DISCRETE STRUCTURES
MAY 19 (CBCS)

Q1 a) Prove using Mathematical Induction
$2+5+8+\ldots.+(3n-1)=n(3n+1)/2$ **(5)**

Solution:
Let $P(n)= 2+5+8+\ldots.+(3n-1)=n(3n+1)/2$

Step1: n=1
LHS=3x1-1
=2

RHS=1(3x1+1)/2
=2
LHS=RHS
P(n) is true for n=1.

Step2:
Let P(n) be true for n=k
$2+5+8+\ldots.+(3k-1)=k(3k+1)/2$ ……………………………….(1)

Now we have to prove that P(n) is true for n=k+1

$2+5+8+\ldots.(3k-1)+[3(k+1)-1]=(k+1)[3(k+1)+1]/2$
LHS= $2+5+8+\ldots.(3k-1)+[3(k+1)-1]$
$=k(3k+1)/2+[3(k+1)-1]$
$=(3k^2+k)/2+[3k+2]$
$=3k^2+7k+4/2$
$=(k+1)(3k+4)/2$

RHS=$(k+1)[3(k+1)+1]/2$
$=(k+1)(3k+4)/2$

LHS=RHS
P(n) is true for n=k+1

Hence from step1 and step2
By the principal of mathematical induction
$2+5+8+\ldots.+(3n-1)=n(3n+1)/2$

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Q1 b) Find the generating function for the following finite sequences

I) 1,2,3,4,……….

II) 2,2,2,2,2 **(5)**

Solution:

I) If $\{a_m\}=\{a_0,a_1,a_2,a_3\ldots.\}$ is a sequence of real numbers and x is a real variable then Ordinary generating function of the sequence is infinite sum

$$g(x)= \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots\ldots.$$

For sequence $\{a_n\}=\{1,2,3,4,\ldots\ldots.\}$

$g(x)= 1+2x+3x^2+4x^3+\ldots\ldots.$

$=(1-x)^{-2}$

$=1/(1-x)^2$

The generating function $g(x)=1/(1-x)^2$

II) If $\{a_m\}=\{a_0,a_1,a_2,a_3\ldots.\}$ is a sequence of real numbers and x is a real variable then Ordinary generating function of the sequence is infinite sum

$$g(x)= \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots\ldots.$$

For sequence $\{a_n\}=\{2,2,2,2,2\}$

$g(x)= 2+2x+2x^2+2x^3+2x^4$ which id GP with first term a=2 number of terms n=5 and common ration r=x

In GP sum of series

$$S_n = \frac{a}{r-1}(r^n - 1)$$

The generating function $g(x)=\frac{2}{x-1}(x^5-1)$

Q1 c) Let A={1,4,7,13} and R={(1,4), (4,7), (7,4),(1,13)}

Find Transitive closure using Warshall's Algorithm. **(5)**

Solution:

Relation matrix W= M_R=

$$\begin{array}{c|c} & \begin{matrix} 1 & 4 & 7 & 13 \end{matrix} \\ \begin{matrix} 1 \\ 4 \\ 7 \\ 13 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

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Step1: First we copy all 1's from W to matrix W1.

We observe W has no 1's at first column and has 1's at 4 and 13 in first row.

$\therefore$ W1 is same as W

$$W1=\begin{array}{cc} & \begin{array}{cccc}1 & 4 & 7 & 13\end{array} \\ \begin{array}{c}1\\4\\7\\13\end{array} & \begin{bmatrix}0 & 1 & 0 & 1\\0 & 0 & 1 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}\end{array}$$

Step 2: We copy all 1's from W1 to W2.
We observe that W1 has 1's at (1,7) position in second column and at(7) position in second row. So we add 1's at (1,7) and (7,7) position in W2.

$$W2=\begin{array}{cc} & \begin{array}{cccc}1 & 4 & 7 & 13\end{array} \\ \begin{array}{c}1\\4\\7\\13\end{array} & \begin{bmatrix}0 & 1 & 1 & 1\\0 & 0 & 1 & 0\\0 & 1 & 1 & 0\\0 & 0 & 0 & 0\end{bmatrix}\end{array}$$

Step 3:we copy all 1's from W2 to W3.
We observe that W2 has 1's at (1,4,7) position in third column and (4,7) position in third row
So we add 1's at (1,4), (1,7),(4,4),(4,7),(7,4)(7,7) in W3.

$$W3=\begin{array}{cc} & \begin{array}{cccc}1 & 4 & 7 & 13\end{array} \\ \begin{array}{c}1\\4\\7\\13\end{array} & \begin{bmatrix}0 & 1 & 1 & 1\\0 & 1 & 1 & 0\\0 & 1 & 1 & 0\\0 & 0 & 0 & 0\end{bmatrix}\end{array}$$

Step4: First we copy all 1's from W3 to matrix W4.

