Q.P Code: 77261

DISCRETE STRUCTURES

DEC 19 (CBCS)

Q1 a) Prove using Mathematical Induction

$1^2+2^2+3^2+\ldots\ldots n^2=n(n+1)(2n+1)/6$ **(5)**

Solution:

Let $P(n)= 1^2+2^2+3^2+\ldots\ldots n^2=n(n+1)(2n+1)/6$

Step1: n=1

LHS=1^2

=1

RHS=1(1+1)(2x1+1)/6

=1(2)(3)/6

= 1

LHS=RHS

P(n) is true for n=1.

Step2:

Let P(n) be true for n=k

$1^2+2^2+3^2+\ldots\ldots k^2=k(k+1)(2k+1)/6$ ……………………………(1)

Now we have to prove that P(n) is true for n=k+1

$1^2+2^2+3^2+\ldots\ldots(k+1)^2=(k+1)(k+1+1)[2(k+1)+1]/6$

LHS= $1^2+2^2+3^2+\ldots\ldots k^2+(k+1)^2$

= $k(k+1)(2k+1)/6 +(k+1)^2$

$=(k+1)[k(2k+1)/6 + (k+1)]$

$=1/6\ (k+1)(2k^2+k+6k+6)$

$=1/6\ (k+1)(2k^2+7k+6)$

$=1/6\ (k+1)\ (k+2)\ (2k+3)$

RHS$=(k+1)(k+1+1)[2(k+1)+1]/6$

$=1/6\ (k+1)(k+2)(2k+3)$

LHS=RHS

P(n) is true for n=k+1

Hence from step1 and step2

By the principal of mathematical induction

$1^2+2^2+3^2+\ldots\ldots n^2=n(n+1)(2n+1)/6$

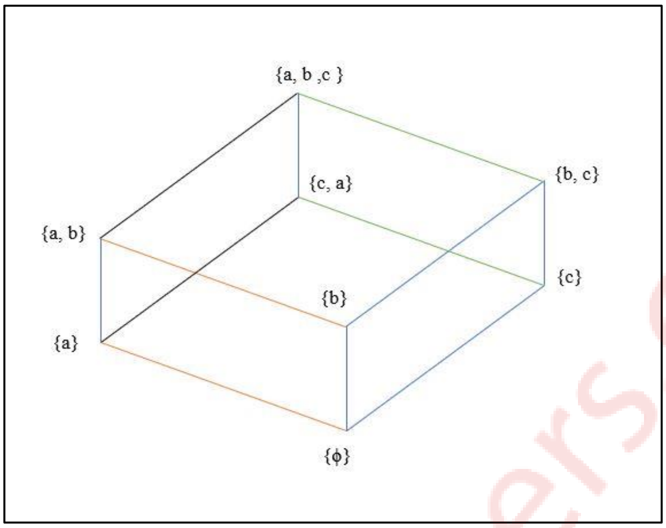
Q1 b) Let A={a, b, c } . Draw Hasse Diagram for {p(A), ⊆} **(5)**

Solution:

A= {a, b, c}

P(A)= {ϕ, {a}, {b}, {c}, {a,b},{b,c},{c,a},{a,b,c} }

Hasse Diagram is as follows:

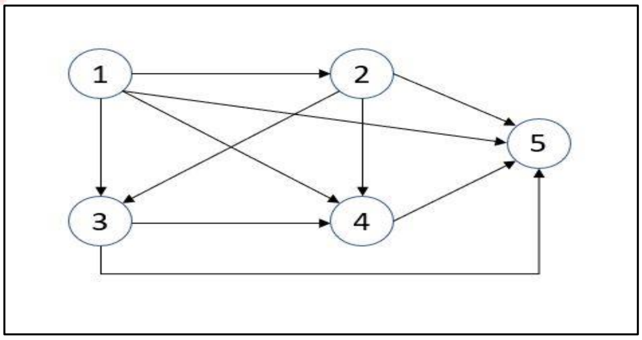


Q1 c)**Let A={1,2,3,4,5}. A relation R is defined on A as aRb iff a<b. Compute R^2 and R ∞** **(5)**

Solution:

R={(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)}

Diagraph of R is as shown below:



