COMPUTER ENGINEERING

DISCRETE MATHEMATICS

(CBCGS - DEC 2017 SEM 3)

Q1.a) Prove that 1.1! + 2.2! + 3.3! + … + n.n! = (n+1)! -1 where n is a positive integer. [5]

Solution:-

Let $p(n)$ = 1.1! + 2.2! + 3.3! + -- + n.n!

Let us prove $p(1)$ is true;

$n = 1$; => LHS : 1.1! = 1

RHS : (n+1)! = -1 => (2)! – 1 => 2-1 =1

Let us assume $p(k)$ is true

1.1! + 2.2! + 3.3! + … + k.k! = (k+1)! -1 ……………………… (1)

LHS :- 1.1! + 2.2! + 3.3! + … + k.k! + (k+1)(k+1)!

=> (k+1)! – 1 + (k+1)(k+1)!

=> (k+1)! + (k+1)(k+1)! - 1

=> (k+1)![1+k+1] – 1

=> (k+1)![k+2] – 1

=> (k+2)! – 1

= RHS

Thus the result is proved.

Q1.b) Let A={a,b,c}. Show that $\{P(A), \subseteq\}$ is a poset and draw its Hasse diagram. [5]

Solution:-

Set contained belongs is always a partial order since for any subset B of A; B is a subset of B is reflexive.

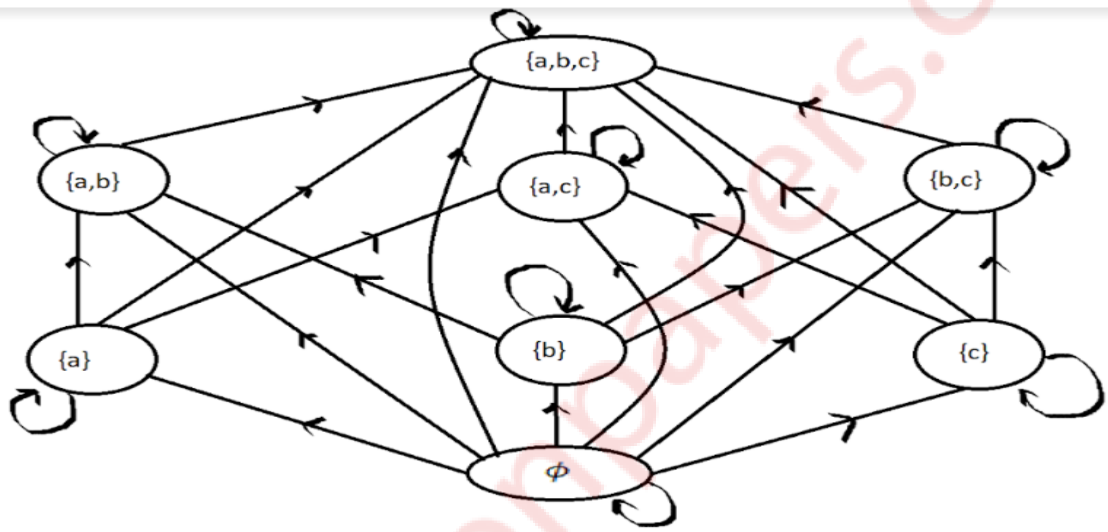
If B ⊆ C and C ⊆ => B = C ; so ⊆ is anti symmetric

If $B \subseteq C$, $C \subseteq D$ then $B \subseteq D$. So $\subseteq$ is transitive

Partial order relation of set containment on set P(A) is as follows:-

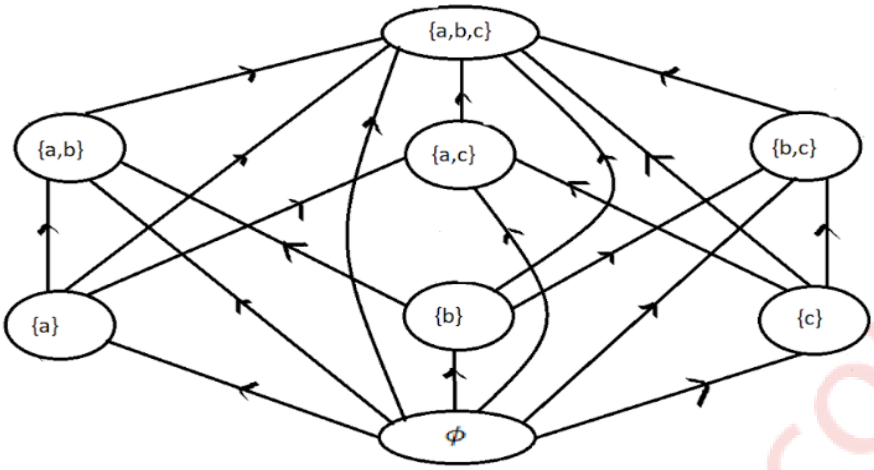
R = {{φ},{φ}}; {{φ},{a}} ; {{φ},{b}}; {{φ},{c}}; {{φ},{a,b}}; {{φ},{a,c}}; {{φ},{b,c}}
{{φ},{a,b,c}};{{a},{a}}; {{a},{a,b}}; {{a},{a,c}}; {{a},{a,b,c}};{{b},{b}};{{b},{a,b}}
{{b},{b,c}};{{b},{a,b,c}};{{c},{c}};{{c},{a,c}}; {{c},{b,c}};{{c},{a,b,c}};{{a,b},{a,b,c}}
{{b,c},{a,b,c}};{{a,b},{a,b}};{{a,c},{a,c}};{{b,c},{b,c}};{{a,b,c},{a,b,c}}