We observe W3 has 1's at (13) position in fourth column and has no 1's at fouth row.

$\therefore$ W3 is same as W4

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$$
W4= \begin{array}{c} \\ 1 \\ 4 \\ 7 \\ 13 \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 4 & 7 & 13 \end{array} \\
\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\end{array}
$$

Hence by Warshall's Algorithm

Transitive closure={(1,4)(1,7)(1,13)(4,4)(4,7)(7,4)(7,7)}

Q1 d) Let f: R O R, where f(x)=2x-1 and f^{-1}(x)=(x+1)/2

Find (f O f^{-1})(x)

Solution: **(5)**

f(x)=2x-1

f^{-1}(x)=(x+1)/2

(f O f^{-1})(x)= f(f^{-1}(x))

$=f(\frac{x+1}{2})$

$= 2(\frac{x+1}{2})-1$

=x+1-1

=x

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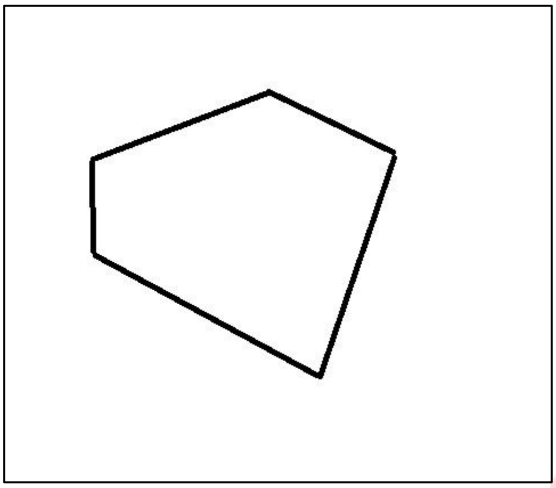
Q2 a) Define lattice. Check if the following diagram is a lattice or not. **(4)**



Solution:

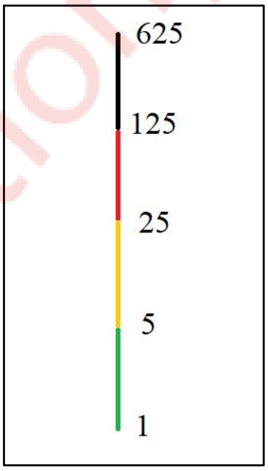
A posset (L,≤) in which every pair (a,b) of L has a LUB(least Upper Bound) and GLB (Greatest Lower bound) is called a lattice.

For eg.

Let R be a relation of divisibility. Consider the set of divisors of 625

i. e. $D_{625}=\{1,5,25,125,625\}$

The hasse diagram for D_{625} is



We observe that every pair of elements of D_{625} has a LUB and GLB.

Also each LUB and GLB ε D_{625}.

The given diagram is lattice because it has LUB and GLB.

Q2 b) Prove that set G={1,2,3,4,5,6} is a finite abelian group of order 6 w.r.t multiplication module 7.

Solution: **(8)**

X_7	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

G1:

Consider any three numbers from table

5 x (6 x 3)=5 x 4=6

(5 x 6) x 3=2 x 3=6

As 5 x (6 x 3)=(5 x 6) x 3

Hence x is associative.

G2:

From table we observe first row is same as header.

I Є G

Hence Identity of x exists.

G3:

Consider any two number from table

4 x 2 =1 and 2 x 4=1

Hence x is commutative.

G4:

Inverse of x exists.

