Max. Marks: 80 **Time: 3 hrs.**

N.B. : 1. Q1 is compulsory
2. Attempt any three questions from Q2 to Q6.
3. Figures to the right indicate full marks.

Q1. (a) A r.v. X assumes the values $-3,-2,-1,0,1,2,3$ such that $P(X=1)=P(X>1)=P(X\le 0)$ and $2P(X=-3)=P(X=-2)=5P(X=-1)$, $P(X=1)=3P(X=2)=2P(X=3)$. Find the pmf and the distribution of X. 5

(b) The following calculations have been made for closing prices of 12 stocks (x) on the Mumbai stock exchange on a certain day, along with the volume of sales in thousands of shares (y). 5
$\sum x=580$, $\sum y=370$, $\sum x^2=41658$, $\sum xy=11494$, $\sum y^2=17205$
From these calculations, find the linear regression equation of volume of sales depending on stock price.

(c) Evaluate the integral $\int_C \frac{z-1}{z^2+3z+2}\,dz$, $C:|z|=\frac{3}{2}$. 5

(d) Convert the given set of vectors into an orthonormal basis using Gram Schmidt process of orthogonalization. $(2,-1,1)$, $(1,-1,3)$, $(1,1,2)$. 5

Q2. (a) Is the following a subspace of a given vector space with usual addition and scalar multiplication? Justify your answer. 6
(i) $W=\{(x,y,z)\mid x,y,z\in\mathbb{R},\ y=x+z\}$
(ii) $W=\left\{\begin{bmatrix}a & b\\ c & d\end{bmatrix}\ \middle|\ a,b,c,d\in\mathbb{R},\ \begin{vmatrix}a & b\\ c & d\end{vmatrix}\neq 0\right\}$

(b) A random variable X has the probability density function 6
$f(x)=k\,x\left(9-x^2\right)$, $0\le x\le 3$. Find k and mean of X.

(c) Fit the least square line to the following data i) line of y on x ii) line of x on y. Also find the correlation coefficient using the regression coefficients. 8

x:	65	63	67	64	68	62	70
y:	68	66	68	65	69	66	68

Q3. (a) At a certain university, 4% of men are over 6 feet tall and 1% of women are over 6 feet tall. The total student population is divided in the ratio 3:2 in favour of women. If a student is selected at random from among all those over six feet tall, what is the probability that the student is a (i) woman (ii) man? 6

(b) Find the extremals of $\int_{x_1}^{x_2}\frac{(y')^2}{x^2}\,dx$. 6

(c) Find a singular value decomposition of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$. 8

Q4. (a) Find the usual inner product of the two vectors $(-4,6,-1,1)$, $(2,1,-2,9)$. 6
Find the norm of each vectors. Are these vectors orthogonal to each other? Verify the triangle inequality and Cauchy Schwarz inequality.

(b) Evaluate the following integrals using Cauchy Residue theorem, 6

$$\int_C \frac{1}{z^5} e^{z^2} \, dz \quad , \quad |z| = 1.$$

(c) The marks of 1000 students in a semester examination of an Engineering college are distributed normally with mean 70% and standard deviation 5%. Estimate the number of students whose marks will be i) between 60% and 75% ii) more than 75% iii) less than 68%. 8

Q5. (a) Find all possible Laurent's series expansion of $\frac{2z+1}{z^2+5z+6}$ about the origin. 6

(b) Find the extremals of $\int_0^1 xy + y^2 - 2y^2 y' \, dx$. 6

(c) Reduce the quadratic form $x_1^2 - 2x_2^2 + 3x_3^2 + 6x_1x_3 - 4x_2x_3$ to a diagonal form using a congruent transformation. Obtain the congruent transformation applied for the reduction. Find the rank, index signature and class value of the quadratic form. 8

Q6. (a) An examination of 11 applicant for an accountant post was taken by a finance company. The marks obtained by the candidate in reasoning (x) and aptitude (y) test are given below. Calculate the rank correlation coefficient between the performance in the reasoning and aptitude test. 6

x :	20	50	25	70	90	50	76	45	30	19	26
y :	30	60	40	50	45	30	68	30	47	39	38

(b) Evaluate using Cauchy integral formula, 6

$$\int_C \frac{2z^3 + z^2 + 4}{z^4 + 4z^2} \, dz \quad , \; C : |z - 2 - 2i| = 3.$$

(c) Using Rayleigh-Ritz method, find an approximate solution for the extremal of 8

$$\int_0^1 2xy - y^2 - (y')^2 \, dx, \; y(0) = 0, \; y(1) = 0.$$