Duration: 3hrs **[Max Marks: 80]**

N.B.: (1) Question No 1 is Compulsory.
(2) Attempt any three questions out of the remaining five.
(3) All questions carry equal marks.

1 **a** Find the continued product of all the roots of $\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^{3/4}$. **[5]**

b If $\tan\left(\frac{\pi}{8}+i\alpha\right)=x+iy$, prove that $x^2+\ y^2+2x=1$. **[5]**

c Find the n^th^ order derivative of $e^{2x}cos\,5\,x.\cos x$. **[5]**

d Prove that $\log\,(1+sinx)=x-\frac{x^2}{2}+\frac{x^3}{6}-$ **[5]**

2 **a** Separate into real and imaginary parts $\cos^{-1}\left(\frac{5i}{12}\right)$. **[6]**

b Investigate for what values of λ and μ do the equations $2x+3y+5z=9$, $7x+3y-2z=8, 2x+3y+\lambda z=\mu$ have **[6]**
(i) no solution
(ii) an unique solution
(iii) an infinite number of solutions.

c Solve the following equations by using Jacobi's iterative method
15x + 2y + z = 18, 2x + 20y – 3z = 19, 3x – 6y + 25z = 22 up to six iterations. **[8]**

3 **a** If $u=f(e^{x-y},e^{y-z},e^{z-x})$, then prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$ **[6]**

b Express $sin^7\,\theta$ in terms of sines and cosines of multiples of θ. **[6]**

c Find the value of $(i)\ tanh(log\sqrt{6})$ $(ii)\ \sin(log(i^i))$ **[8]**

4 **a** Show that $\frac{\partial^2 z}{\partial x^2}=a^2\frac{\partial^2 z}{\partial y^2}$, if $z=\tan(y+ax)+(y-ax)^{3/2}$, **[6]**

b Use the test of rank to solve completely the system of equations
4x − y + 2z + w = 0, 2x + 3y − z – 2w = 0, 7y – 4z – 5w = 0,
2x – 11y + 7z + 8w = 0. **[6]**

c If $y=\sin(m\,sin^{-1}x)$ then prove that
$(1-x^2)y_{n+2}-\ (2n+1)xy_{n+1}+\ (n^2-m^2)y_n=0$. **[8]**

5 **a** Find the extreme values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. **[6]**

b Check whether the matrix $A = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$ is orthogonal or not. If yes, then find the inverse of A. **[6]**

c Determine the root using Regular-Falsi method of $x^4 + x^3 - 7x^2 - x + 5 = 0$ which lies between 2 and 3 correct up to 3 places of decimal. **[8]**

6 **a** Find two non-singular matrices P and Q such that PAQ is in normal form for $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 2 \\ 7 & 4 & 10 \\ 8 & 5 & 8 \end{bmatrix}$ **[6]**

b If $u = tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2sin^3u\, cosu$$ **[6]**

c Find the roots of the equation $x^4 + 1 = 0$ and $x^5 - 1 = 0$ **[8]**

****