difference between the means

$$SE = sp \times \sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 1.8261\sqrt{\frac{1}{9}+\frac{1}{7}} = 0.9135$$

$$t = \frac{\overline{X_1}-\overline{X_2}}{SE} = \frac{196-199}{0.9135} = -3.2841$$

|t| = 3.2841

Table value of t at $\alpha = 0.05$ for

$v = 9 + 7 - 2 = 14$ degree of freedom is 2.145

Decision : since $t_{calc} > t_{table}$

Null hypothesis is rejected

The sample cannot be considered to have been drawn from same population.

Q5] C) Use the dual simplex method to solve the following L.P.P (8)

Minimize : $\mathbf{z = 6x_1 + 3x_2 + 4x_3}$

Subject to : $\mathbf{x_1 + 6x_2 + x_3 = 10}$

$\mathbf{2x_1 + 3x_2 + x_3 = 15}$

$\mathbf{x_1, x_2, x_3 \geq 0}$

SOLUTION :-

Minimize : $z = 6x_1 + 3x_2 + 4x_3$

Subject to : $x_1 + 6x_2 + x_3 = 10$

$2x_1 + 3x_2 + x_3 = 15$

$x_1, x_2, x_3 \geq 0$

We first express the given problem using $\leq$ in the given constraints.

$x_1 + 6x_2 + x_3 \leq 10$ and $x_1 + 6x_2 + x_3 \geq 10$

$2x_1 + 3x_2 + x_3 \leq 15$ and $2x_1 + 3x_2 + x_3 \geq 15$

Multiply these equations by -1

$x_1 + 6x_2 + x_3 \leq 10$ and $-x_1 - 6x_2 - x_3 \leq -10$

$2x_1 + 3x_2 + x_3 \leq 15$ and $-2x_1 - 3x_2 - x_3 \leq -15$

Introducing stack variables,

Minimise : $z = 6x_1 + 3x_2 + 4x_3 - 0s_1 - 0s_2 - 0s_3 - 0s_4$

i.e. $z - 6x_1 - 3x_2 - 4x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = 0$

subject to : $x_1 + 6x_2 + x_3 + s_1 + 0s_2 + 0s_3 + 0s_4 = 10$

$-x_1 - 6x_2 - x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = -10$

$2x_1 + 3x_2 + x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = 15$

$$-2x_1 - 3x_2 - x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 = -15$$

Iteration no	Basic variable	Coefficients of							RHS solution
		x_1	x_2	x_3	s_1	s_2	s_3	s_4	
0	Z	-6	-3	-4	0	0	0	0	0
s_4 leaves	s_1	1	6	1	1	0	0	0	10
x_2 enters	s_2	-1	-6	-1	0	1	0	0	-10
	s_3	2	3	1	0	0	1	0	15
	s_4	-2	-3	-1	0	0	0	1	-15
Ratio		3	1	4					
1	Z	-4	0	-3	0	0	0	-1	15
s_4 leaves	s_1	-3	0	-1	1	0	0	0	-20
x_1 enters	s_2	3	0	0	0	1	0	-2	20
	s_3	0	0	0	0	0	1	1	0
	x_2	2/3	1	1/3	0	0	0	-1/3	5
Ratio		4/3		3					
2	Z	0	0	-5/3	-4/3	0	0	-11/3	125/3
	x_1	1	0	1/3	-1/3	0	0	-2/3	20/3
	s_2	0	0	0	0	1	0	0	0
	s_3	0	0	0	0	0	1	1	0
	x_2	0	0	1/2	1/2	0	0	1/9	5/9

$x_1 = \frac{20}{3};\quad x_2 = \frac{5}{9};\quad x_3 = 0$

$$\mathbf{Z = 6\left(\frac{20}{3}\right) + 3\left(\frac{5}{9}\right) + 0 \;=\; \frac{125}{3}}$$

Q6] A) Show that the matrix A satisfies Cayley- Hamilton theorem and hence find $\mathbf{A^{-1}}$ **(6)**

Where $\mathbf{A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}}$

SOLUTION :-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$$

Characteristics equation,

$\lambda^3 - (1-1-1)\lambda^2 + (-3-10-5)\lambda - 40 = 0$

$\lambda^3 - \lambda^2 - 18\lambda - 40 = 0$ .... (1)

By Cayley Hamilton theorem;

Matrix A satisfies characteristics equation

$A^3 + A^2 - 18A + 40I = 0$

$$A^3 = \begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & -1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & -1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & -1\end{bmatrix} = \begin{bmatrix}44 & 33 & 46\\ 24 & 13 & 74\\ 52 & 14 & 8\end{bmatrix}$$

$$A^2 = \begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & -1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & -1\end{bmatrix} = \begin{bmatrix}14 & 3 & 8\\ 12 & 9 & -2\\ 2 & 4 & 14\end{bmatrix}$$

$$\begin{bmatrix}44 & 33 & 46\\ 24 & 13 & 74\\ 52 & 14 & 8\end{bmatrix} + \begin{bmatrix}14 & 3 & 8\\ 12 & 9 & -2\\ 2 & 4 & 14\end{bmatrix} - \begin{bmatrix}18 & 36 & 54\\ 36 & -18 & 72\\ 52 & 14 & 8\end{bmatrix} - \begin{bmatrix}40 & 0 & 0\\ 0 & 40 & 0\\ 0 & 0 & 40\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Thus the theorem is verified.

