3/12/2024 FE ALL BRANCHES SEM-II C SCHEME EM-II QP CODE: 10069278

(Time: 3 hours) **Max Marks: 80**

Note: (1) Question No. 1 is Compulsory.
(2) Answer any three questions from Q.2 to Q.6.
(3) Figures to the right indicate full marks.

Q1.

a) Solve: $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$ 5

b) Solve: $(D^2 - 4D + 4)y = e^x + \cos 2x$ 5

c) Compute the value of $\int_0^{\frac{\pi}{2}} \sqrt{sinx + cosx}\, dx$ using Simpson's 3/8th rule by dividing into six sub intervals 5

d) Change the order of Integration: I=$\int_0^4 \int_{y/2}^{9-y} f(x,y)\,dx\,dy$ 5

Q2.

a) Solve: $\frac{d^2y}{dx^2} - y = x^2 sin3x$ 6

b) Solve: $(ylogy)dx + (x - logy)dy = 0$ 6

c) Apply Runge- Kutta method of fourth order to find an approximate value of y when $x = 0.4$ by taking $h = 0.2$ for $\frac{dy}{dx} = \frac{y-x}{y+x}$; $y(0) = 1$ 8

Q3.

a) Using DUIS Rule, show that $\int_0^1 \frac{x^a - 1}{\log x}\, dx = \log(1+a)$; $a \geq 0$ 6

b) Evaluate $\iint y\, dx\, dy$ over a region bounded by $y = x^2$ and $x + y = 2$ 6

c) Evaluate: $\int_0^2 \int_0^x \int_0^{2x+2y} e^{x+y+z}\, dz\, dy\, dx$ 8

Q4.

a) Find using double integration the area of the cardioid $r = a(1 - cos\theta)$ lying outside the circle $r = acos\theta$ 6

b) Using Euler's method find $y(0.5)$ for $\frac{dy}{dx} = 1 + xy$; $y(0) = 1$ with $h = 0.1$ 6

c) Prove that $\int_0^\infty x^2 e^{-x^4}\, dx * \int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}\, dx = \frac{\pi}{4\sqrt{2}}$ 8

Q5.

a) Evaluate $\iiint xyz\, dxdydz$ over a region D by changing into cylindrical polar coordinates where region is bounded by coordinate planes, plane $z = 1$ and a cylinder $x^2 + y^2 = 1$ 6

b) Solve $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$ 6

c) Solve by method of Variation of parameter: $(D^2 + 1)y = \sec x \ \tan x$ 8

Q6.

a) Using modified Euler's method find $y(0.2)$ for $\frac{dy}{dx} = x - y^2$; $y(0) = 1$ with $h = 0.2$ 6

b) Prove that length of the arc of the curve $y = log\left(\frac{e^x-1}{e^x+1}\right)$ from $x = 1$ to $x = 2$ is $log\left(e + \frac{1}{e}\right)$ 6

c) Evaluate by changing to polar coordinates: 8

$$\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} \log(x^2 + y^2)\, dx\, dy \quad ; a > 0$$

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