Diagraph:-



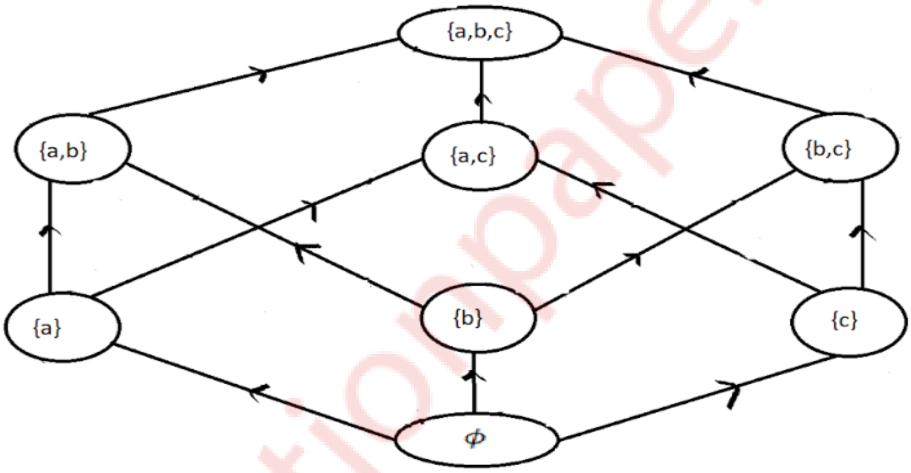
	{φ}	{a}	{b}	{c}	{a,b}	{a,c}	{b,c}	{a,b,c}
{φ}	1	1	1	1	1	1	1	1
{a}	0	1	0	0	1	1	0	1
{b}	0	0	1	0	1	0	1	1
{c}	0	0	0	1	0	1	1	1
{a,b}	0	0	0	0	1	0	0	1
{a,c}	0	0	0	0	0	1	0	1
{b,c}	0	0	0	0	0	0	1	1
{a,b,c}	0	0	0	0	0	0	0	1

Step 1:- remove loops:



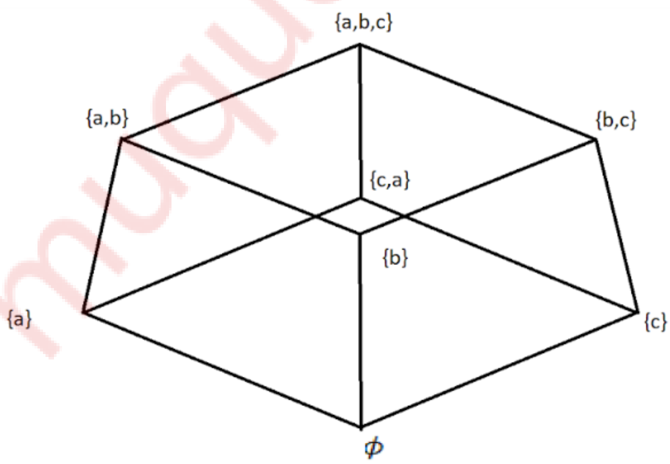
Remove transitive edges:-

{{φ},{a,b}},{{φ},{a,c}},{{φ},{b,c}},{{φ},{a,b,c}},{{a},{a,b,c}},{{b},{a,b,c}},{{c},{a,b,c}}



All edges are pointing upwards. Now replace circles by dots and remove arrows from edges.

Hasse Diagram:-



Q1.c) Explain the following terms:

[5]

i. **Lattice**

ii. **Poset**

iii. **Normal Subgroup**

iv. **Group**

v. **Planar Graph**

Solution:-

1. **<u>Lattice</u>** :- It is a poset (L, ≤) in which every subset {a,b} consisting of two elements has a least upper bound and a greatest lower bound. We denote LUB({a,b}) by a v b and call it the join of a,b. similarly we denote GLB ({a,b}) by a ∧ b and call it the meet of a and b.

2. **<u>Poset:-</u>** A relation R on a set A is called partial order if R is reflexive, anti symmetric and transitive poset. The set A together with the partial order R is called a partial order set or simply a poset.

3. **<u>Normal Subgroup :-</u>** A subgroup μ of G is said to be a normal subgroup of G if for every a ∈ G , aH = Ha. A subgroup of an Abelian group is normal.

4. **<u>Group :</u>**- Let (A,*) be an algebraic system where * is a group if the following conditions are satisfied.

    1. * is closed operation.
    2. * is an associative operation
    3. There is an identity operation.
    4. Every element in A has a left inverse.

Because of associativity ,a left inverse of an element is also a right inverse of the element in a group.

5. **Planar Graph:-** A graphic is said to be planar if it can be drawn on a plane in such a way that no edges cross one another except of course at common vertices.

Q1.d) Comment whether the function f is one to one or onto. Consider the function [5]

Solution:-

f:N⟶N where N is set of natural numbers including zero.

$f(j) = j^2 + 2$

Solution:-

$f(j) = j^2 + 2$

N = {0,1,2,3 ……….}

$f(0) = 2$

$f(1) = 3$

$f(2) = 6$

$f(3) = 11$

$f(4) = 18$

For every number n ∈ N we can find another n. so the given function is one to one.

But not every element of n ∈ N is image of some element n. so the given function is not onto.

Q2.a) Find the number of ways a person can be distributed Rs 601 as pocket money to his three sons, so that no son should receive more than the combined total of the other two. (Assume no fraction of a rupee is allowed)

Solution:- [6]

Let A,B and C be the 3 sons and a,b and c be the money given to them respectively. By the given conditions;0

$a \le b + c$, $a + b + c = 601c=601$

-a

$a = 300$

Similarly, we can deduce that

$b \le 30000$and $c \le 300$

So we have,

$a \le 300$, $b \le 300$, $c \le 300300$and $a + b + c = 601$

The corresponding multinomial is

$\left(1+x+x^2+x^3+ \ldots\ldots + x^{300}\right)^3$

The total number of distribution is the coefficient of x^{601} in the expansion of $\left(1+x+x^2+x^3+ \ldots\ldots + x^{300}\right)^3$

$$\left(1+x+x^2+x^3+ \ldots\ldots + x^{300}\right)^3 = \left(\frac{x^{301}-1}{x-1}\right)^3 = -(x^{301}-1)^3(1-x)^{-3}$$

$$\left(1+x+x^2+x^3+ \ldots\ldots + x^{300}\right)^3 = -\left(x^{903} - 3x^{602} + 3x^{301} - 1\right)\times\left(1+\binom{3}{1}x+\binom{4}{2}x^2+\ldots..+\binom{603}{601}x^{601}\right)$$

