

02/06/2025 SE MECHANICAL SEM-III C-SCHEME EM-III QP CODE: 10083689

(3 Hours)

Note: 1) Question No.1 is compulsory.**Total Marks: 80**2) Attempt **any THREE** from the remaining.

3) Figures to the right indicate full marks.

- Q.1** A) Find the values of constants a,b,c and d if $f(z) = (x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2)$ is analytic **5**
- B) Find the Eigen Value of $A^3 - 3A^2$ **5**
- Where $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$
- C) Find the Laplace Transform of t sin at **5**
- D) Find the Fourier series expansion for $f(x) = x$ defined in $(-1,1)$ **5**
- Q.2** A) If $L[f(t)] = \frac{s}{s^2+s+4}$ find $L[e^{-3t}f(2t)]$ **6**
- B) Find the Fourier series expansion for $f(x) = x$ defined in $(-\pi, \pi)$ with period 2π **6**
- C) Find the analytic function $f(z)$ with the real part $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ **8**
- Q.3** A) Show that the function $u = x^3 - 3xy^2$ is harmonic function. Hence find the corresponding analytic function and harmonic conjugate. **6**
- B) A string is stretched and fastened to two points distance L apart motion is started by displacing the string in the form $u = a \sin\left(\frac{\pi x}{L}\right)$ from which it is released at time $t = 0$. Show that the displacement of a point at a distance X from one end at time t is given by $u(x,t) = a \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi ct}{L}\right)$ **6**
- C) Obtain the Fourier series expansion of $f(x) = |x|$ where $-\pi \leq x \leq \pi$ **8**
- Q.4** A) Find Laplace transform of $e^{-4t} \int_0^t u \sin 3u \, du$ **6**
- B) Find Inverse Laplace transform of $\frac{2s+3}{s^2+2s+2}$ **6**
- C) Verify Cayley – Hamilton theorem for the matrix A and hence find A^{-1} & A^4 **8**
- where $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$
- Q.5** A) Solve by Crank-Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - 16 \frac{\partial u}{\partial t} = 0$, $0 \leq x \leq 1$ subject to the condition $u(0,t) = 0, u(1,t) = 100t$, $u(x,0) = 0$, $h = \frac{1}{4}$ for one –time step. **6**
- B) Find the inverse Laplace transform of $\log\left(\frac{s+a}{s+b}\right)$ **6**
- C) Show that the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 7 \end{bmatrix}$ is diagonalizable. **8**
- Find transforming matrix and diagonal Matrix.
- Q.6** A) Evaluate $\int_0^\infty e^{-3t} t \sin t \, dt$ using Laplace transform. **6**
- B) Find the solution $u_t = u_{xx}$ subject to $u(0,t) = 0, u(5,t) = 0, u(x,0) = x^2(25 - x^2)$ using Schmidt method taking $h = 1$ up to 3 seconds. **6**
- C) Find the inverse Laplace transform of $\frac{s}{(s^2+1)^2}$ using convolution theorem. **8**
