

(3 Hours)

Total Marks : 80

- Note: (1) Question No. 1 is Compulsory.
 (2) Answer any three questions from Q.2.to Q.6
 (3) Figures to the right indicate full marks.

- Q1. a) Find $L[te^{3t} \sin t]$ 5
 b) Find a, b, c, d if $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$ is analytic. 5
 c) Find the Fourier expansion of $f(x) = x^2, -\pi \leq x \leq \pi$ 5
 d) Find the eigen values of $A^2 - 5A + 4I$ if $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ 5
- Q2. a) i) If $L\{f(t)\} = \frac{s}{s^2 + s + 4}$, find $L\{e^{-2t} f(2t)\}$ 3
 ii) Find $L(t^2 \sin at)$ 3
 b) Determine the Half Range Sine Series for $f(x) = \frac{x(\pi^2 - x^2)}{12}$, where $0 < x < \pi$. 6
 c) Find analytic function $f(z)$ whose imaginary part is $e^x \cos y + x^3 - 3xy^2$ 8
- Q3. a) Solve $\frac{\partial^2 u}{\partial x^2} - 32 \frac{\partial u}{\partial t} = 0$ by Bender-Schmidt method subjected to the conditions $u(0, t) = 0, u(x, 0) = 0, u(1, t) = t$, taking $h = 0.25, 0 < x < 1$, upto $t = 5$. 6
 b) Determine the Harmonic Conjugate of u if $u + iv$ is analytic $3x^2y - y^3 = u$ 6
 c) Determine the Fourier Series $f(x) = \left(\frac{\pi - x}{2}\right)^2$ over $[0, 2\pi]$. Hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$ 8
- Q4. a) Evaluate the following Integral using Laplace Transforms. 6

$$I = \int_0^\infty e^{-t} \left(\int_0^t u \cos^2 u \, du \right) dt$$

 b) Determine inverse Laplace Transform of $\frac{s}{(s^2+1)(s^2+4)}$, using Convolution theorem. 6
 c) Is the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ diagonalizable? If so find the diagonal form of A and transforming matrix of A. 8

- Q5. a) Find the Eigen value and the eigen vector of $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -6 \\ 2 & 2 & 2 \end{bmatrix}$ **6**
- b) Determine the Inverse Laplace Transform of $\log \left[\frac{s^2+a^2}{(s+b)^2} \right]$ **6**
- c) Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, by Crank-Nicholson simplified formula, where $u(0, t) = 0, u(4, t) = 0, u(x, 0) = \frac{x}{3}(16 - x^2)$, find u_{ij} , for $i = 0, 1, 2, 3, 4$ and $j = 0, 1, 2$ taking $h = 1$. **8**
- Q6. a) Find the Laplace Transform of $f(t) = \frac{\cos at - \cos bt}{t}$ **6**
- b) A tightly stretched string with fixed end points $x = 0$, and $x = l$, in the shape defined by $y = kx(l - x)$, where k is a constant, is released from this position of rest. Find $y(x, t)$ the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ **6**
- c) Determine the Inverse Laplace Transform of i) $\frac{s+2}{s^2-4s+13}$ **4**
 ii) $\tan^{-1}(s)$ **4**