Hence the coefficient of x^{601} in the above expression is

$$\binom{603}{601} - 3\times\binom{302}{300} = 45150$$

Q2.b) Let A = $\{a_1,a_2,a_3,a_4,a_5\}$ and let R be a relation on A whose matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find M_R * by Warshall's algorithm. **[6]**

Solution:-

A = $\{a_1 , a_2, a_3 , a_4, a_5 \}$

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$M_R = W_0$

$$W_0 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Computing W_1 :

C_1 = 1 is present at 1,4

R_1 = 1 is present at 1,4

Put 1 in (1,1);(1,4);(4,1);(4,4)

$$W_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Computing W_1

C_2 = 1 is present at 2,5

R_2 = 1 is present at 2

Put 1 in (2,2) and (5,2)

$$W_2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Computing W_3

C_3 = 1 is present at no position

R_3 = 1 is present at 4,5

No new ordered pair , therefore $W_3 = W_2$

$$W_3 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Computing W_4

C_4 = 1 is present at 1,3,4

R_4 = 1 is present at 1,4

Put 1 in (1,1);(1,4);(3,1);(3,4);(4,1);(4,4)

$$W_4 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Computing W_5

C_5 = 1 is present at 3,5

R_5 = 1 is present at 2,5

Put 1 in (3,2);(3,5);(5,2);(5,5)

$$W_5 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$M_R^{\infty} = W_5$

Transitive closure = $\{(a_1,a_1),(a_1,a_4),(a_2,a_2),(a_3,a_1),(a_3,a_2),(a_3,a_4),(a_3,a_5),(a_4,a_4),(a_5,a_2),(a_5,a_5)\}$

Q2.c) Find the complete solution of the recurrence relation : [4]

$a_n + 2a_{n-1} = n + 3$ for n≥1 and with $a_0 = 3$

Solution:-

$a_0 = 3$

Recurrence relation :- $a_n + 2a_{n-1} = n + 3$

The characteristics equation is:-

$\alpha + 2 = 0$

$\alpha = -2$

Homogenous solution is :

$a_n^{(n)} = A_1(-2)^n$

For particular solution:

RHS is linear polynomial, the particular solution will be of the form $P_0 + P_1^n$

$a_n = p_0 + p_1 n$

$a_{n-1} = p_0 + p_1(n-1)$

Substituting these values in given equation

$P_0 + P_1 n + 2[P_0 + P_1(n-1)] = n + 3$

$P_0 + P_1 n + 2P_0 + 2P_1 n - 2P_1 = n + 3$

$(3P_0 - 2P_1) + 3P_1 n = n + 3$

Comparing coefficient on both sides

$3P_0 - 2P_1 = 3$; $3P_1 = 1$

$P_1 = \frac{1}{3}$

$3P_0 - \frac{2}{3} = 3$

$3P_0 = \frac{11}{3}$

$P_0 = \frac{11}{9}$

Thus the general solution is:

$a_n = a_n^{(n)} + a_n^{(p)}$

$a_n = A_1(-2)^n + \frac{11}{9} + \frac{1}{3}$

Using initial condition ; $a_0 = 3$

$$a_n = 1.78(-2)^n + \frac{11}{9} + \frac{1}{3}n$$

Q2.d) Let f:R→R defined as $f(x) = x^3$ and g:R→R defined as $g(x) = 4x^2 + 1$
Find out gof,fog,f^2,g^2 **[4]**

Solution:-

1. gof = g(f(x)) = g(x^2) = $4(x^3)^2 + 1 = 4x^6 + 1$

2. fog = f(g(x)) = f($4x^2 + 1$) = $(4x^2+1)^3$

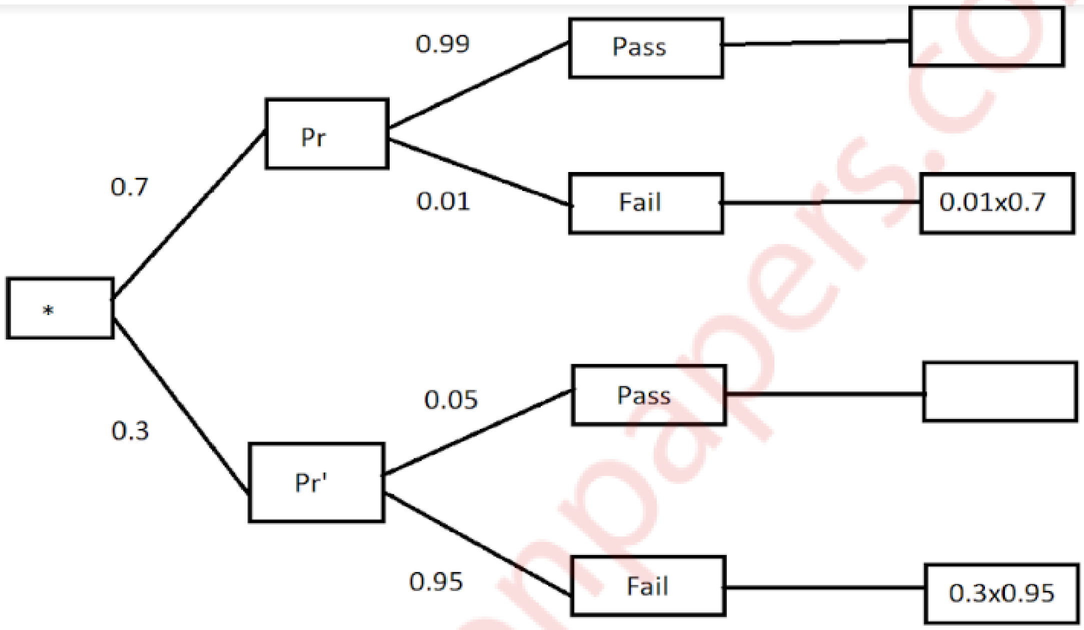
3. f^2 = fof = f(f(x)) = f(x^3) = $(x^3)^3$