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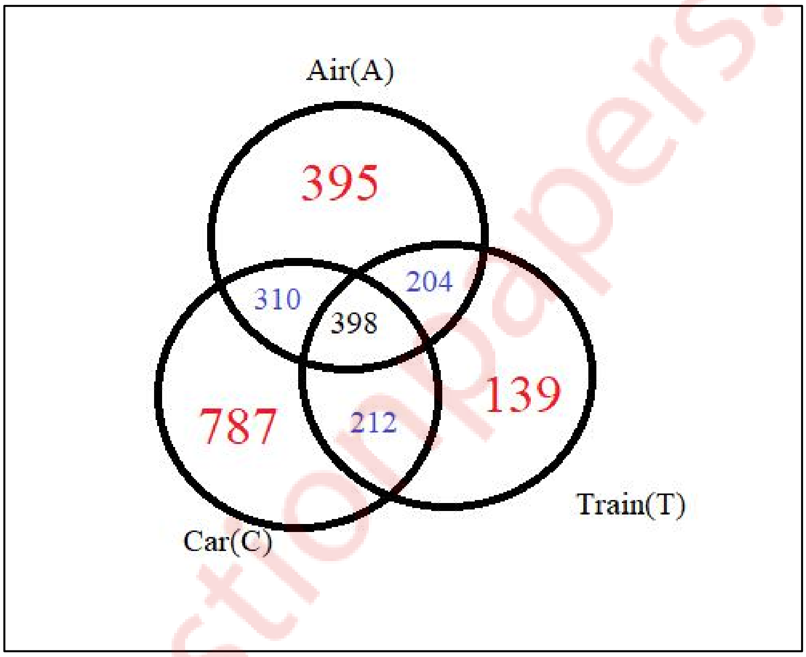
Q2 c) A travel company surveyed it's travelers, to learn how much of their travel is taken with an Airplane, a Train or a car. The following data is known; make a complete Venn Diagram with all the data. The number of people who flew was 1307. The number of people who both flew and used a train was 602. The people who used all three were 398 in number. Those who flew but didn't drive came to total 599. Those who drove but did not use train totaled 1097. There were 610 people who used both trains and cars. The number of people who used either a car or train or both was 2050. Lastly, 421 people used none of these .Find out how many people drove but used neither a train nor an airplane, and also, how many people were in the entire survey.

Solution: **(8)**



Let A be the set of people who flew
Let C be the set of people who travelled by car
Let T be the set of people who travelled by train
$N(A \cap T \cap C) = 398$

$N(A) = 1307$

$N(A \cap T) = 602$
The people who flewed and travelled by train but didn't travelled by car
= $N(A \cap T) - N(A \cap T \cap C)$
= 602-398

= 204

People who flew but didn't drive are 599.
Therefore people who only flew are 599- 204=395

$N(C) = 1097$
$N(C \cap T) = 610$
The people who travelled by car and train but didn't flew
$= N(C \cap T) - N(A \cap T \cap C)$
$= 610 - 398$
$= 212$

The people who flewed and travelled by car but didn't travelled by train are =310

Those who drove but did not use train totaled 1097
The people who travelled by only car = 1097-310=787
The people who travelled by only train= 139

The no of people who drove but used neither a train nor an airplane= **787**
The no of people in entire survey were =**2866**

Q3 a) Prove $\neg(p \vee (\neg p \wedge q))$ and $(\neg p \wedge \neg q)$ are logically equivalent by developing a series of logical equivalences.
Solution: **(4)**

$\neg(p \vee (\neg p \wedge q))$

$\neg p \wedge \neg(\neg p \wedge q)$	By Demorgan's law
$\neg p \wedge (p \vee \neg q)$	By double negation
$(\neg p \wedge p) \vee (\neg p \wedge \neg q)$	By distributive law
$F \vee (\neg p \wedge \neg q)$	$(\neg p \wedge p) = F$
$(\neg p \wedge \neg q)$	By Identity law

Hence $\neg(p \vee (\neg p \wedge q))$ and $(\neg p \wedge \neg q)$ are logically equivalent.