Multiplying (i) by A^{-1}

$40A^{-1} = A^2 + A - 18I$

$A^{-1} = \frac{1}{40}(A^2 + A - 18I)$

$$A^{-1} = \frac{1}{40}\begin{bmatrix}14 & 3 & 8\\ 12 & 9 & -2\\ 2 & 4 & 14\end{bmatrix} + \begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & -1\end{bmatrix} - \begin{bmatrix}18 & 0 & 0\\ 0 & 18 & 0\\ 0 & 0 & 18\end{bmatrix}$$

$$\mathbf{A^{-1} = \frac{1}{40}\begin{bmatrix}-3 & 5 & 11\\ 14 & -10 & 2\\ 5 & 5 & -5\end{bmatrix}}$$

Q6] B) The probability Distribution of a random variable X is given by (6)

X	**-2**	**-1**	**0**	**1**	**2**	**3**
P(X=x)	**0.1**	**k**	**0.2**	**2k**	**0.3**	**k**

Find k, mean and variance.

SOLUTION :-

X	-2	-1	0	1	2	3
P(X=x)	0.1	k	0.2	2k	0.3	k

$\sum P_i = 1$

$0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$0.6 + 4k = 1$

$4k = 0.4$

$k = 0.1$

X	-2	-1	0	1	2	3
P(X=x)	0.1	0.1	0.2	0.2	0.3	0.1

$\text{mean} = E(X) = \sum p_i x_i = -0.2 - 0.1 + 0.2 + 0.6 + 0.3 = 0.8$

$E(X^2) = \sum p_i {x_i}^2 = (0.1 \times 4) + 0.1 + 0 + 0.2 + (4 \times 0.3) + (9 \times 0.1) = 0.4 + 0.1 + 0.2 + 1.2 + 0.9 = 2.8$

$\text{variance} = E(X^2) - [E(X)]^2 = 2.8 - 0.8^2 = 2.8 - 0.64 = 2.16$

Variance = 2.16

Mean = 0.8

Q6] C) Using Kuhn-Tucker conditions, solve the following NLPP (8)

Maximise $\mathbf{Z = {x_1}^2 + {x_2}^2}$

Subject to ; $\mathbf{x_1 + x_2 - 4 \leq 0}$

$\mathbf{2x_{1+}x_2 - 5 \leq 0}$

$\mathbf{x_1, x_1 \geq 0}$

SOLUTION :-

$Z = {x_1}^2 + {x_2}^2$

Subject to ; $x_1 + x_2 - 4 \leq 0$

$$2x_{1+}x_2 - 5 \leq 0$$

$$x_1, x_1 \geq 0$$

We write the problem as

$f(x_1, x_2) = x_1{}^2 + x_2{}^2$

$h_1(x_1, x_2) = \ x_1 + x_2 - 4$

$h_2(x_1, x_2) = \ 2x_1 + x_2 - 5$

The kuhn jucker conditions for maxima are:

$\frac{\partial f}{\partial x_1} - \lambda \frac{\partial h_1}{\partial x_1} - \lambda_2 \frac{\partial h_2}{\partial x_1} = 0$

$2x_1 - \lambda_1 - 2\lambda_2 = 0$ .... (1)

Also, $\frac{\partial f}{\partial x_2} - \lambda \frac{\partial h_1}{\partial x_2} - \lambda_2 \frac{\partial h_2}{\partial x_2} = 0$

$2x_2 - \lambda_1(1) - \ \lambda_2(1) = 0$ .... (2)

$\lambda_1(x_1 + x_2 - 4) = 0$ .... (3)

$\lambda_2(2x_1 + x_2 - 5) = 0$ .... (4)

$x_1 + x_2 - 4 \leq 0$ .... (5)

$2x_1 + x_2 - 5 \leq 0$ .... (6)

$x_1, x_2 > 0$ .... (7)

$\lambda_1, \lambda_2 \geq 0$ ....(8)

Case 1: $\lambda_1 = \ \lambda_2 = 0$

From 1 and 2 we get,

$2x_1 = 0 \qquad \text{and} \quad 2x_2 = 0$

$x_1 = \ x_2 = 0$

This is a trivial solution

Case 2: $\lambda_1 = 0; \quad \lambda_2 \neq 0$

To find x_1 and x_2, we get

$2x_1 = 2\lambda_2$ and $2x_2 = \lambda_2$

From (4) we get, $2x_1 + 2x_2 = 5$

$2\lambda_2 - \frac{\lambda_2}{2} = 5$, $\frac{5\lambda_2}{2} = 5$

$\lambda_2 = 2$

$x_1 = 2;\quad x_2 = 1$

But these values do not satisfy 5 and 6

Thus reject the pair

Case 3: $\lambda_1 \neq 0;\ \lambda_2 = 0$

From (1) and (2) we get $2x_1 = \lambda_1;\quad 2x_2 = \lambda_2$

$x_1 = x_2$

From (3) we get, $x_1 + x_2 = 4$

Put $x_1 = x_2$

$2x_2 = 4 \qquad x_2 = 2 \qquad x_1 = 2$

These values satisfy the equation,

$z_{max} = {x_1}^2 + {x_2}^2 = 2^2 + 2^2 = 8$

Case 4: $\lambda_1 \neq 0, \lambda_2 \neq 0$

From 3 and 4, $x_1 + x_2 = 4$ and $2x_1 + x_2 = 5$

On solving these simultaneously we get,

$x_1 = 1$ and $x_2 = 3$

Putting in equation (1) and (2)

$\lambda_1 + 2\lambda_2 = 2$ and $\lambda_1 + \lambda_2 = 6$

Solving these, $\lambda_2 = -4$ and $\lambda_1 = 10$

$\lambda_1 > 0$ and $\lambda_2 < 0$ we reject the pair

Required solution = $\mathbf{x_1 = x_2 = 2 \quad and \quad z_{max} = 8}$