4. g^2 = gog = g(g(x)) = g($4x^2 + 1$) = $4\left(4x^2+1\right)^2 + 1$

Q3.a) Given that a student had prepared, the probability o passing a certain entrance exam is 0.99. Given that a student did not prepare, the probability of the passing the entrance is exam is 0.5. Assume that the probability o preparing is 0.7. The student fails in the exam. What is the probability that he or she did not prepare. [6]

Solution:-

Probability tree diagram



We will find:

P(Fail | Pr') = P(Fail ∩ Pr')

P(Fail | Pr') = P(Fail ∩ Pr') + P(Fail ∩ Pr)

$$P(\text{Fail} \mid Pr') = \frac{0.3 \times 0.95}{0.3\times0.95+0.7\times0.01} = 0.976$$

Probability = 0.976

Q3. b) Define an equivalence relation with example. Let 'T' be a set of triangles in a plane and define R as the set R = {(a,b)|a,b ∈ T and a is congruent to b} then show that R is an equivalence relation. [6]

Solution:-

A relation R on a set A is called an equivalence relation if it is reflexsive symmetric and transitive.

eg:- let A = IR and R be ‘quality of numbers ‘.

Consider all subsets of a universal set and R be the relation ‘equality of sets’.

A is set of triangles and R is ‘similarity’ of triangles.

Digraph of equivalence relation will have a loop. Edge from b,a if a,b is present and if are from a,b and are from b,c; there should be are from a to c.

Let T be set of triangles in a place.

Since every triangle is congruent to itself; R is reflexive.

If Δa is congruent to Δb, then Δb is congruent to Δa; R is symmetric

If $\Delta a \cong \Delta b$ and $\Delta b \cong \Delta c$ implies $\Delta a \cong \Delta c$;

R is transitive.

Therefore R is equivalence relation.

Q3.c) Let A=B=R, the set of real numbers. Let f:A→B be given by the formula $f(x) = 2x^3 - 1$ and let g:B→A be given by the formula

$g(x) = \sqrt[3]{\frac{1}{2}y+\frac{1}{2}}$. Show that f is bijection between A and B and g is a bijection between B and A. **[4]**

Solution:-

f is bijection if it is one to one and onto $f(x) = 2x^3 - 1$ to be one to one and onto

If $a,b \in A$

Such that $f(a) = f(b)$

$2a^3 - 1 = 2b^3 - 1$

$a^3 = b^3$

$a = b$

f is one to one

now for $y = 2x^3 - 1$

$1 + y = 2x^3$

$\frac{1+y}{2} = x^3$

$x = \sqrt[3]{\frac{y+1}{2}}$

For each y ∈ B;there exist unique x in A

Such that $f(x) = y$

f is onto

f is bijective

y is bijective if it is one to one and onto

$g(a) = g(b)$

$$\sqrt[3]{\frac{a}{2}+\frac{1}{2}} = \sqrt[3]{\frac{b}{2}+\frac{1}{2}}$$

Cubing both sides;

$$\frac{a}{2} + \frac{1}{2} = \frac{b}{2} + \frac{1}{2}$$

a = b

g is one to one.

Now for $x \in A$; there exists unique y in B such that g(y) = x

g is onto.

So g is bijective.

Q3.d) Let z_n denote the set of the integers {0, 1, 2, …, n-1}. Let O be a binary operation on z_n denote such that a O b = the reminder of ab divided by n.

i) Construct the table for the operation O for n=4

ii) Show that (z_n,O) is a semigroup for any n. [4]

Solution:-

1. Table for the operation * for n =4

$*_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3

2	0	2	0	2
3	0	3	2	1

2. The set z_n is closed under the operation * because for any a,b,∈ z_n

   $(a * b) \in z_n$

   $(a *_4 b) *_4 c = a *_4 (b *_4 c)$

   Let a=1 ; b =2; c = 3

   $(1 *_4 2) *_4 3 = 1 *_4 (2 *_4 3)$

   $2 *_4 3 = 1 *_4 (2)$

   2 = 2

Is associative operation

From above deduction ; $(z_n, *)$ is semigroup for n.

Q4.a) **[6]**

1. **Among 50 students in a class, 26 got an A in the first examination and 21 got an A in the second examination. If 17 students did not get an A in either examination, how many students got an A in both examinations?**
2. **If the number of students who got an A in the first examination is equal to that in the second examination. If the total number of students who got an A in exactly one examination is 40 and if 4 students did not get an A in either examination then determine the number of students who got an A in the first examination only, who got A in the second examination only and who got an A in both the examination.**

Solution:

a) Let T be no of students

   Let F be students who got A in 1^{st} exam

   Let S be students who got A in 2^{nd} exam

$n(T) = 50$; $n(F) = 26$; $n(S) = 21$

no of students who did not get on A in either examination = 17

no of students got an A in at least one examination is 50-17 = 33

no of students got A in both exams is $n(F \cap S)$

$33 = n(F) + n(S) - n(F \cap S)$

$N(F \cap S) = 47 - 33 = 14$

b) Number of students who got an A in 1^{st} exam equal to that in 2^{nd} exam; $n(F) = n(S)$

Total no of students who got an A in exactly one examination is 40

$N(F) + n(S) - 2n(F \cap S) = 40$ ....(1)

4 students did not get an A in at least one examination is 50-4 = 46

From (i);

$n(F) + n(S) - 2n(F \cap S) = 40$

$n(F) + n(S) - n\ F \cap S) - n(F \cap S) = 40$

$46 - n(F \cap S) = 40$

$n(F \cap S) = 6$ .... (2)

6 students got an A in both examinations

Using equation (i)

$n(F) + n(S) - 2n(F \cap S) = 40$

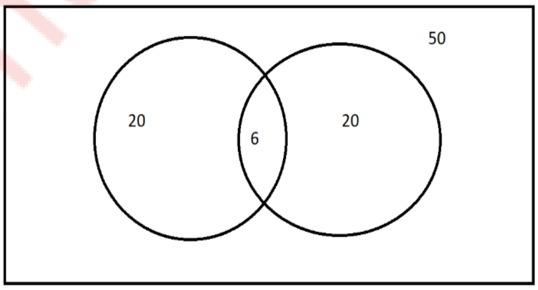
$n(F) + n(S) - (2 \times 6) = 40$

$n(F) + n(S) = 52$

$n(F) = n(S) = 52/2 = 26$

$n(F) - n(F \cap S) = 26 - 6 = 20$ got A in first exam

$n(S) - n(F \cap S) = 26 - 6 = 20$ got A in first exam.



