26/11/24

Time: 2 ½ Hours. **Total Marks: 75**

N.B.:

1. **All** questions are **compulsory**.
2. **Figures** to the **right** indicate **full** marks.
3. Draw **neat** diagrams wherever **necessary**.
4. Symbols have usual meaning unless otherwise stated.
5. Use of **non-programmable** calculator is allowed.

Constants: Boltzmann Constant: $k = 1.38 \times 10^{-23}$ J/°K
Planck's Constants: $h = 6.63 \times 10^{-34}$ J-sec

1. (A) Attempt any **one: -**
   (i) State and explain probability theorems. **10**
   (ii) Explain the Normal or Gaussian distribution. **10**
   (B) Attempt any **one: -**
   (i) Write a short note on "Sample Space". **5**
   (ii) Three coins are tossed; what is the probability that two are tails and one head? What is the probability of getting two consecutive tails and then a head? If there was at least one tail, what is the probability of all tails? **5**
2. (A) Attempt any **one: -**
   (i) Explain terminology and notation of a complex number. Find x, y, r, θ of given complex number z. Plot the number and label it and find its complex conjugate. **10**
   a) $z = 3 + 3i$
   b) $z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
   (ii) Explain hyperbolic functions of complex numbers. Using these prove that **10**
   a) $\cosh^2 z - \sinh^2 z = 1$
   b) $\frac{d}{dz}\cosh z = \sinh z$
   (B) Attempt any **one: -**
   (i) Prove: $\cos z = 2$ **5**
   (ii) A particle moves in (x, y) plane so that its position (x, y) as a function of time t is given by **5**
   $$z = 5e^{i\omega t}$$
   Calculate the magnitude of velocity and acceleration.
3. (A) Attempt any **one: -**
   (i) Define partition function. Obtain an expression for translational partition function. Find translational partition function for A_r (mass 6.63×10^{-26} kg) confine to a volume of 1 liter at 298 K. **10**
   (ii) Derive an expression for total energy for two-level system. Determine the total energy of an ensemble consisting of N particles that have only two energy levels separated by energy $h\nu$. **10**

(B) Attempt any **one: -**

(i) Write a short note on degeneracy of energy states. **5**

(ii) What is the weight associated with the configuration corresponding to observing 40 heads after flipping a coin 100 times? How does this weight compare to that of the most probable outcome? **5**

4. (A) Attempt any **one: -**

(i) Imagine N identical but distinguishable balls are randomly distributed within a large box that is divided into k cells, each with a different area. How can we demonstrate that the number of balls in each cell will be proportional to the cell's area? **10**

(ii) How can we derive the Bose-Einstein distribution law that describes how bosons occupy various energy states in a thermodynamic system? **10**

(B) Attempt any **one: -**

(i) Derive the expressions for the mean velocity of gas molecules that adhere to the Maxwell-Boltzmann distribution law." **5**

(ii) Three identical particles can be in any of the five states. What are the number of possible ways of distributing them in various states according to Maxwell-Boltzmann (MB), Bose-Einstein (BE), and Fermi-Dirac (FD) statistics? **5**

5. Attempt any **Five: -**

(i) There are 10 chairs in a row and 8 people to be seated. In how many ways can this be done? **3**

(ii) Define probability of an event. A three-digit number is selected "at random". What is the probability that all three digits are same? **3**

(iii) Find the absolute value of given complex number z. **3**

$$z = \frac{25}{3+4i}$$

(iv) Evaluate: $i^{\ln i}$ **3**

(v) What is the difference in energy between $n = 2$ and $n = 1$ states for molecular oxygen ($mass = 5.31 \times 10^{-26}$ kg) constrained by a one-dimensional box having length of 1.00 cm? **3**

(vi) Using the given partition function $q = 1.58$, calculate the probability of an oscillator occupying the first three energy levels ($n = 0, 1$ & 2) **3**

(vii) A large box of area 200 m^2 is divided into small square cells. If 10^6 balls are thrown at random in the box, find the most probable number of balls which fall in the square cell of side 10 cm. **3**

(viii) A black body cavity is cubical with each side of length 10 cm. calculate the number of modes in the wavelength region 6000 A° to 6002 A°. **3**
