Time: 3 hrs. **M. M.: 100**

N.B.:

1. **All** questions are **compulsory**.
2. **Figures** to the **right** indicate **full** marks.
3. Draw **neat** diagrams wherever **necessary**.
4. Symbols have usual meaning unless otherwise stated.
5. Use of **non-programmable** calculator is allowed.

Q1. Attempt any **two**

(i) State Kepler's three laws of planetary motion and prove second and third laws. **10**

(ii) Show that when body moves in a central force field its motion is confined to a plane. **10**

(iii) Obtain the equation of motion of a particle of mass m as related to the rotating earth. **10**

(iv) A starred system rotates with a variable angular velocity ω with respect to inertial system fixed in space. Show that **10**

$$\frac{dr}{dt} = \frac{d^*r}{dt} + \boldsymbol{\omega} \times \mathbf{r}$$

Hence obtain the Coriolis theorem.

Q2 Attempt any **two**

(i) a) What is virtual displacement? **10**

b) State and derive an expression for the principle of virtual work.

(ii) a) Define the generalized force Q_k. Using that definition, show that, **10**

$$Q_k = -\frac{\partial V}{\partial q_k}$$

Where V is potential and q_k are generalized coordinates.

b) Show that, for Cartesian coordinates x_i and general coordinates q_k,

$$\frac{\partial \dot{x}_i}{\partial \dot{q}_k} = \frac{\partial x_i}{\partial q_k}$$

(iii) Consider a particle constrained to move on the inner surface of the cone with half angle α. Set up the Lagrangian and get the equations of motion. **10**

(iv) a. What is canonical momentum? For a Lagrangian given by **10**

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - q\phi(x, y, z) + q\left(\vec{A} \cdot \vec{v}\right)$$

b. Calculate all the components of canonical momentum.

c. Is this canonical momentum a conserved quantity? Give reason to your answer.

Q3 Attempt any **two**

(i) Derive Euler's equation of motion for a rigid body .Discuss its cases for spin, moment of inertia and angular velocity is at constant ,zero and nonzero **10**

(ii) Obtain an equation of continuity in kinematics of moving fluid .Write the equation if the fluid is incompressible **10**

(iii) Derive Bernoulli's theorem from conservation of linear momentum and energy. Write the meaning of every term in its equation **10**

(iv) What is an Ideal fluid. Obtain the Eulers equation equation of motion for an ideal fluid. state the assumptions **10**

Q4 Attempt any **two**

(i) What is an Anharmonic oscillator? Write down the general expression for restoring force indicating harmonic and anharmonic terms.Draw potential energy curves for (i) $\propto < 0, K > 0$ (ii) $\propto < 0, K < 0$. Infer on the confined motion in each case, If potential energy of Anharmonic oscillator is $V(x) = K\left(\frac{x^2}{2} + \frac{\propto x^4}{2}\right)$. Where $K \rightarrow spring\ constant$, $\alpha \rightarrow anharmonic\ coefficient.$ **10**

(ii) What is logistic map? Find its fixed points, showthat the fixed point x=0 is an attractor for $\lambda < 1$ and a repeller for $\lambda > 1$. **10**

(iii) Obtain reduced Duffing's equation by suitable rescalling. Discuss numerical solutions of Duffing's equation for γ =0.1, f = 0.5 graphically showing odd and even harmonics. Explain the mechanical hysteresis. **10**

(iv) Show that the undamped duffing's equation $\ddot{x} + x + x^3 = f\cos wt$ can have the exact solution $x(t) = A_0 \cos\frac{wt}{3}$. find the conditions under which such simple subharmonic solution occurs. **10**

Q5. Attempt any **four**

(i) Define central force. Explain types of central forces. **05**

(ii) Interpret the various terms involved in the Coriolis's theorem **05**

(iii) Set up the Lagrangian for a simple pendulum. Using that derive the equation of motion. **05**

(iv) Define constraints. Give three examples **05**

(v) Consider a liquid flowing through horizontal tube of non-uniform cross-section. The pressure is 1600 N/m^2 at a point where the velocity of flow is 0.5 N/m^2 what is the Velocity of flow at a point where pressure is 1000 N/m^2 .Density of liquid is uniform and is equal to 1000 kg/m^3 . **05**

(vi) A rigid body consists of three particles of masses 2, 1 and 4 units located at (1, −1,1), (2,0,2) and (−1,1,0) respectively. Determine the elements of the moment of inertia matrix for the rigid body. **05**

(vii) Write down three properties of deterministic chaos **05**

(viii) What is phase space diagram? Plot phase space diagram for one - dimentional damped oscillator and driven oscillator. **05**
