

(3 Hours)

Total Marks: 80

- N.B. :** 1) Question No.1 is **Compulsory**.
 2) Attempt any **THREE** from the remaining **FIVE** questions.
 3) Figures to the right indicate full marks.
 4) Scientific Calculator is allowed.

Q1. (a) Let X be a random variable for which $E(X) = 24$ and $V(X) = 2$. Find the values of a and b such that $Y = aX - b$ has expectation 20 and variance 8. [5]

(b) The following table gives the number of accidents in a city during 10 days of time. Find whether the accidents are uniformly distributed over that period. (given for 9 degrees of freedom at 5% level of significance, the table value of χ^2 is 16.9) [5]

Day	1	2	3	4	5	6	7	8	9	10
No. of accidents	8	8	10	9	12	8	10	14	10	11

(c) The ranking of ten students in two subjects A and B as follows: Find Spearman's rank correlation coefficient. [5]

Subject A	3	5	8	4	7	10	2	1	6	9
Subject B	6	4	9	8	1	2	3	10	5	7

(d) In a random arrangement of the letters of the word "COMMERCE", find the probability that all the vowels come together. [5]

Q2. (a) Prove with example that three events may be mutually independent but need not be pair wise independent [10]

(b) Calculate Karl Pearson's coefficient of correlation of the following. [10]

Price	8	10	15	17	20	22	24	25
Supply	25	30	32	35	37	40	42	45

Q3. (a) The number of accidents in a year attributed to taxi drivers in a Dadar follows Poisson Distribution with mean 3. Out of 1000 taxi drivers, find approximately the number of drivers with i) no accident in a year ii) more than 3 accident in a year (Given that $e^{-1} = 0.3679$, $e^{-2} = 0.1353$, $e^{-3} = 0.0498$) [10]

(b) A coin is tossed four times. Calculate the expectation value and variance of the number of heads obtained. [10]

- Q4. (a) The regression line of y on x for a certain bivariate data is $5y + 3x = 52$ and the regression line of x on y is $2x + y = 30$. [10]

- 1) Find the arithmetic mean of x and y
- 2) Find the coefficient of correlation between x and y
- 3) Find the most probable value of y when $x = 10$

- (b) A man with 'n' keys wants to open his door and tries the keys independently and a random. Find the mean and variance of the number of trials required to open the door a) if unsuccessful keys are not eliminated from the further selection b) if unsuccessful keys are eliminated from the further selection [10]

- Q5. (a) Calculate the Bowley's coefficient of skewness of the following [10]

Age	30-35	35-40	40-45	45-50	50-55	55-60
No. of people	5	10	30	35	15	5

- (b) The probability mass function of a random variable X is zero except at points $x=0, 1, 2$. At these points it has the values $P(0) = 3C^2$, $P(1) = 4C - 10C^2$ and $P(2) = 5C - 1$, for some $C > 0$ [10]

- (i) Determine the value of C
- (ii) Compute the following probabilities $P[x < 2]$ and $P[1 < x \leq 2]$
- (iii) Find the largest x such that $F(x) < 1/2$
- (iv) Find the smallest x such that $F(x) \geq 1/3$

- Q6. (a) The joint probability density function of a two dimensional random variable (X, Y) is given by [10]

$$f(x, y) = \begin{cases} 2 & 0 < x < 1, 0 < y < x \\ 0 & \text{elsewhere} \end{cases}$$

- i) Find the marginal density function of X and Y
 - ii) Find the conditional density function of Y given $X=x$ and conditional density function of X given $Y=y$
- (b) From the following frequency distribution of marks of students in an examination, calculate the value of Karl Pearson's coefficient of skewness [10]

Marks less than	10	20	30	40	50
No. of Students	5	12	32	44	50