Q3 b) Consider the (3,5) group encoding function defined by **(8)**
e(000)=00000
e(010)=01001
e(100)=10011
e(110)=11010
e(001)=00110
e(011)=01111

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e(101)=10101
e(111)=11000
Decode the following words relative to maximum likelihood decoding functions.
i)11001 ii)01010 iii)00111

Solution:
The decoding table is as follows:

00000	00110	01001	01111	10011	10101	11010	11000
00001	00111	01000	01110	10010	10100	11011	11001
00010	00100	01011	01101	10001	10111	11000	11010
00100	00010	01101	01011	10111	10001	11110	11100
01000	01110	00001	00111	11011	11101	10010	10000
10000	10110	11001	11111	00011	00101	01010	01000
10001	10111	11000	11110	00010	00100	01011	01001
10010	10100	11011	11101	00001	00111	01000	01010

1) Encoded word= 11001
Corresponding encode word belongs to the column 01001
Therefore d(11001)=010

2) Encoded word= 01010
Corresponding encode word belongs to the column 11010
Therefore d(01010)=110

3) Encoded word= 00111
Corresponding encode word belongs to the column 00110
Therefore d(00110)=001

Q3c) Mention all the elements of set D_{36} also specify R on D_{36} as aRb if a|b. Mention Domain and range of R. Explain if the relation is equivalence relation or a Partially Ordered Relation. If it is A Partially Ordered Relation, draw its Hasse Diagram.
Solution: **(8)**

D_{36}={1,2,3,4,6,9,12,18,36}

R={(1,1)(1,2)(1,3)(1,4)(1,6)(1,9)(1,12)(1,18)(1,36)(2,2)(2,3)(2,4)(2,6)(2,9)(2,12)(2,18)(2,36)(3,3)(3,4)(3,6)(3,9)(3,12)(3,18)(3,36)(4,4)(4,6)(4,9)(4,12)(4,18)(4,36)(6,6)(6,9)(6,12)6,18)(6,36)(9,9)(9,12)(9,18)(9,36)(12,12)(12,18)(12,36)(18,18)(18,36)(36,36)}

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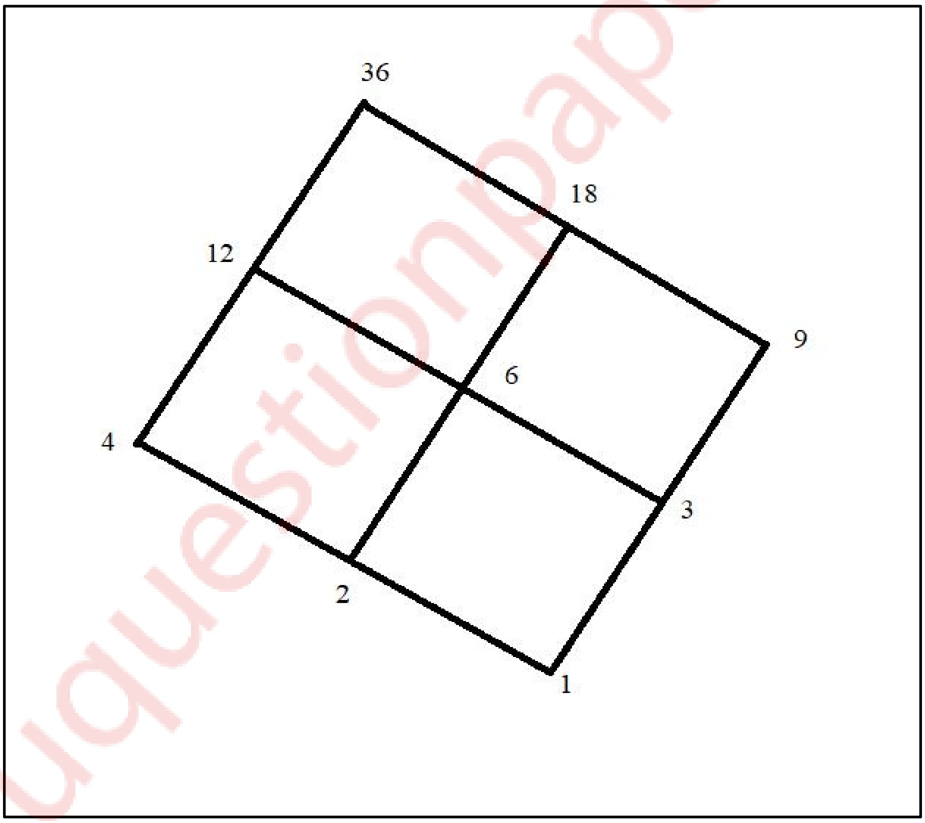
Domain of R ={1,2,3,4,6,9,12,18,36}
Range of R={1,2,3,4,6,9,12,18,36}

A relation R on a set A is called a partial order relation if it satisfies the following three properties:

- Relation R is Reflexive, i.e. aRa $\forall$ a$\in$A.
- Relation R is Antisymmetric, i.e., aRb and bRa $\Longrightarrow$a = b.
- Relation R is transitive, i.e., aRb and bRc $\Longrightarrow$aRc.