R^2:

$1R^23$ since 1R2 and 2R3

$1R^24$ since 1R2 and 2 R 4

$1R^25$ since 1R2 and 2R5

$2R^24$ since 2R3 and 3R4

$2R^25$ since 2R4 and 4R5

$3R^25$ since 3R4 and 4R5

R^2={(1,3),(1,4),(1,5),(2,4),(2,5),(3,5)}

R∞={(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)}

Q1 d) Let f: R O R, where f(x)=2x-1 and f^{-1}(x)=(x+1)/2

Find (f O f^{-1})(x)

Solution: **(5)**

f(x)=2x-1

f^{-1}(x)=(x+1)/2

(f O f^{-1})(x)= f(f^{-1}(x))

$=f(\frac{x+1}{2}$

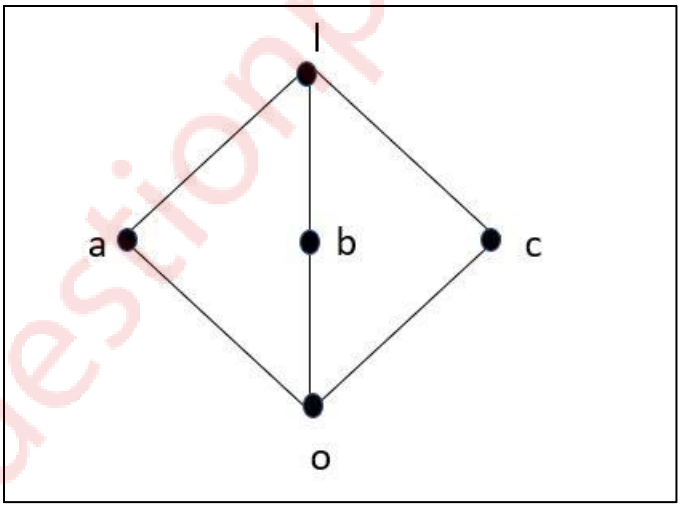
$$= 2\left(\frac{x+1}{2}\right)-1$$

$$=x+1-1$$

$$\mathbf{=x}$$

$(f \circ f^{-1})(x)=x$

Q2 a) Define Distributive lattice. Check if the following diagram is a distributive lattice or Not. (4)



Solution:

A lattice L is called distributive if for any elements a, b and c in L we have the following distributive properties

1. $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

2. $a \vee (b \wedge c)=(a \vee b) \wedge (a \vee c)$

From given figure :

$a \wedge (b \vee c) = a \wedge 1 = a$

While,

$(a \wedge b) \vee (a \wedge c)= o \vee o = o$

Therefore given figure is **non-distributive.**

Q2 b) Prove that set G={1,2,3,4,5,6} is a finite abelian group of order 6 w.r.t multiplication module 7.

Solution: **(8)**

X_7	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

G1:

Consider any three numbers from table

$5 \times (6 \times 3)=5 \times 4=6$

(5 x 6) x 3=2 x 3=6

As 5 x (6 x 3)=(5 x 6) x 3

Hence x is associative.

G2:

From table we observe first row is same as header.

I €G

Hence Identity of x exists.

G3:

Consider any two number from table

4 x 2 =1 and 2 x 4=1

Hence x is commutative.

G4:

Inverse of x exists.

Q2 c) Find the number of positive integers not exceeding 100 that are not divisible by 5 or 7. Also draw corresponding venn diagram. (8)

Solution:

Let

A: All positive integers not exceeding 100

A1: Divisible by 5

A2:Divisible by 7

There are 100 integers not exceeding 100

|A|=100

There are 100 integers not exceeding 100, while a number divisible by5 is every 5^{th} element in the lost of positive integers. Use the division rule:

|A1|= $\frac{|A|}{d}$ =100/5=20

Similarly we obtain for numbers divisible by 7 (round down)

|A2|= $\frac{|A|}{d}$ =100/7=14

Numbers divisible by 5 and 7 are divisible by 35(round down)

|A1 U A2|= $\frac{|A|}{d}$ =100/35=2

By principal of inclusion-exclusion

|A1 U A2|= |A1|+ |A2|- |A1 ∩ A2|

= 20+14-2

= 32

Thus 32 of the 100 integers are divisible by 5 or 7 , then the number of integers not divisible by 5 or 7 are

