(3 Hours) **Total Marks : 80**

Note: **1) Question 1 is compulsory.**
2) Attempt any 3 questions from Question 2 to Question 6
3) Figures to the right indicate full marks.

Q1 **Attempt All questions**

A If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ then find the Eigen values of $A^3 + 5A + 8I$ **5**

B Find Laplace transform of $f(t) = te^{3t} \sin 4t$ **5**

C Find the half range Fourier sine Series for $f(x) = x^2 + 1$, where $x \in (-\pi, \pi)$ **5**

D Prove that f(z) = $x^2 - y^2 + 2ixy$ is analytic and also find its derivative **5**

Q2

A Using Green's theorem in a plane to evaluate the line integral **6**

$$\oint_C (x^2 - y)dx + (2y^2 + x)dy$$

Around the boundary of the region defined by $y=x^2$and y=4

B Find the Eigen values and Eigen vectors of the matrix **6**

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

C Show that the function $v = 3x^2y+6xy-y^3$ is harmonic function and find the corresponding analytic function. **8**

Q3

A If $\bar{F} = x^2zi\text{-}2y^3z^3j + xy^2z^2k$ find div$\bar{F}$ and curl$\bar{F}$ **6**

B Find the orthogonal trajectories of the family of curves $3x^2y - y^3 = c$ **6**

C Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1} and A^4 **8**

where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Q4

A Using Stokes theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ **6**

Where $\bar{F} = 4xzi - y^2j + yzk$ and C is the area in the plane z=0 bounded by x=0, y=0 and $x^2 + y^2 = 1$

B Evaluate $\int_0^\infty \frac{e^{-t}\sin t}{t} dt$, using Laplace transforms **6**

C Using Convolution theorem find $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$ **8**

Q5

A Find $L\{t\cos^3 t\}$ **6**

B Consider the vector field $\bar{F}$ on $\mathbb{R}^3$ defined by **6**

$$\bar{F}(x,y,z) = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$$

Show that $\bar{F}$ is irrotational.

C Expand $f(x) = lx - x^2, 0 \le x \le l$ **8**

in a half-range (i)cosine series (ii) sine series

Q6

A Obtain Fourier series expansion of $f(x) = 4 - x^2$ in (-2, 2) **6**

B Prove that the matrix A is diagonalisable **6**

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

C **4**

i) Find $L^{-1}\left\{\log\left(\sqrt{\frac{s+2}{s+3}}\right)\right\}$

ii) Find $L^{-1}\left\{\frac{s}{s^2+2s+5}\right\}$ **4**