For D36 Relation R is reflexive, antisymmetric and transitive
Hence it is partial order relation.

Hasse diagram of D36 is



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Q4a) Explain Extended pigeonhole principle. How many friends must you have to guarantee that at least five of them should will have birthdays in the same month.
Solution: **(4)**

Extended pigeonhole principal

It states that if n pigeons are assigned to m pigeonholes (The number of pigeons is very large than the number of pigeonholes), then one of the pigeonholes must contain at least [(n-1)/m]+1 pigeons.

Here Number of pigeons = n = ?

No. of pigeonholes = m = 12 (months)

$\therefore$ [(n−1)/m]+1=5

[(n−1)/12]+1=5

n - 1 = 48

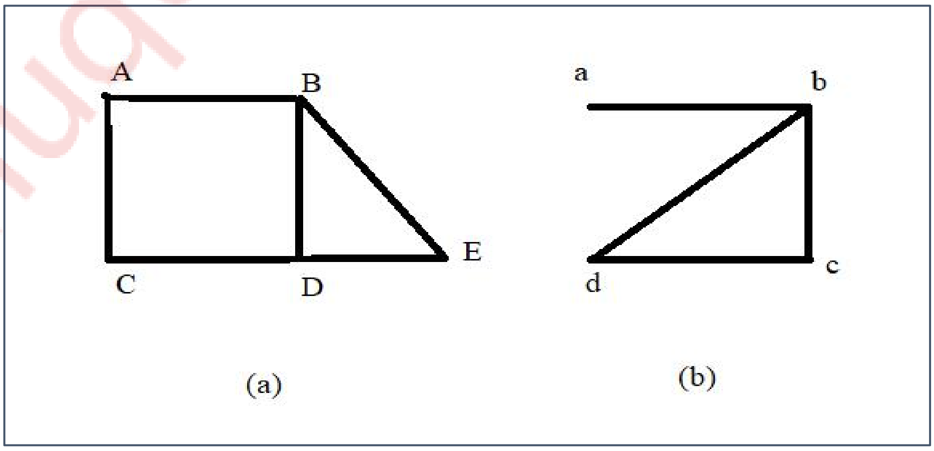
n = 49 [No. of pigeons]

$\therefore$ 49 friends should be their to guarantee that at-least five of them must have birthday in a same month of year.

Q4b) Define Euler path and Hamilton path.

I) Determine Euler cycle and path in graph shown in (a)

II) Determine Hamiltonian cycle and path in graph shown in (b) **(8)**



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Solution:

Euler path: An Euler path is a path that uses every edge of a graph exactly once.

Euler circuit: An Euler circuit is a circuit that yses edges of a graph exactly once and which starts and ends with same vertex.

Criteria for Euler cycle:

If a connected graph G has a Euler circuit,then all vertices Of G must have a even degree

In fig(a)

Since vertices B and D have odd degree.

Therefore there is no Euler cycle for fig(a).

Criteria for Euler path:

If a connected graph G has a Euler path then it must have exactly two vertices with odd degree. The two endpoits of Euler path must be the vertices with odd degree.

The Euler path- BACDBED

Hamilton path: Hamiltonian path is a graph that visits each vertex exactly once.

Hamiltonian circuit: Hamiltonian circuit is a path that visitsevery vertex exactly once and which starts and ends on the same vertex.

Criteria for Hamiltonian circuit:

The given condition are necessary but not sufficient

A) A simple graph with n vertices (n>=3) is Hamiltonian if every vertex has degree n/2 or greater.

B) A graph with n vertices (n>=3) is hamiltonian if for every pair of non-adjacent , the sum of their degrees is n or greater.

For graph (b)

Hamiltonian path= abcdb

There is no hamiltonian cycle.