Q4.b) Consider the (2,5) group encoding function **[6]**

e:$B^2 \rightarrow B^5$ defined by :

e(00)=000000 **e(01)=01110**

e(10)=10101 **e(11)=11011**

Decode the following words relative to a maximum likelihood decoding function:

i) 11110 ii) 10011 iii)10100

Solution:-

e : $B^2 \rightarrow B^5$ defined by;

e(00) = 00000 e(01) = 01110

e(10) = 10101 e(11) = 11011

decoding table;

00	01	10	11
00000	01110	10101	11011
00001	01111	10100	11010
00010	01100	10111	11001
00100	01010	10001	11111
01000	00110	11101	10011
10000	11110	00101	01011

1. We receive the word 11110 we first locate it in 2nd column. The word at top is 01110. We decode 11110 as 01
2. We receive the word 10011 we first locate it in 4th column. The word at top is 11011. We decode it as 11
3. We receive the word 10100 we first locate it in 3rd column. The word at top is 10101. We decode it as 10.

Q4.c) (i) Is every Eulerian graph a Hamiltonian? [4]

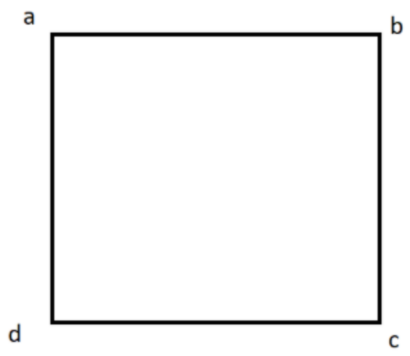
3. Is every Hamiltonian graph a Eulerian?

Explain with the necessary graph.

Solution:-

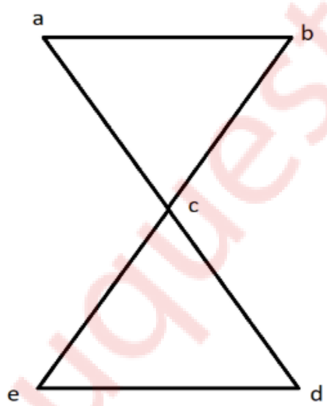
1. Let G = (V,E) be a graph. A eulerian graph is a graph which passes through every edge exactly once.

   Let $G_1(V_1,E_1)$ be a graph. A Hamiltonian circuit is one which passes through every vertex exactly one. A graph is called Hamiltonian if it posses a Hamiltonian circuit.



Eulerian : a,b,c,d,a

Hamiltonian : a,b,c,d,a



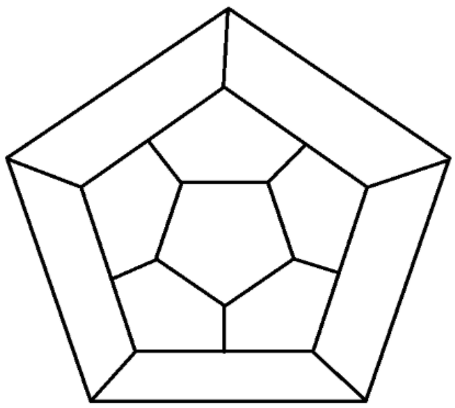
Eulerian : c,a,b,c,e,d,c

No Hamiltonian

Hence every eulerian graph is not Hamiltonian.

2. In Hamiltonian graph, we need to visit each vertex once except last vertex.

Repetition of edge is not necessary. Therefore Hamiltonian graph may not be Eulerian.



Hamiltonian but not eulerian(since it is not possible to cover all edges at once).

Q4.d) Given the parity check matrix. [4]

$$\mathbf{H} = \begin{vmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{vmatrix}$$

Find the minimum distance of the code generated by H. How many errors it can detect and correct?

Solution:-

$$H = \begin{vmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{vmatrix}$$

In the given parity check matrix , all columns are distinct and non zero.

So d >= 3

We can use the property that the minimum distance of a binary linear code is equal to the smallest no of columns of the parity check matrix H that sum upto zero.

We can so sum of first three columns is zero so minimum dist = 3

It can correct $(d_{min} - 1)/2 = 1$ error

It can detect $d_{min} - 1 = 2$ errors.

Q5 a) Explain pigeonhole principle and extended pigeonhole principle. Show that in any room of people who have been doing some handshaking there will always be at least two people who have shaken hands in the same number of times. [6]

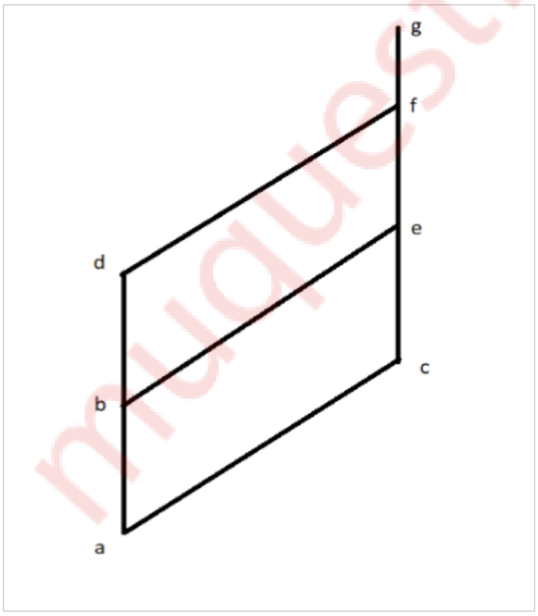
Solution:-

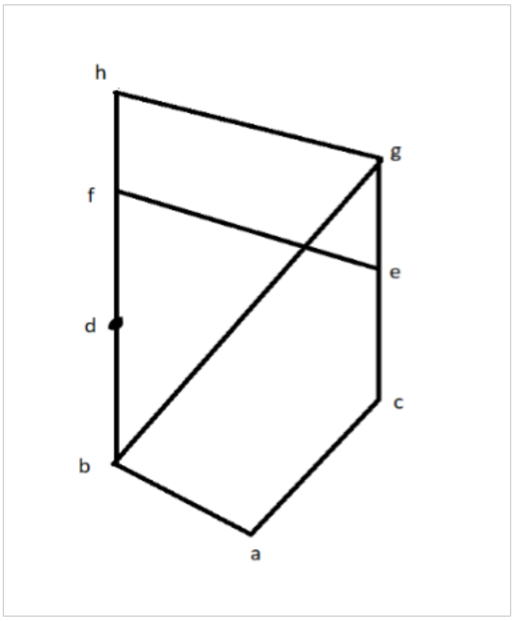
Pigeon hole principle : If n pigeons are assigned to m pigeonholes and m < n then at least one pigeonhole contains two or more pigeons.

