June 3, 2024 10:30 am - 01:30 pm 1T01831 - F.E.(SEM I)(ALL BRANCHES) (Rev - 2019 C Scheme) / 58651 - Engineering Mathematics - I QP CODE: 10057182

(Duration: 3 hours) **Max.Marks:80**

N.B: (1) Question No.1 is compulsory
(2) Answer any three questions from Q.2 to Q.6
(3) Figures to the right indicate full marks.

1	a)	Prove that $\log\left(\frac{2+3i}{2-3i}\right) = 2i\tan^{-1}\left(\frac{3}{2}\right)$	5
	b)	Prove that every square matrix can be uniquely expressed as sum Hermitian and skew Hermitian matrix.	5
	c)	If $z = x^2y + y^2, x = \log t, y = e^t$, find $\frac{dz}{dt}$ at $t = 1$.	5
	d)	Find the n^{th} derivative of $\frac{x}{(2x+3)(x+2)}$	5
2	a)	Prove that $\sin^5\theta = \frac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$	6
	b)	If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, then show that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$	6
	c)	Test for consistency the following system & solve them if consistent $x_1 - 2x_2 + x_3 - x_4 = 2,\quad x_1 + 2x_2 + 2x_4 = 1,\quad 4x_2 - x_3 + 3x_4 = -1$	8
3	a)	Prove that $\left(1 + i\sqrt{3}\right)^n + \left(1 - i\sqrt{3}\right)^n = 2^{n+1}\cos\frac{n\pi}{3}$	6
	b)	Find the extreme values of the function $x^2y - 3x^2 - 2y^2 - 4y + 3$	6
	c)	Find the real root of $x^3 - 2x - 5 = 0$ correct up to three places of decimal using Newton-Raphson Method.	8

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4 **a)** If $x + iy = \cot\left(\frac{\pi}{6} + i\alpha\right)$ P.T $x^2 + y^2 - 2\frac{x}{\sqrt{3}} = 1$ **6**

b) Expand $tan^{-1}(x)$ in powers of $(x - \frac{\pi}{4})$. **6**

c) If $\cos^{-1}\frac{y}{b} = \log\left(\frac{x}{n}\right)^n$, then prove that **8**

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$$

5 **a)** Separate real and imaginary parts of $(1+i)^{1-i}$ **6**

b) Solve the following equations by Gauss Jacobi's Iteration method: **6**

15x + 2y + z = 18, 2x + 20y - 3z =19,3x - 6y + 25z = 22.

c) Solve: $x^{10} + 8x^5 + 15 = 0$. **8**

6 **a)** If $u = tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ Prove that **6**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2sin^3u\,cosu$$

b) i) Prove that $\sinh^{-1}(\tan\theta) = \log\left[\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]$ **6**

ii) If $u = \log\left(\frac{x}{y}\right)$, then find $xu_x + yu_y$

c) If $A = \begin{pmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{pmatrix}$ find non-singular matrices P and Q such that PAQ is in normal form and find its rank. **8**

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