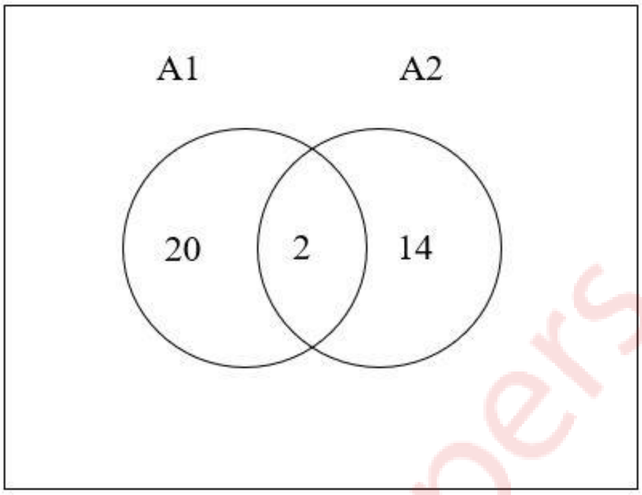
$|(A1 \cup A2)^C|$=|A|-|A1 U A2|

= 100 - 32

= 68

Thus, there are 68 integers not divisible by 5 or 7.

Venn Diagram:



Q3 a) Construct Truth Table and check if the following statement is tautology.

$(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$

Solution: **(4)**

P	Q	$((P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P))$
F	F	**T**
F	T	**T**
T	F	**T**
T	T	**T**

Hence the given statement is tautology.

Q3 b) Consider the (2,5) group encoding function defined by **(8)**

e(00)=00000

e(01)=01110

e(10)=10101

e(11)=11011

Decode the following words relative to maximum likelihood decoding functions.

i)11110 ii)10011 iii)10100

Solution:

The decoding table is as follows:

00000	01110	10101	11011
00001	01111	10100	11010
00010	01100	10111	11001
00100	01010	10001	11111
01000	00110	11101	10011
10000	11110	00101	01011

1) Encoded word= 11110

Corresponding encode word belongs to the column 01110

Therefore d(11110)=01

2) Encoded word= 10011

Corresponding encode word belongs to the column 11011

Therefore d(01010)=11

3) Encoded word= 10100

Corresponding encode word belongs to the column 10101

Therefore d(00110)=10

Q3c)How many four digits can be formed out of digits 1,2,3,5,7,8,9 if no digits repeated twice? How many of these will be greater than 3000?

Solution: **(8)**

We have to make 4 digit number without repetition using 1,2,3,5,7,8,9

For this we have to fill 4 spaces (_ _ _ _) with required numbers.

1^{st} space can be filled in 7 ways. (7 _ _ _)

2^{nd} space can be filled in 6 ways because we already used one digit in previous space so only 6 digits are remaining now. (7 6 _ _)

Similarly 3^{rd} and 4^{th} space can be filled in 5 and 4 respectively. (7 6 5 4)

So the no of four digits can be formed out of 1,2,3,5,7,8,9 =7*6*5*4

=840 digits

The four digit number greater than 3000 are:

The first place can have number 3,5,7,8,9 i.e 5 digits. 1^{st} space can be filled in 5 ways.

(5 _ _ _)

2^{nd} space can be filled in 6 ways because we already used one digit in previous space so only 6 digits are remaining now. (5 6 _ _)

Similarly 3^{rd} and 4^{th} space can be filled in 5 and 4 respectively. (5 6 5 4)

So the no of four digits can be formed out of 1,2,3,5,7,8,9 which are greater than