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Q4c) In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

Solution: **(8)**

In a group of 6 boys and 4 girls, four children are to be selected such that at least one boy should be there.
So we can have

(four boys) or (three boys and one girl) or (two boys and two girls) or (one boy and three girls)

This combination question can be solved as

$=({}^6C_4)+({}^6C_3 * {}^4C_1)+({}^6C_2 * {}^4C_2)+({}^6C_1 * {}^4C_3)$

$=[6\times5/2\times1]+[(6\times5\times4/3\times2\times1)\times4]+[(6\times5/2\times1)(4\times3/2\times1)]+[6\times4]$

$=15+80+90+24$

=209

Q5a) Let G be a group. Prove that the identity element e is unique.

Solution: **(4)**

As the identity element $e \in G$ is defined such that $ae=a\ \forall a \in G$.

While the inverse does exist in the group and multiplication by the inverse element gives us the identity element, which assumes that the identity element is unique.

A more standard way to show this is suppose that e,f are both the identity elements of a group GG.

Then, e= e o f since f is the identity element.

=f since e is the identity element.

This shows that the identity element is indeed unique.

Q5b) A pack contains 4 blue, 2 red and 3 black pens. If 2 pens are drawn at random from the pack, Not replaced and then another pen is drawn. What is the probability of drawing 2 blue pens and 1 black pen?

(8)

Solution:

Total blue pens: 4

Total red pens: 2

Total black pens: 3

Total pens: 4+2+3= 9

Probability of drawing 2 blue pens= ${}^{4}C_{2}$ / ${}^{9}C_{2}$ =4 x 3/ 9 x 8= 1/6

After these the pens are not replaced. Therefore there are only 7 pens left.

Probability of drawing 1 black pen from 7 pens= ${}^{3}C1$ / ${}^{7}C1$ = 3/7

Probability of drawing 2 blue pen and 1 black pen= 1/6 x 3/7 = 1/14

Q5c) Let A be a set of integers, let R be a relation on AxA defined by (a,b) R (c,d) if and only if a+d=b+c.

Prove that R is an equivalent relation.

Solution: **(8)**

Relation R is defined by (a,b) R (c,d) if and only if a+d=b+c.

I) Put a=c and b=d in a+d=b+c

$\therefore$ c+d=d+c, which is true

$\therefore$ (c,d) R (c,d)

Therefore R is reflexive.

II) Let (a,b) R(c,d)

$\therefore$ a+d=b+c

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∴ b+c=a+d

∴ c+b=d+a

∴ (c,d)R(a,b)

∴ R is symmetric.

III) Let (a,b)R(c,d) and (c,d)R(e,f)

∴ a+d=b+c ……………………………….(1)

And c+f=d+c ……………………………….(2)

Adding (1) and (2)

(a+d)+(c+f)=(b+c)+(d+c)

∴ a+f=b+e

∴ (a,b)R(e,f)

Hence R is an equivalence relation.

Q6a) Define reflexive closure and symmetric closure of a relation. Also find reflexive and symmetric closure of R.

A={1,2,3,4}

B={(1,1),(1,2),(1,4),(2,4),(3,1),(3,2),(4,2),(4,3),(4,4)}

Solution: **(4)**

Reflexive closure: A relation R' is the reflexive closure of a relation R if and only if
(1) R' is reflexive,
(2) $R \subseteq R'$, and
(3) for any relation R'', if $R \subseteq R''$ and R'' is reflexive, then $R' \subseteq R''$, that is, R' is the smallest relation that satisfies (1) and (2).

Symmetric closure: A relation R' is the symmetric closure of a relation R if and only if
(1) R' is symmetric,
(2) $R \subseteq R'$, and
(3) for any relation R'', if $R \subseteq R''$, and R'' is symmetric, then $R' \subseteq R''$, that is, R' is the smallest relation that satisfies (1) and (2).

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A={1,2,3,4}

B={(1,1),(1,2),(1,4),(2,4),(3,1),(3,2),(4,2),(4,3),(4,4)}

If Δ is euality relation then Reflexive closure R1 = Δ ∪ R ……….(1)

Symmetric closure R2 = R U R^{-1} …………..(2)

Reflexive closure:

Δ={(1,1),(2,2),(3,3),(4,4)}

From (1)

Reflexive closure R1=Δ ∪ R=
{(1,1),(1,2),(1,4),(2,2),(2,4),(3,1),(3,2),(3,3),(4,2),(4,3),(4,4)}

Symmetric closure:

R^{-1}= {(1,1),(2,1),(4,1),(4,2),(1,3),(2,3),(2,4),(3,4)}

R2=R U R^{-1}= {(1,1),(1,2),(1,3),(1,4),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,1),(4,2),(4,3)}

Q6b) let H= $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Be a parity matrix. Determine the group code B^3 ---> B^6

Solution: **(8)**

Let e_H: B^m --->B^n be encoding function.

