

(3 Hours)

[Total Marks: 80]

N.B.: 1) Question No. 1 is **Compulsory**.

2) Answer **any THREE** questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) Fit a straight line for following data **(5)**

| | | | | | | | |
|---|----|----|----|----|----|----|----|
| X | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Y | 11 | 14 | 14 | 15 | 12 | 17 | 16 |

(b) Let X be a continuous random variable with probability density function **(5)**

$$f(x) = kx^2(1 - x), \quad 0 \leq x \leq 1$$

Find k , mean and variance.

(c) Evaluate $\int_c \bar{z} \, dz$ where c is unit circle $|z| = 2$. **(5)**

(d) Can it be concluded that the average life span of an Indian is more than 70 years, if a random sample of 100 Indians has average life span of 71.8 years with standard deviation of 8.9 years? **(5)**

Q.2 (a) The probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20 find the number of packets containing at least 2 defective items in consignment of 1000 packets. **(6)**

(b) Evaluate $\int_c \frac{3z^2+z}{(z^2-1)} \, dz$ where c is $|z - 1| = 1$ **(6)**

(c) Obtain two lines of regression and coefficient of correlation **(8)**

| | | | | | | | | |
|---|-----|-----|-----|-----|-----|-----|-----|-----|
| X | 62 | 64 | 65 | 69 | 70 | 71 | 72 | 74 |
| Y | 126 | 125 | 139 | 145 | 165 | 152 | 180 | 208 |

Also estimate Y when X=73.

Q.3 (a) Calculate Correlation coefficient between the variables x and y for the following data **(6)**

| | | | | | |
|---|----|----|----|----|----|
| X | 10 | 20 | 30 | 40 | 50 |
| Y | 35 | 42 | 55 | 60 | 70 |

(b) Find the work done in moving a particle in the force field **(6)**

$$\vec{F} = 3xyi - 5zj + 10xk \quad \text{along } x = t^2 + 1, y = 2t^2, z = t^3 \text{ from } (2,2,1) \text{ to } (5,8,8).$$

(c) The sizes of 10000 items are normally distributed with mean 20cm and standard deviation 4 cm. Find the expected number of items having size: **(8)**
 i) between 18cm and 23 cm, ii) above 26cm.

- Q.4** (a) Following result were obtained from two samples each drawn from two different populations A and B (6)

| Population | A | B |
|-------------|----|----|
| Sample Size | 20 | 17 |
| Sample SD | 60 | 50 |

Test the hypothesis that variance of A greater than variance of B.
Given $(F(0.05) = 2.21 \text{ for } d. o. f. 16 \text{ and } 19)$

- (b) Use Green's theorem to evaluate $\int_c (2x^2 - y^2) dx + (x^2 + y^2) dy$ where c (6)
is the boundary of the surface enclosed by the lines $x = 0, y = 0, x = 2, y = 2$
- (c) Find all possible Laurent's series expansion of the function (8)
 $f(z) = \frac{2-z^2}{z(1-z)(2-z)}$ about $z = 0$.

- Q.5** (a) The sales-data of an item in six shops before & after a special promotional campaign is as follows- (6)

| Shops | A | B | C | D | E | F |
|-----------------|----|----|----|----|----|----|
| Before campaign | 53 | 28 | 31 | 48 | 50 | 42 |
| After campaign | 58 | 29 | 30 | 55 | 56 | 45 |

Can the campaign be judged to be a success at 5% level of significance?

- (b) Using Stoke's Theorem evaluate $\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = yi + zj + xk$ and c (6)
is the boundary of the surface of $x^2 + y^2 = 1 - z, z > 0$.
- (c) The theory predicts the proportion of beans in the four groups A, B, C, D (8)
should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory?

- Q.6** (a) A newly constructed flyover is likely to collapse. The chance that design is (6)
faulty is 0.5. The chance that the flyover will collapse if the design is faulty is 0.95 otherwise it is 0.30. If the flyover collapsed what is the probability that it collapsed because of faulty design?
- (b) Three fair coins are tossed. Find the expectation and the variance of the (6)
number of heads. Also find moment generating function.
- (c) Show that $\vec{F} = (2xyz^2)i + (x^2z^2 + z \cos(yz))j + (2x^2yz + y \cos(yz))k$ (8)
is conservative. Find scalar potential such that $\vec{F} = \nabla\phi$ and hence, find the work done in displacing a particle from $(0,0,1)$ to $(1,\pi/4,2)$ along the straight line.