3000 are =5*6*5*4

=600 digits

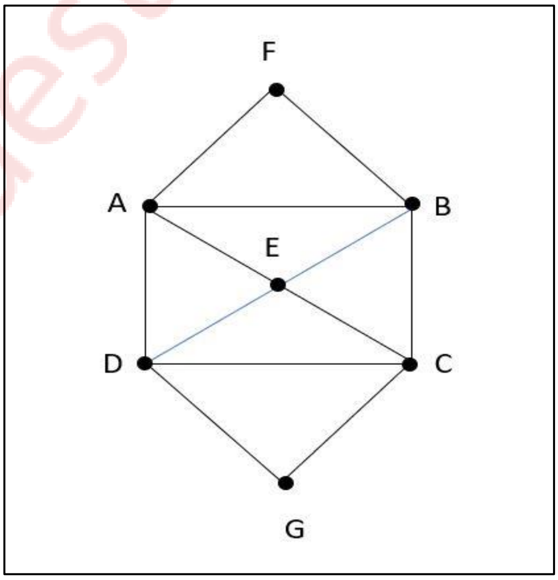
Q4a) A bag contains 10 red marbles, 10 white marbles and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color?

Use pigeonhole Principle.

Solution: **(4)**

Apply pigeonhole principle.
No. of colors (pigeonholes) n = 3
No. of marbles (pigeons) K+1 = 4
Therefore the minimum no. of marbles required = Kn+1
By simplifying we get Kn+1 = 10.
Verification: ceil[Average] is [Kn+1/n] = 4
[Kn+1/3] = 4
Kn+1 = 10
i.e., 3 red + 3 white + 3 blue + 1(red or white or blue) = 10

Q4b) Define Euler path, Euler circuit, Hamiltonian path and Hamiltonian circuit. Determine if following diagram has Euler path, Euler circuit, Hamiltonian path and Hamiltonian circuit and state the path/circuit. **(8)**



Solution:

Euler path: An Euler path is a path that uses every edge of a graph exactly once.

Euler circuit: An Euler circuit is a circuit that uses edges of a graph exactly once and which starts and ends with same vertex.

Criteria for Euler cycle:

If a connected graph G has a Euler circuit, then all vertices Of G must have a even degree

Criteria for Euler path:

If a connected graph G has a Euler path then it must have exactly two vertices with odd degree. The two endpoints of Euler path must be the vertices with odd degree.

In above graph all the vertex has even degree.

Hence there is no Euler path but it has Euler circuit.

Euler circuit: **BCEADGCDEBAFB**

Hamilton path: Hamiltonian path is a graph that visits each vertex exactly once.

Hamiltonian path: **FABEDCG**

Hamiltonian circuit: Hamiltonian circuit is a path that visits every vertex exactly once and which starts and ends on the same vertex.

Criteria for Hamiltonian circuit:

The given condition are necessary but not sufficient

A) A simple graph with n vertices (n>=3) is Hamiltonian if every vertex has degree n/2 or greater.

B) A graph with n vertices (n>=3) is Hamiltonian if for every pair of non-adjacent , the sum of their degrees is n or greater.

Hamiltonian circuit: **FADGCEBF**

Q4c) In how many ways a committee of three faculty members and 2 students can be formed from 7 faculty members and 8 students. (8)

Solution:

A committee of 3 faculty and 2 students need to be formed.

Available faculty and students are 7 and 8 respectively.

Out of 7 faculty members 3 faculty members can be chosen in 7C3 ways.

Out of 8 students 2 students can be chosen in 8C2 ways.

Total number of ways of forming a committee =(7C3) * (8C2)

=(7 * 6 * 5 / 1 * 2 * 3) x (8 * 7 / 1 * 2)

=980 ways.

We can form a committee of three faculty members and 2 students from 7 faculty and 8 students in 980 ways.

Q5a)Let Z_n denote the set of integers {0,1,2,…….n-1}. Let Θ be a binary operation on Z_n such that a Θ b=reminder of ab divided by n

I) Construct table for the operation Θ for n=4

II) Show that (Z_n,Θ) is a semi group for any n

Solution: (4)

i)The table for the operation ʘ for n=4

Θ	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

ii) The set Z_n is closed under the operation Ꙩ because for any

a,b € Z_n , a Ꙩ b € Z_n ……………………..1)

Now check for associativity for any a,b,c€ Z_n

(a $Ꙩ_4$b) $Ꙩ_4$c = a $Ꙩ_4$(b $Ꙩ_4$c)

Let a=1,b=2,c=3

(1 $Ꙩ_4$2) $Ꙩ_4$3 = 1 $Ꙩ_4$(2 $Ꙩ_4$3)

2 $Ꙩ_4$3=1 $Ꙩ_4$2

2=2

∴ Ꙩ is an associative operation. ……………………….2)

From 1) and 2) we conclude that (Z_n,Ꙩ) is a semigroup for any 'n'.

