TY 26/11/24

Time: 3 hrs. **M. M.: 100**

N.B.:

1. **All** questions are **compulsory**.
2. **Figures** to the **right** indicate **full** marks.
3. Draw **neat** diagrams wherever **necessary**.
4. Symbols have usual meaning unless otherwise stated.
5. Use of **non-programmable** calculator is allowed.

Constants: Boltzmann Constant: $k = 1.38 \times 10^{-23}$ J/k
Planck's Constants: $h = 6.63 \times 10^{-34}$ J-s

Q1. Attempt any **two**

(i) What is binomial distribution. Write the formula for finding the probability of an event using binomial distribution. Explain each term . The overall percentage of failures in a certain examination is 20. If six candidates appear for the examination,what is the probability that at least five pass the examination. **10**

(ii) Define average value μ , variance and standard deviation of a random variable x with f(x) as probability function. **10**
When a die is showing 5 , ₹ 5 are paid, if a die shows 2 or 3 , ₹ 2 are paid and nothing otherwise. Prepare sample space and find the average value μ , variance and standard deviation

(iii) State and Explain the multiplication law of probability. **10**
An article manufactured by a company have two parts A and B. In the process of manufacture of part A, 9 out of 100 are likely to be defective. while for part B, 5 out of 100 are likely to be defective. Calculate the probability that assembled article will not be defective.

(iv) Explain Poisson's distribution Law. Derive an expression for the same. **10**

Q2 Attempt any **two**

(i) What do you mean by hyperbolic function in complex number? Using these prove the following formulas. **10**
(a) $\sin z = \sin x \cosh y + i \cos x \sinh y$
(b) $\sinh 2z = 2 \sinh z \cosh z$

(ii) A particle moves in (x, y) plane so that its position (x, y) as a function of time t is given by$z = (1 + i)e^{it}$. Describe the motion of the particle and find the magnitude of velocity and acceleration of the particle. **10**

(iii) Consider the equation of the form **10**

$$y'' + p_0 y' + q_0 y = f(x)$$

Obtain its general solution using method of successive integration.

(iv) Consider the one-dimensional heat equation 10

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\sigma}\frac{\partial T}{\partial t}$$

If b^2 is the separation constant, discuss the nature of the solution for

a) $b^2 > 0$
b) $b^2 = 0$
c) $b^2 < 0$

Q3 Attempt any **two**

(i) Define partition function. Obtain an expression for translational partition function. Find translational partition function for Ar (mass 6.63×10^{-26} kg) confine to a volume of 1 liter at 298 K. 10

(ii) Derive an expression for root mean square deviation in occupation number of dominant configuration. (ii)

(iii) With reference to canonical Ensemble and average energy, show that total energy of an Ensemble is given by; $E = NkT^2\left(\frac{\partial lnq}{\partial T}\right)$. For an Ensemble comprised of particles with two energy levels separated by energy $h\nu$, Show that, it is given by; $E = \frac{Nh\nu}{e^{\beta h\nu}+1}$. (iii)

(iv) Prove the first law of thermodynamics: $Tds = dU + pdV$ for infinitesimal general interaction. (iv)

Q4 Attempt any **two**

(i) Given a large box divided into k cells with varying areas, if N identical but distinguishable balls are thrown into the box, show that the number of balls in each cell is directly proportional to the area of that cell? 10

(ii) What are the key steps to derive Planck's radiation formula, which defines the distribution of energy emitted by a black body as a function of temperature and wavelength? 10

(iii) How can we derive the Fermi-Dirac distribution law, which governs the occupancy of energy states by fermions in a thermodynamic system? 10

(iv) Derive Expression for mean velocity and root mean square velocity for a system of gas molecules obeying Maxwell-Boltzmann distribution law. 10

Q5. Attempt any **four**

(i) Explain the following terms with suitable examples. **05**

1. Sample Space
2. Conditional probability
3. Trial and Event
4. Mutually Exclusive events

(ii) Find the probability of throwing an even number with an ordinary sixfaced die,hence. Find the probability of throwing twodice to obtain sum as 8. **05**

(iii) Write any five important partial differential equations in Physics. **05**

(iv) Evaluate: $e^{-\left(i\frac{\pi}{4}\right)+\ln 3}$ **05**

(v) What is the difference in energy between $n = 2$ and $n = 1$ states for molecular oxygen (mass = 5.31×10^{-25} kg) constrained by a one-dimensional box having length of 1 cm? **05**

(vi) Determine the total energy of an ensemble consisting of N particles that have only two energy levels separated by energy hν. **05**

(vii) Determine the most probable speed, average speed, and root mean square speed of nitrogen molecules at 310 K, given that the molar mass of nitrogen is 28 gm/mol, the gas constant R is 8.314 J/(mol-K) $N_A = 6 \text{ X } 10^{23}$. **05**

(viii) Three identical particles can occupy any of the four states. How many possible ways can they be distributed among these states according to Maxwell-Boltzmann (MB), Bose-Einstein (BE), and Fermi-Dirac (FD) statistics? **05**