Extended pigeonhole : If n pigeons are assigned for m pigeonholes then one of the pigeonholes must obtain at least $\left[\frac{(n\text{-}1)}{m}\right] + 1$ pigeons.

There are n people in a party.(n ≥ 2). If no two people have shaken hands with equal number of people then their handshake count must differ by at least 1. So the possible choice for hand shake count would be 0,1 …… n-1. These are exactly n choices and n people. If there exit a person with (n-1) handshake count, there can be a person with 0 handshake count. Thus reducing the possible choices to (n-1). Now due to pigeonhole principle , we have that at least two person will have same number of handshake count.

Q5.b) Determine whether the poset with the following Hasse diagrams are lattice or not. Justify your answer. [6]

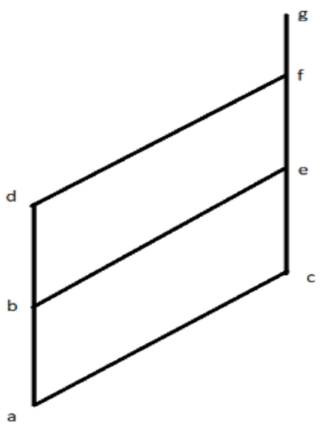




Solution:-

1. LUB :-

V	a	b	c	d	e	f	g
a	a	b	e	d	e	f	g
b	b	b	e	d	e	f	g
c	c	e	c	f	e	f	g
d	d	d	f	d	f	f	g
e	e	e	e	f	e	f	g
f	f	f	f	f	f	f	g
g	g	g	g	g	g	g	g



GLB:-

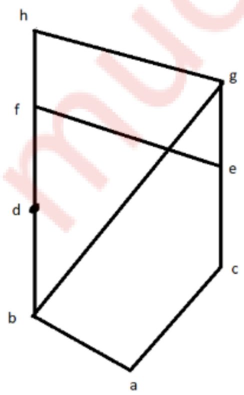
^	a	b	c	d	e	f	g
a	a	a	a	a	a	a	a
b	a	b	a	b	b	b	b
c	a	a	c	a	c	c	c
d	a	b	a	d	b	d	d
e	a	b	c	b	e	e	e
f	a	b	c	d	e	f	f
g	a	b	c	d	e	f	g

2. LUB:-

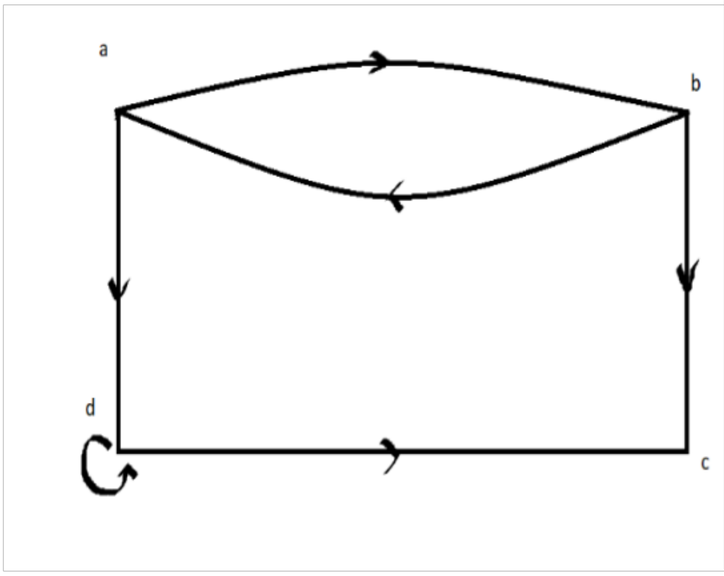
V	a	b	c	d	e	f	g	h
a	a	b	c	d	e	f	g	h
b	b	b	f	d	f	f	g	h
c	c	f	c	f	e	f	g	h
d	d	d	f	d	f	f	h	h
e	e	f	e	f	e	f	g	h
f	f	f	f	f	f	f	h	h
g	g	g	g	h	g	h	g	h
h	h	h	h	h	h	h	h	h

