

(Time: 3 hours)

Max Marks: 80

Note: (1) Question No. 1 is Compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks.

Q1.

- a) Solve: $\frac{dy}{dx} = \frac{\tan y - 2xy - y}{x^2 - x \tan^2 y + \sec^2 y}$ 5
- b) Solve: $[(D - 2)^2(D + 1)]y = e^{2x} + \sin 3x$ 5
- c) Using Euler's method find $y(0.5)$ for $\frac{dy}{dx} = 1 + xy$; $y(0) = 1$ with $h = 0.1$, correct up to 3 decimal places. 5
- d) Change the order of Integration for $I = \int_0^5 \int_{2-x}^{2+x} f(x, y) dy dx$ 5

Q2.

- a) Solve: $(D^3 - 4D + 4)y = x^2 + \cos 2x$ 6
- b) Solve: $x \frac{dy}{dx} + (1 + x)y = e^x$ 6
- c) Evaluate $I = \int_0^3 \frac{dx}{1+x}$ using 8
 1) Simpson's 3/8th rule 2) Simpson's 1/3rd rule 3) Trapezoidal rule.

Q3.

- a) Using DUIS rule show that $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)} dx = \frac{\pi}{2} \log(1+a)$ 6
- b) Evaluate $\iint y dx dy$ over a region bounded by $y = x^2$ and $x + y = 2$ 6
- c) Evaluate: $I = \int_0^2 \int_0^x \int_0^{2x+2y} e^{x+y+z} dz dy dx$ 8

Q4.

- a) Find using double integration the area inside the circle $r = a \sin \theta$ and outside the cardioid $r = a(1 - \cos \theta)$ **6**
- b) Using Runge-Kutta Method of order four find $y(0.1)$ for $\frac{dy}{dx} = \sqrt{x+y}$; $y(0) = 1$ with $h = 0.1$, correct up to 3 decimal places. **6**
- c) Show that: $\int_0^{\infty} x^2 e^{-x^4} dx * \int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}} dx = \frac{\pi}{4\sqrt{2}}$ **8**

Q5.

- a) Evaluate $I = \iiint (x^2 y^2 + y^2 z^2 + z^2 x^2) dx dy dz$ throughout the volume of the sphere $x^2 + y^2 + z^2 = a^2$ **6**
- b) Solve: $(x \sec^2 y - x^2 \cos y) dy = (\tan y - 3x^4) dx$ **6**
- c) Solve by method of variation of parameter $(D^2 + 9)y = \frac{1}{1 + \sin 3x}$ **8**

Q6.

- a) Using modified Euler's method find $y(0.2)$ for $\frac{dy}{dx} = x - y^2$; $y(0) = 1$ with $h = 0.2$, correct up to 3 decimal places. **6**
- b) Find the length of the cardioid $r = a(1 + \cos \theta)$ which lies outside the circle $r + a \cos \theta = 0$ **6**
- c) Change into polar coordinates and evaluate **8**

$$\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2 - y^2}} \log(x^2 + y^2) dx dy$$
