

S.E./SEM-III/ Mechanical / NEP / Dec-25

17/12/25

Time: 2 Hour

Q.P. Code:- 95407

Marks : 60

 $\frac{1}{2}$ 

- Note: 1. Question No. 1 is Compulsory.  
2. Attempt any 3 Questions from the remaining questions.  
3. Scientific Calculator is allowed to use

- Que. 1 Attempt **any Five** questions of the following.
- Find Laplace transform of  $e^{-3t} \cosh 5t \sin 4t$  3
  - Find inverse Laplace transform of  $\frac{s+2}{s^2-2s+17}$  3
  - Obtain half range sine series for  $f(x) = x$  in  $(0, \pi)$  3
  - Show that  $v = e^x \sin x$  is a harmonic function. 3
  - If  $\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$  then find eigen values of  $4A^{-1} + 3A + 2I$  3
  - An elastic string is stretched between two points in its equilibrium position a point, it displayed through a distance transversely & then released from this position, State its equation. 3
- Que. 2
- Find analytic function whose real part is  $\frac{1}{2} \log(x^2 + y^2)$  4
  - Find half range cosine series for  $f(x) = x$  in  $(0, 2)$  5
  - By Laplace transform, prove that  $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt = \frac{1}{4} \log 5$  6
- Que. 3
- Find inverse Laplace transform of  $\log\left(1 + \frac{1}{s^2}\right)$  4
  - Determine Constants a, b, c and d, if  $f(z) = (x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2)$  is an analytic function. 5
  - Find the values of  $u(x, t)$  satisfying differential equation  $\frac{\partial^2 u}{\partial x^2} - \frac{1}{4} \frac{\partial u}{\partial t} = 0$  6  
with boundary conditions  $u(0, t) = 0$ ,  $u(8, t) = 0$   $u(x, 0) = 4x - \frac{x^2}{2}$  at the point  $x = i$ , where  $i = 0, 1, 2, 3, 4, 5, 6, 7, 8$  and  $t = \frac{j}{8}$  where  $j = 0, 1, 2, 3, 4, 5$ , using Bender- Smith relation

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Que. 4 a. Find Fourier series of  $f(x) = x|x|$  in  $(-1, 1)$  4

b. Using Crank- Nicholson simplified formula, solve 5

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0 \text{ with boundary conditions } u(0, t) = 0, u(4, t) = 0$$

$$u(x, 0) = \frac{x}{3}(16 - x^2) \text{ at the point } x=i, \text{ where } i=0, 1, 2, 3, 4, \text{ and } t=j \text{ where } j=0, 1$$

c. Find eigen values and eigen vectors of  $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$  6

Que. 5 a. Verify Cayley- Hamilton theorem for matrix 4

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

b. Find Laplace transform of  $g(4t)$  where  $g(t) = t\sqrt{1 + \sin t}$  5

c. Expand Fourier series for  $f(x) = x^2$  in  $(0, 2\pi)$  6

Que. 6 a. Find Laplace transform of  $\int_0^t \int_0^t \int_0^t e^{-2u} \cos^2 u (du)^3$  4

b. Show that the matrix  $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$  is diagonalisable and find its diagonalising matrix 5

c. By using convolution theorem, find inverse Laplace transform of  $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$  where  $a > 0$  and  $b > 0$  are constants. 6