If b=$b_1b_2b_3$……b_m then

$e_H(b)=b_1b_2b_3……b_m\ x_1x_2x_3……x_r$……………(1)

Where r=n-m and

$X_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}\ldots b_m.h_{mr}$……………….(2)

Let B={0,1}

$\therefore B^3$={000,001,010,011,100,101,110,111}

Given $B^3 \dashrightarrow B^6$ and $H=\begin{vmatrix}1&0&0\\0&1&1\\1&1&1\end{vmatrix}$ $\begin{vmatrix}1&0&0\\0&1&0\\0&0&1\end{vmatrix}$

Here m=3, n=6

$h_{11}=1$, $h_{12}=0$, $h_{13}=0$, $h_{21}=0$, $h_{22}=1$, $h_{23}=1$, $h_{31}=1$, $h_{32}=1$, $h_{33}=1$

$\therefore r=n-m=6-3=3$

◆ For b=000, $b_1=0,b_2=0,b_3=0$

$\therefore$From (1), $e_H(000)=b_1b_2b_3x_1x_2x_3$

$=000x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=0.h_{1r}+ 0.h_{2r}+0.h_{3r}=0$

$\therefore$ $x_1=0$, $x_2=0$, $x_{3}=o$

$\therefore e_H(000)=000000$

◆ For b=001, $b_1=0,b_2=0,b_3=1$

$\therefore$From (1), $e_H(001)=b_1b_2b_3x_1x_2x_3$

$=001x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=0.h_{1r}+0.h_{2r}+1.h_{3r}=h_{3r}$

$\therefore x_1=h_{31}=1, x_2=h_{32}=1, x_3=h_{33}=1$

$\therefore e_H(001)=000111$

- For $b=010$, $b_1=0, b_2=1, b_3=0$

$\therefore$From (1), $e_H(010)=b_1b_2b_3x_1x_2x_3$

$=010x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+b_2.h_{2r}+b_3.h_{3r}$

$=0.h_{1r}+1.h_{2r}+0.h_{3r}=h_{2r}$

$\therefore x_1=h_{21}=0, x_2=h_{22}=1, x_3=h_{23}=1$

$\therefore e_H(010)=010011$

- For $b=011$, $b_1=0, b_2=1, b_3=1$

$\therefore$From (1), $e_H(011)=b_1b_2b_3x_1x_2x_3$

$=011x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+b_2.h_{2r}+b_3.h_{3r}$

$=0.h_{1r}+1.h_{2r}+1.h_{3r}=h_{2r}+h_{3r}$

$\therefore x_1=h_{21}+h_{31}=0+1=1, x_2=h_{22}+h_{32}=1+1=0, x_3=h_{32}+h_{33}=1+1=0$

$\therefore e_H(011)=011100$

- For $b=100$, $b_1=1, b_2=1, b_3=0$

$\therefore$From (1), $e_H(100)=b_1b_2b_3x_1x_2x_3$

$=100x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+b_2.h_{2r}+b_3.h_{3r}$

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$=1.h_{1r}+ 0.h_{2r}+0.h_{3r}=h_{1r}$

$\therefore x_1=h_{11}=1 ,x_2=h_{12}=0, x_3=h_{13}=0$

$\therefore e_H(100)=100100$

- For b=101, $b_1=1,b_2=0,b_3=1$

$\therefore$From (1), $e_H(101)=b_1b_2b_3x_1x_2x_3$

$=101x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=1.h_{1r}+ 0.h_{2r}+1.h_{3r}=h_{1r}+h_{3r}$

$\therefore x_1=h_{11}+h_{31}=1+1=0 ,x_2=h_{12}+h_{32}=0+1=1, x_3=h_{13}+h_{33}=0+1=1$

$\therefore e_H(101)=101011$

- For b=110, $b_1=1,b_2=1,b_3=0$

$\therefore$From (1), $e_H(110)=b_1b_2b_3x_1x_2x_3$

$=110x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=1.h_{1r}+ 1.h_{2r}+0.h_{3r}=h_{1r}+h_{2r}$