Q5b)Find Transitive Closure of R represented by M_R as follows Using Warshall's algorithm set {a,b,c,d}

$$M_R = \begin{vmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

Solution: **(8)**

$$W = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Step1: First we copy all 1's from W to matrix W1.

We observe W has 1's at b and c position in first column and has 1's at b and d in first row.

So add 1's at (b,b),(b,d),(c,b),(c,d)

$\therefore$W1 is same as W

$$W1= \begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Step 2: We copy all 1's from W1 to W2.

We observe that W1 has 1's at (a,b,c) position in second column and at(a,b,c,d) position in second row. So we add 1's at (a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d),(c,a),(c,b),(c,c),(c,d) position in W2.

$$W2= \begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Step 3:we copy all 1's from W2 to W3.

We observe that W2 has 1's at (a,b,c) position in third column and (a,b,c,d) position in third row

So W2 and W3 are same.

$$W3=\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Step4: First we copy all 1's from W3 to matrix W4.

We observe W3 has 1's at (a,b,c) position in fourth column and has no 1's at fouth row.

$\therefore$W3 is same as W4

$$W4=\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Hence by Warshall's Algorithm

Transitive closure={(a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d),(c,a),(c,b),(c,c),(c,d)}

Q5c) Let A={1,2,3,4,5} and let

R={(1,1),(1,3),(1,4),(2,2),(2,5),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4),(5,2),(5,5)}. Check if R is a equivalence relation. Justify your answer . Find equivalence classes of A.

Solution: **(8)**

Equivalence relation: A relation R on set A is called equivalence relation if it is reflexive, symmetric and transitive.

A={1,2,3,4,5}

R={(1,1),(1,3),(1,4),(2,2),(2,5),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4),(5,2),(5,5)}

R is reflexive since (1,1),(2,2),(3,3),(4,4),(5,5) € R.

R is symmetric.

R is transitive.

Hence given relation is equivalence relation.

The equivalence classes of elements A are:

[1]={1,3,4}

[2]={2,5}

[3]={1,3,4}

[4]={1,3,4}

[5]={2,5}

Q6a)How many vertices are necessary to construct a graph with exactly 6 edges in which each vertex is of degree 2.

Solution: **(4)**

Handshaking Lemma: The sum of the degrees of all vertices of any graph is equal to twice number of edges.

Here, number of edges (e) =6

Degree of each vertex= d(v) =2

As per the lemma $\sum_{i=1}^{n} d(V\text{i})$ =2e

$\therefore$ n x 2= 2x6

$\therefore$ n=6

Hence, there are 6 vertices in the graph.

Q6b)What is the solution of the recurrence relation a_n= -a_{n-1} +4a_{n-2} + 4a_{n-3} with a_0=8,a_1=6 and a_2=26?

Solution: **(8)**

Since it is a linear homogeneous recurrence, we find the characteristic equation of:

$$r^3 + r^2 - 4r - 4 = 0$$

which we can rewrite as:

$$r^3 + -4r + r^2 - 4 = r(r^2 - 4) + (r^2 - 4) = 0$$

which factors as $(r + 1)(r - 2)(r + 2)$.

From our theorem, we know that

$$an = \alpha 1(-1)^n + \alpha 2(-2)^n + \alpha\, 3\,(2^n) \text{ is a solution.}$$

We need to use the initial conditions to solve this.

$$a0 = \alpha 1 + \alpha 2 + \alpha 3 = 8 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

