(3 Hours) *Total Marks :80*

Note: 1) Question No.1 is compulsory

2) Attempt any Three from the remaining

Q1

A) Find $L\left\{\int_0^t e^{-u}\, u^n\, du\right\}$ **5**

B) Prove that $f(z) = e^z$ is analytic everywhere. **5**
Hence find $f'(z)$

C) Find half range sine series of $f(x) = x$ in $(0, \pi)$ **5**

D) If A=$[a_{ij}]$ is a matrix of order 3×3 such that $a_{ij} = \begin{cases} 1, & if\ i \neq j \\ 0, & if\ i = j \end{cases}$ **5**

Find an eigen value of
- i) A
- ii) adjoint of A
- iii) $A^2 - 2A + 2I$

Q2

A) If L[f(t)] = $\frac{1}{9s^2-3s+1}$ then Find L[te^t f(3t)] **6**

B) Find Fourier series for $f(x) =$x $, if\ 0 < x < 2\pi$ **6**
and $f(x + 2\pi) = f(x)$

C) Find analytic function f(z) in terms of z where **8**
$u = y^2 - x^2$

Q3

A) A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string in the form **6**
y=a sin(πx/ l) from which it is released at time t=0.Show that the displacement of a point at a distance x from one end at time t is given by y = a sin(πx/ l) cos (πct / l)

B) Prove that **6**
$u = e^x cosy$ is harmonic function hence find it's harmonic conjugate function

C) Find the Fourier Series for $f(x)$ in $(-\pi, \pi)$where **8**

$$f(x) = |x|$$

Q4

A) Evaluate $\int_0^\infty \left[\frac{cos2t-cos4t}{t}\right] dt$ **6**

B) Find Inverse Laplace transform of $\frac{s+1}{(s-1)^2(s-2)}$ **6**

C) $Is\ the\ matrix\ A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ Diagonalizable? If so find the Diagonal form of A **8**
and transforming matrix of A

Q5

A) If A=$[a_{ij}]$ is a matrix of order 3 × 3 such that **6**

$$a_{ij} = \begin{cases} 2, & if\ i = j \\ -1 & ,if\ i + j = 3\ or\ 5 \\ 1\ , & if\ i + j = 4\ and\ i \neq j \end{cases}$$

Compute: $A^9 - 6A^8 - 9A^7 - 4A^6 + A^5 - 12A^4 - 18A^3 - 8A^2 + 2A + I$

B) Solve by Crank-Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0$, **6**

$0 \leq x \leq 1$ subject to the condition $u(0,t) = 0, u(1,t) = 100t$,

$u(x,0)$=0 h=0.25 for one-time step

c) Find inverse Laplace transform of $(i)\ log[z^2 - 4]$ $(ii)\ \frac{s+2}{(s+16)^2}$ **8**

Q6

A) Find the Laplace Transform of $\int_0^t \cos(u)\sin(u)du$ **6**

B) Find the solution of **6**

$4\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0, 0 < x < 8, \quad u(x,0) = 4x - \frac{1}{2}x^2$, u(0,t)=0, u(8,t)=0

Taking $h = 1, k = \frac{1}{8}$ $for\ 0 \leq t \leq 5/8$

Where h is the step length for $x\ axis$ and k is the step size in time direction using Bender –Schmidt method

C) Find inverse Laplace transform of $\frac{1}{(s^2+16)((s^2+49)}$ using convolution theorem **8**

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