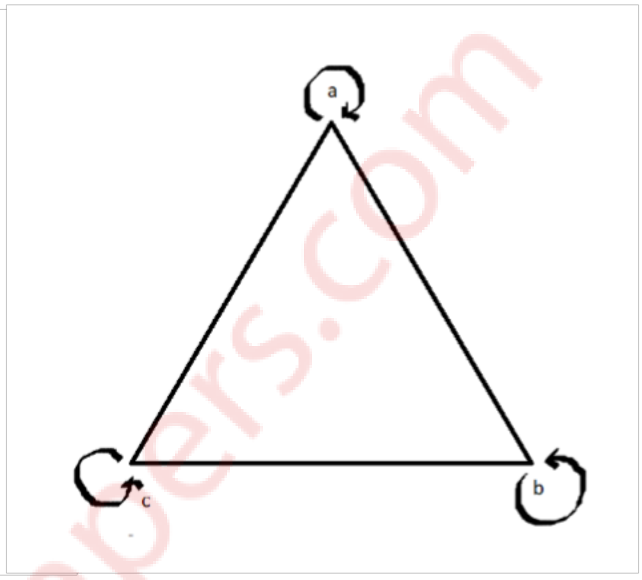
GLB:-

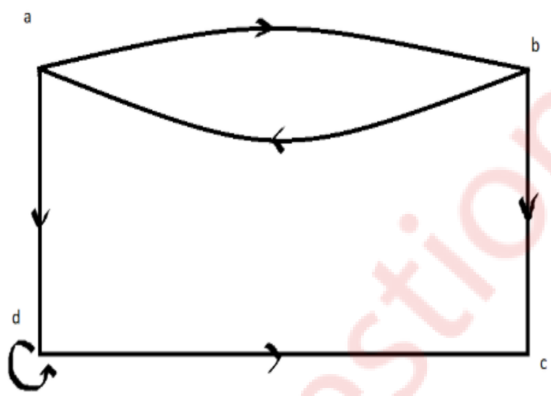
^	a	b	c	d	e	f	g	h
a	a	a	a	a	a	a	a	a
b	a	b	a	b	a	b	b	b
c	a	a	c	a	c	a	c	a
d	a	b	a	d	a	d	b	d
e	a	a	c	a	e	e	e	e
f	a	b	a	d	e	f	-	f
g	a	b	c	b	e	-	g	g
h	a	b	a	d	e	f	g	h



Q5.c) From the following digraphs, write the relation a set of ordered pairs. Are the relations equivalence relations? [4]





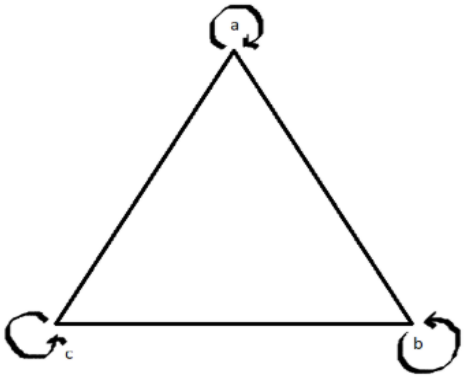


An equivalence relation is reflexive, symmetric and transitive.

R_1 = {(a,b), (a,c), (b,a), (b,d),(c,c),(c,d)}

R_1 is not reflexive because (a,a) ,(b,b), and (d,d) does not exist

R_1 is not equivalence.



R_2 = $\{(a,a), (b,b), (c,c), (a,b), (b,c), (c,a)\}$

Relation R_2 is reflexive and transitive but not symmetric because (a,b) exists but (b,a), (c,b) (a,c) do not belong to R_2

Hence R_2 is not equivalence relation.

Q5.d) For the set X = {2, 3, 6, 12, 24, 36}, a relation ≤ is defined as x≤y if x divides y. Draw the Hasse diagram for (X, ≤). Answer the following:

i)What are the maximal and the minimal elements?

ii)Give one example of chain and antichain

iii)Is the poset a lattice. [4]

Solution:-

$$R = \begin{Bmatrix} (2,2), (2,6), (2,12), (2,24),(2,36), (3,3), (3,6), (3,12), (3,24), (3,36), (6,6), (6,12) \\ (6,24), (6,36), (12,12), (12,24), (12,36), (24,24), (36,36) \end{Bmatrix}$$

Hasse diagram:-

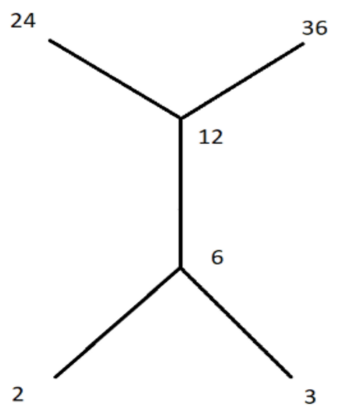
1. Maximal : 24,36

   Minimal : 2,3

2. Chain = {2,6,12,24}

   Antichain ={2,3}

   This poset is not a lattice



Q6.a) Prove that the set (1, 2, 3, 4, 5, 6} is a group under multiplication modulo 7. [6]

Solution:-

Multiplication module 7 table for set A is

X_7	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

1 is identity element of algebraic system

a X_7 1 = a = 1 X_7 a

eg:- 1 X_7 1 = 1; 2 X_7 1 = 2 ; 3 X_7 1 = 3 ………………. 6 X_7 1 = 6

recall that a^{-1} is that element of G such that a * a^{-1}

2 X_7 4 = 1 inverse of 2 = 4

3 X_7 5 = 1 inverse of 3 = 5

6 X_7 6 = 1 inverse of 6 = 6

We have $2'$ = 2

$2^2 = 2 \ X_7 \ 2 = 4$

$2^3 = 2 \ X_7 \ 2 = 4 \ X_7 \ 2 = 1$

$2^4 = 8 \ X_7 \ 2 = 1 \ X_7 \ 2 = 2$

Hence $|2| = 3$

2 is not generator

We have $3^{'} = 3$

$3^2 = 3 \ X_7 \ 3 = 2$

$3^3 = 9 \ X_7 \ 3 = 2 \ X_7 \ 3 = 6$

$3^4 = 27 \ X_7 \ 3 = 6 \ X_7 \ 3 = 4$

$3^5 = 3^4 \ X_7 \ 3 = 4 \ X_7 \ 3 = 5$

$3^6 = 3^5 \ X_7 \ 3 = 5 \ X_7 \ 3 = 1$

3 is generator of this group and is cyclic

Subgroup generated by {3,4} is denoted by <{3,4}> since 3,4 are element of this set they have to be there in <3,4>

Inverse of 3 is 5 and 4 is 2

$3,4,5,2 \in$ <{3,4}>

$3 \ X_7 4 = 5$ X_74 =5 $\qquad 5 \ X_7 \ 4 = 6$

$3 \ X_7 \ 3 = 2$ X_7 3 = 2 $\qquad 6 \ X_7 \ 6 = 1$

$3 \ X_7 \ 5 = 1$ X_7 5 =1 $\qquad 5 \ X_7 \ 1 = 5$

$4 \ X_7 \ 4 = 2$ X_7 4 = 2 $\qquad 1 \ X_7 \ 1 = 1$

$3 \ X_7 \ 2 = 6$ X_7 2 =6 $\qquad 5 \ X_7 \ 2 = 3$

$5 \ X_7 \ 5 = 4 \qquad 3 \ X_7 \ 6 = 4$

$5 \ X_7 \ 6 = 2 \qquad 2 \ X_7 \ 2 = 4$

<3,4> = <1,2,3,4,5,6>

Subgroup generated by <{3,4}> is the set A itself.