$\therefore x_1=h_{11}+h_{21}=1+0=1 ,x_2=h_{12}+h_{22}=0+1=1, x_3=h_{13}+h_{23}=0+1=1$

$\therefore e_H(110)=110111$

- For b=111, $b_1=1,b_2=1,b_3=1$

$\therefore$From (1), $e_H(111)=b_1b_2b_3x_1x_2x_3$

$=111x_1x_2x_3$

$\therefore$From (2), $x_r=b_1.h_{1r}+ b_2.h_{2r}+b_3.h_{3r}$

$=1.h_{1r}+ 1.h_{2r}+1.h_{3r}=h_{1r}+h_{2r}+h_{3r}$

$\therefore\ x_1=h_{11}+h_{21}+h_{31}=1+0+1=0$,

$x_2=h_{12}+h_{22}+h_{32}=0+1+1=0$,

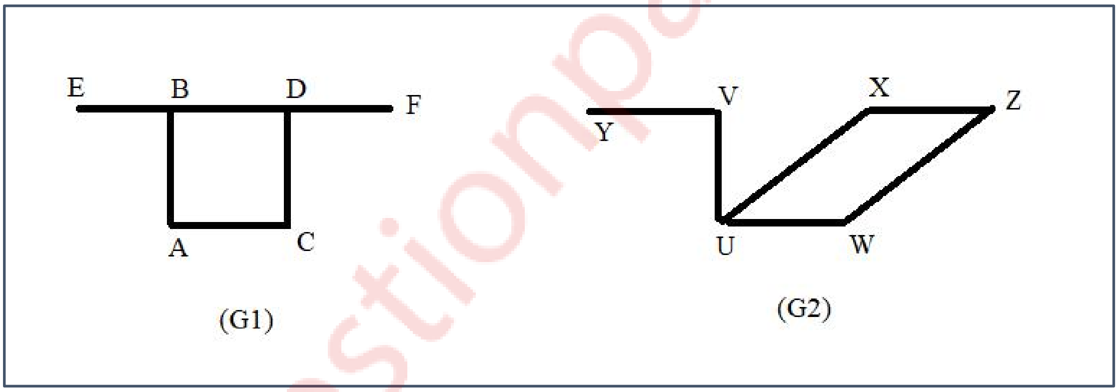
$x_3=h_{13}+h_{23}+h_{33}=0+1+1=0$

$\therefore e_H(111)=111000$

Hence the(3,6) encoding function e_H:B^3 ---> B^6 is defined as:

e(000)=000000,e(001)=001111,e(010)=010011,e(011)=010011,e(100)=100100,e(101)=101011,e(110)=110111,e(111)=111000.

Q6c) Determine if following graphs G1 and G2 are isomorphic or not. (8)



Solution:

Two graphs G(V,E) and H(V',E') isomorphic if

1) There is one to one correspondence f from V to V' such that f(V1)=V1' and f(V2)=V2' for every (v1, v2)Є V and (v1', v2')Є V'
2) G and H should have equal no. Of edges.
3) G and H should have equal no. Of vertices.
4) G and H should have same degree of vertices
5) Adjacency property is observed in each vertex.

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From the above two graphs:

Graph G1		
Number of vertices	6	
Number of Edges	6	
Vertex	**Degree of vertex**	**Adjacent vertices**
A	1	B(3)
B	3	A(1), C(3), E(2)
C	3	B(3), D(1), F(2)
D	1	C(3)
E	2	B(3), F(2)
F	2	E(3),C(2)

Graph G2		
Number of vertices	6	
Number of Edges	6	
Vertex	**Degree of vertex**	**Adjacent vertices**
Y	1	V(2)
V	2	Y(1), U(3)
X	2	Z(2), U(3)
Z	2	X(2),W(2)
U	3	V(2), X(2),W(2)
W	2	U(3),Z(2)

We observe that,

There are equal no of edges and vertices for both graph.

Graph G1 has 2 vertices with degree 1, two vertices with degree 2 and two vertices with degree 2.

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Graph G2 has one vertices with degree 1, four vertices with degree 2 and one vertex with degree 3.

Therefore, the degree in two graphs are not equal.

Hence the two graphs are not isomorphic.

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