02/06/2025 SE IT SEM-III C-SCHEME EM-III QP CODE: 10081973

TIME: 03 HOURS **MAX. MARKS : 80**

Note:
1. Question **No. 1** is compulsory.
2. Attempt **any three** questions out of remaining **five** questions.
3. Assume suitable data wherever necessary.
4. Figures to right indicate full marks.

Q.1 Answer the following **(Any four)** **Marks**

a. Find the Laplace transform of t sin^3t. **05**

b. Calculate the Spearman's rank correlation coefficient R. **05**

X	10	12	18	18	15	40
Y	12	18	25	25	50	25

c. Find the constants a, b, c, d ,e if f (z) = **05**
(a $x^3 + bx\,y^2 + 3x^2 + cy^2 + x$) + i ($dx^2y - 2y^3$ + e x y + y) is analytic.

d. Find inverse Laplace transform of $tan^{-1}\left(\frac{s+a}{b}\right)$. **05**

Q.2 a. Evaluate by using Laplace transform of $\int_0^\infty \left(\frac{\sin 3t + sn\,2t}{te^t}\right)$ dt. **06**

b. If the mean of the following distribution is 16 find m, n and variance **06**

X :	8	12	16	20	24
P (X= x) :	$\frac{1}{8}$	m	n	$\frac{1}{4}$	$\frac{1}{12}$

c. Obtain the Fourier expansion of f (x) = $\left(\frac{\pi - x}{2}\right)^2$ in (0, 2π) **08**

Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}$ .....

Q.3 a. Find the analytic function f (z) = u + i v in terms of z **06**
if u + v = e^x (cos y + sin y) + $\frac{x-y}{x^2+y^2}$.

b. Find the coefficient of regression and hence the equations of the lines of regression for the following data **06**

X	78	36	98	25	75	82	90	62	65	39
Y	84	51	91	60	68	62	86	58	53	47

c. Using convolution theorem Find the inverse Laplace transform of **08**
$\frac{1}{(s^2+4s+13)^2}$

Q.4 a. Obtain Fourier series of f (x) = $|\sin x|$ in $((-\pi, \pi)$. **06**

b. If X denotes the outcome when a fair die is tossed, find the moment generating function of x and hence find the mean and variance of X. **06**

c. Evaluate by using Laplace transforms of $\int_0^\infty e^{-t}$ (t $\int_0^t e^{-4u} \cos u \, du$)dt . **08**

Q.5 a. Find the orthogonal trajectories of family of curves **06**
$3x^2y + 2x^2 - y^3 - 2y^2 = c$.

b. Find the inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$. **06**

c. Fit a second-degree parabolic curve to the following data and estimate the **08**
Production in 1982.

Year (X)	1974	1975	1976	1977	1978	1979	1980	1981
Production (y)(in tons)	12	14	26	42	40	50	52	53

Q.6 a. Obtain half range Sine series for f (x) = x- x^2 in $0 \leq x \leq 1$. **06**
Hence show that $\frac{\pi^3}{32} = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3}$ ……..

b. Show that the function v = $e^{2x}(y \cos 2y + x \sin 2y$ *is* harmonic. **06**
And find its corresponding analytic function f (z) = u + i v.

c. Find the value of k if the function f (x) = k $x^2(1 - x^3)$, $0 \leq x \leq 1$ **08**
f (x) = 0 otherwise .
Is a probability density function. Also find p ($0 \leq x \leq \frac{1}{2}$) find mean and variance.
