SE Mechanical - Sem-IV DEC. 2025

Paper / Subject Code: 41221 / Engineering Mathematics - IV

SE/IV/ C scheme/ Mechanical / Dec. 2025.

Time: (3 Hours) Date: 17/12/2025 [Total Marks: 80]

Q.P. Code. 93866 (1/2)

N.B.: 1) Question No. **1** is **Compulsory**.

2) Answer **any THREE** questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) Calculate Correlation coefficient between the variables x and y for the following data **(5)**

X	1	2	4	5	3
Y	3	3	5	8	6

(b) A random variable x has the following probability function **(5)**

X	1	2	3	4	5
P(x)	3c	2c	2c	c	2c

Find i) C ii) P(x<3) iii) E(X) iv) V(X)

(c) A random sample of 50 items gives the mean 6.2 and variance 10.24 .Can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance ? **(5)**

(d) Find a,b, and c if $\bar{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x - cy + 2z)k$ is irrotational. **(5)**

Q.2 (a) Fit a straight line to the following data **(6)**

X	1	2	3	4	5
Y	5	8	3	9	6

(b) Find the work done in moving a particle in the force field $\bar{F} = (3x^2 + 6y)i - 14yzj + 20xz^2k$ along $x = t, y = t, z = t$ from $(0,0,0)$ *to* $(1,1,1)$. **(6)**

(c) Find all possible Laurent's series expansion of the function $f(z) = \frac{3}{(z+2)(z+5)}$ about $z = 0$ indicating region of convergence. **(8)**

Q.3 (a) Given: $2x + 6y = 90$, $9x + 3y = 130$ *are regression lines and* $\sigma_x^2 = 16$ then find (i) *mean of* X *and* Y (ii)*correlation coefficient* (r) $(iii)\sigma_y^2$ **(6)**

(b) Use Green's theorem to evaluate $\int_c (x^2 - y)\,dx + (2y^2 + x)dy$ where c is the boundary of the region enclosed by $y = x^2$ *and* $y = 4$. **(6)**

(c) Investigate the association between the darkness of eye colour in father and son from the following table using χ^2 -test (use 5% LOS) **(8)**

Colour of son's eyes	Colour of father's eyes		
	Dark	Not Dark	Total
Dark	48	90	138
Not Dark	80	782	862
Total	128	872	1000

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Q.4 (a) Let X be a continuous random variable with probability density function $f(x) = ke^{-x},\ x \geq 0$ Find k, mean and variance. (6)

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(b) Following result were obtained from two samples each drawn from two different populations A and B (6)

Group	A	B
Sample Size	25	17
Sample SD	4	3

Test the hypothesis that variance of A is less than or equal to variance of B. Given $(F(0.05) = 2.24\ for d.o.f. 24\ and\ 16)$

(c) Show that $\bar{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is conservative. (8)
Find scalar potential such that $\bar{F} = \nabla\emptyset$ and hence, find the work done by in displacing a particle from (1,2,0) to (3,3,2) .

Q.5 (a) A fair coin is tossed till a head appears. What is the expectation of the number of tosses required? (6)

(b) Using Stoke's Theorem to evaluate $\int_c \bar{F}.d\bar{r}$ where $\bar{F} = x^2i + xyj$ and c is the boundary of the rectangle $x = 0, y = 0, x = a, y = b$. (6)

(c) Evaluate $\int_c \frac{2z}{z^2-4} dz$ where c is (i) $|z-2| = 1$ (ii) $|z+2| = 1$. (8)

Q.6 (a) Four roads lead away from a jail. A prisoner trying to escape from the jail selected a road at random. If road A is selected, the probability of escaping is 1/8, for road B it is 1/6, for road C it is ¼ and for road D it is 9/10. What is the probability that a prisoner will succeed in escaping from the jail (6)

(b) Use Gauss Divergence theorem to evaluate $\iint_s \bar{F}.\hat{n}ds$ where (6)
$\bar{F} = 4x\hat{i} + 3y\hat{j} - 2z\hat{k}$ and s is the surface bounded by
$x = 0, y = 0, z = 0\ and\ 2x + 2y + z = 4$

(c) Monthly salary X in a big organization is normally distributed with mean Rs 3000 and standard deviation of Rs 250. What should be the minimum salary of a worker in this organization, so that the probability that he belongs to top 5% workers? (8)