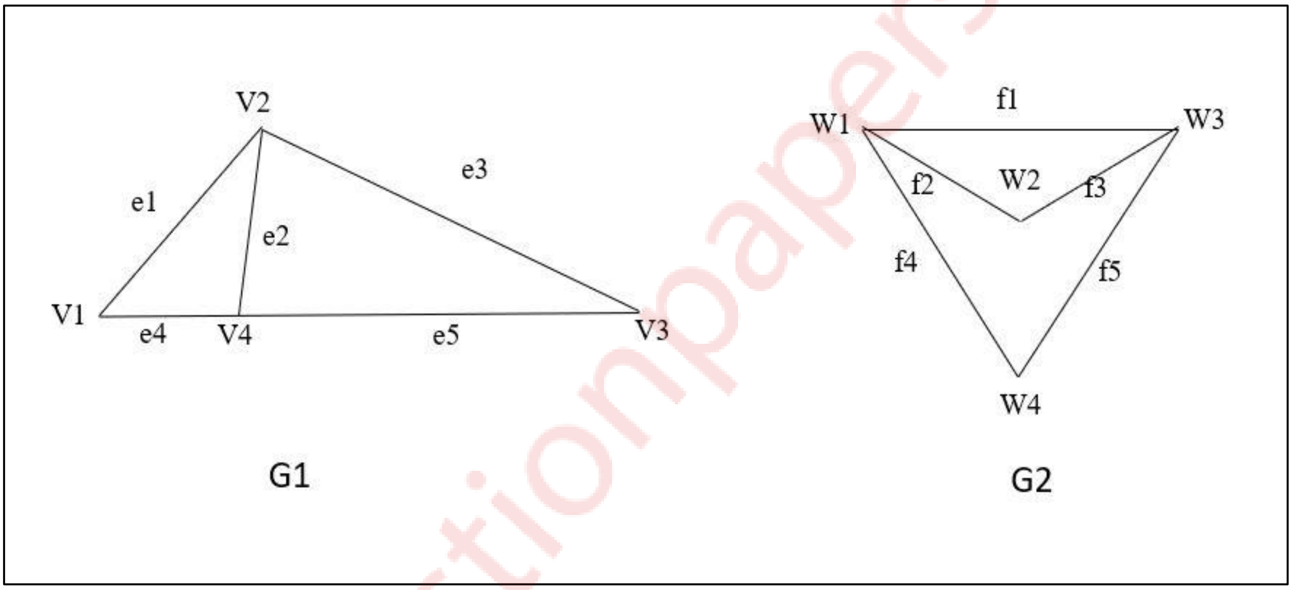
$$a1 = -\alpha 1 - 2\alpha 2 + 2\alpha 3 = 6 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

$$a2 = \alpha 1 + 4\alpha 2 + 4\alpha 3 = 26 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)}$$

After multiplying the first equation by 4 and then subtracting the third equation, we get $\alpha 1 = 2$. Substituting and combine the second and third equations, we get $\alpha 2 = 1$, and then $\alpha 3 = 5$.

Thus the closed form of the recurrence is: an = $2 \quad (-1)^n + (-2)^n + 5 \times (2^n)$.

Q6c) Determine if following graphs G1 and G2 are isomorphic or not. (8)



Solution:

Two graphs G(V,E) and H(V',E') isomorphic if

1) There is one to one correspondence f from V to V' such that f(V1)=V1' and f(V2)=V2' for every (v1, v2)€V and (v1', v2')€ V'
2) G and H should have equal no. Of edges.
3) G and H should have equal no. Of vertices.
4) G and H should have same degree of vertices
5) Adjacency property is observed in each vertex.

From the above two graphs:

Graph G1		
Number of vertices	4	
Number of Edges	5	
Vertex	**Degree of vertex**	**Adjacent vertices**
V1	2	V2(3),V4(3)
V2	3	V1(2),V4(3),V3(2)
V3	2	V2(3),V4(3)
V4	3	V1(2),V2(3),V3(2)

Graph G2		
Number of vertices	4	
Number of Edges	5	
Vertex	**Degree of vertex**	**Adjacent vertices**
W1	3	W2(2),W3(3),W4(2)
W2	2	W1(3),W3(3)
W3	3	W1(3),W2(2),W4(2)
W4	2	W1(3),W3(3)

We observe that,

There are equal no of edges and vertices for both graph.

Graph G1 has 2 vertices with degree 2, two vertices with degree 3.

Graph G2 has 2 vertices with degree 2, two vertices with degree 3.

Adjacency property is observed in each vertex.

Hence the two graphs are isomorphic.
