

2/12/2024 MECH SEM-IV C SCHEME EM-IV QP CODE: 10065695

(3 Hours)

[Total Marks: 80]

N.B.: 1) Question No. 1 is Compulsory.**2) Answer any THREE questions from Q.2 to Q.6.****3) Figures to the right indicate full marks.****Q.1 (a)** Calculate Correlation coefficient between the variables x and y for the following data (5)

X	18	20	34	52	12
Y	39	23	35	18	46

(b) A random variable x has the following probability function (5)

X	0	1	2	3	4	5
P(x)	0	C	2C	2C	3C	2C

Find i) C ii) $P(x < 3)$ iii) $E(X)$ iv) $V(X)$ **(c)** The mean life time of a sample of 25 bulbs is found as 1550 hours with standard deviation of 120 hours. The company manufacturing the bulbs claims that the average life of their bulbs is 1600 hours. Is the claim acceptable at 5% LOS? (5)**(d)** Prove that $\vec{F} = (x + 2y + 4z)\mathbf{i} + (2x - 3y - z)\mathbf{j} + (4x - y + 2z)\mathbf{k}$ is solenoidal and irrotational. (5)**Q.2 (a)** Fit a straight line to the following data (6)

X	1	2	3	4	5
Y	25	28	33	39	46

(b) Find the work done in moving a particle in the force field $\vec{F} = (3x^2 + 6y)\mathbf{i} - 14yz\mathbf{j} + 20xz^2\mathbf{k}$ along $x = t, y = t, z = t$ from $(0,0,0)$ to $(1,1,1)$. (6)**(c)** Find all possible Laurent's series expansion of the function $f(z) = \frac{z-1}{(z+1)(z-3)}$ about $z = 0$ indicating region of convergence. (8)**Q.3 (a)** The regression lines of a sample are $x + 6y = 6$ and $3x + 2y = 10$ Find (a) \bar{x} and \bar{y} (6)**(b)** Use Green's theorem to evaluate $\int_c (3x^2 - 8y^2) dx + (4y - 6xy) dy$ where c is the boundary of the region enclosed by the lines $x = 0, y = 0, x + y = 1$. (6)**(c)** A certain drug is claimed to be effective in curing cold in an experiment on 500 persons with cold. 300 of them were given drug and 200 of them were given the sugar pills. The patients reaction to the treatment are recorded in the following table using χ^2 -test (use 5% LOS) (8)

	Helped	Harmed	No Effect	Total
Drug	200	40	60	300
Sugar pills	120	30	50	200
Total	320	70	110	500

Test the hypothesis that the drug is effective in curing cold.

Q.4 (a) Let X be a continuous random variable with probability density function $f(x) = k(x - x^2)$, $0 \leq x \leq 1$ Find k , mean and variance. (6)

(b) Following result were obtained from two samples each drawn from two different populations A and B (6)

Group	A	B
Sample Size	25	17
Sample SD	4	3

Test the hypothesis that variance of A is less than or equal to variance of B.

Given $(F(0.05) = 2.24 \text{ for d.o.f. } 24 \text{ and } 16)$

(c) Show that $\vec{F} = (6xy + z^3)\mathbf{i} + (3x^2 - z)\mathbf{j} + (3xz^2 - y)\mathbf{k}$ is conservative. (8)
Find scalar potential such that $\vec{F} = \nabla\phi$ and hence, find the work done by in displacing a particle from $(1,2,0)$ to $(3,3,2)$.

Q.5 (a) If X denotes the outcome when a fair die is tossed, find MGF of X about origin and hence find the mean of X . (6)

(b) Using Stoke's Theorem to evaluate $\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x^2 - y^2)\mathbf{i} + 2xy\mathbf{j}$ and c is the boundary of $x = 0, y = 0, x = 4, y = 2$. (6)

(c) Evaluate $\int_c \frac{z^2+3}{z^2-1} dz$ where c is (i) $|z - 1| = 1$ (ii) $|z + 1| = 1$. (8)

Q.6 (a) Three factories A, B, C produce 30%, 50% & 20% of the total production of an item. Out of their production 80%, 50% & 10% are defective. An item is chosen at random and found to be defective. Using Bayes theorem find the probability that it was produced by the factory A. (6)

(b) Use Gauss Divergence theorem to evaluate $\iint_s \vec{F} \cdot \hat{n} ds$ where $\vec{F} = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ and s is the surface of the sphere $x^2 + y^2 + z^2 = 1$ (6)

(c) In an intelligence test administered to 1000 students, the average was 42 and standard deviation was 24. Find the number of students (i) exceeding the score 50 (ii) between 30 and 54 (iii) less than 30. (8)