Hence the set (1, 2, 3, 4, 5, 6} is a group under multiplication modulo 7.

Q6.b) Given a generating function, find out corresponding sequence **[6]**

i) $\frac{1}{3-6x}$ ii) $\frac{x}{1-5x+6x^2}$

Solution:-

1. $\frac{1}{3-6x} = \frac{1}{3(1-2x)}$

The simple geometric function that gives the sum of geometric series

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

Replace x by 2x

$\frac{1}{1-2x} = \sum_{n=0}^{\infty} 2^n . x^n$

Multiply this by 1/3

$\frac{1}{3(1-2x)} = \frac{1}{3}\sum_{n=0}^{\infty} 2^n . x^n$

The associated sequence is $\left(0, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \ldots\ldots\right)$

2. $\frac{x}{1-5x+6x^2}$

We know : $1 - 5x + 6x^2 = (1 - 2x)(1 - 3x)$

Therefore $\frac{x}{1-5x+6x^2} = \frac{x}{(1-2x)(1-3x)}$

$x = A(1-3x) + B(1 - 2x)$

Put x = ½ and 1/3

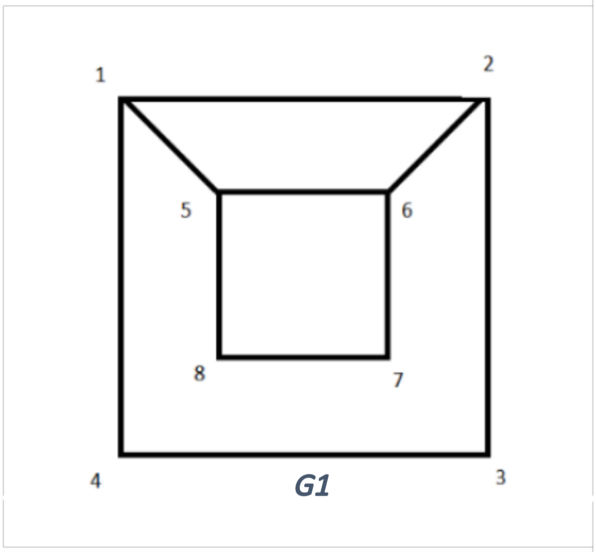
We get A = -1 and B = 1

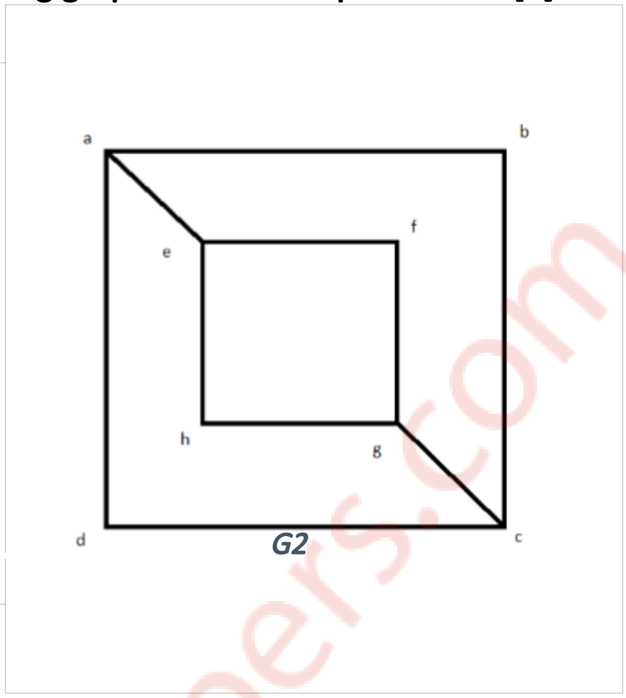
$f(x) = \frac{1}{(1-3x)} + \frac{1}{(1-2x)} = \sum_{n=0}^{\infty} (3^n x^n - 2^n x^n)$

$a_n = 3^n - 2^n$ for $n \geq 0$

Sequence is (0, 1, 5, 19, 65 ,.....)

Q6.c) Determine whether the following graphs are isomorphic or not.[4]





Solution:-

Hence both graph G_1 and G_2 contain 8 vertices and 10 edges no of vertices of degree 2 in both graphs are 4. The number of vertices of degree 3 in both graphs are 4.

For adjacency , consider the vertex 1 of degree 3. In G_1 it is adjacent to two vertices of degree 3 and 1 vertex of degree 2. But in G_2 there does not exist any vertex of degree 3 which is adjacent to degree 3 and 1 vertex of degree 2. Hence adjacency is not presented. Hence given graphs are not isomorphic.

Q6.d) Prove the following (use laws of set theory) [4]

$$A \times (X \cap Y) = (A \times X) \cap (A \times Y)$$

Solution:-

$A \times (X \cap Y) = (A \times X) \cap (A \times Y)$

Let (a,x) $\in A \times (X \cap Y)$ .... (1)

By definition of the Cartesian product

a $\in A$ and x $\in X \cap YY$

ince, x $\in X \cap Y$

x $\in X$ and x $\in Y$

(a,x) $\in A \times X$ and (a,x) $\in A \times Y$

$(a,x) \in (A \times X) \cap (A \times Y)$

$A \times (X \cap Y) \subseteq (A \times X) \cap (A \times Y)$ .... (2)

Again let; $(a,x) \in (A \times X)$ & $(a,x) \in (A \times Y)$

$a \in A$, $x \in X$ & $x \in Y Y$

$\in AA$ & $x \in X \cap Y$

$(a,x) \in A \times (X \cap Y)$

$(A \times X) \cap (A \times Y) \subseteq A \times (X \cap Y)$ .... (3)

From (1) and (2) we get,

$$\mathbf{A \times (X \cap Y) = (A \times X) \cap (A \times Y)}$$
