

(Time: 2 hours)

Max Marks: 60

- Note: 1. Question No. 1 is Compulsory.
2. Attempt any 3 Questions from the remaining questions.
3. Scientific Calculator is allowed to use

- Que. 1 Attempt any Five questions.
- Solve $(x - 2e^y)dy + (y + x \sin x)dx = 0$ 3
 - Using Euler's method find approximate value of y when $\frac{dy}{dx} = x^2 + y^2$; and $y(0) = 1$ at $x = 0.6$ take $h = 0.2$. 3
 - Evaluate: $\int_0^2 \int_0^z \int_0^{yz} xyz \, dx dy dz$. 3
 - Evaluate Gamma function $\int_0^\infty x^2 e^{-x^8} dx$. 3
 - Evaluate: $\int_0^1 \int_0^x e^{x+y} dy dx$. 3
 - Solve $(\frac{d^4 y}{dx^4} + 2\frac{d^2 y}{dx^2} + y) = 0$ 3
- Que. 2
- Solve $\frac{dy}{dx} = x + y$ with $x_0 = 0, y_0 = 1$ by Euler's modified method for $x = 0.05$ correct to three places of decimals. (take $h = 0.05$) 4
 - Show that $\int_0^{2a} x^2 \sqrt{2ax - x^2} dx = \frac{5}{8} a^4 \pi$. 5
 - Use method of variation of parameters to solve the differential equation $(D^2 - 2D + 2)y = e^x \tan x$. 6
- Que. 3
- Evaluate $\int_0^\pi \int_0^a \sin \theta r dr d\theta$. 4
 - Solve the differential equation $(x^2 + y^2 + 1)dx - 2xydy = 0$ 5
 - Solve $\frac{dy}{dx} = \frac{1}{x+y}, x_0 = 0, y_0 = 1$ for the interval $(0,1)$ choosing $h=0.5$ by Runge-Kutta method of 4th Order (up to 3 decimal places). 6
- Que. 4
- Solve $(D^3 + D)y = x^2$ 4
 - Find the mass of the lamina bounded by the curves $ay^2 = x^3$ and the line $by = x$ if the density at a point varies as the distance of the point from the x-axis. 5
 - Solve $\frac{dy}{dx} = 2y(1 - 2xy)$ 6
- Que. 5
- Prove that $\int_0^\infty \frac{\log(1+ax^2)}{x^2} dx = \pi\sqrt{a}, (a > 0)$. 4
 - Find the area of the cardioid $r = a(1 + \cos\theta)$ by double integration. 5
 - Evaluate by changing into polar coordinates $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}} dx dy$. 6
- Que. 6
- Change the order of integration $\int_0^1 \int_0^{\sqrt{1-y^2}} f(x,y) dx dy$. 4
 - Solve the differential equation $(D^2 - 3D + 2)y = e^{2x} x^2$ 5
 - Evaluate $\int_0^1 \frac{x^2}{1+x^3} dx$ by using Trapezoidal rule and Simpson's (1/3)rd rule. (choose $h = 0.25$) 6