

Time: 3 hour

Max. Marks: 80

Note: 1) Question. 1 is compulsory.

2) Attempt any 3 questions from Question 2 to Question 6

3) Figures to the right indicate full marks.

- Q1 Attempt All questions** **Marks**
- A** If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ then find the eigen values of $A^{-1} + A^2$ 5
- B** Find Laplace transform of $f(t) = t\{\sqrt{1 + \sin t}\}$ 5
- C** Find the Fourier Series for $f(x) = x^2$, where $x \in (-\pi, \pi)$ 5
- D** Prove that $f(z) = \log z$ is analytic, also find its derivative. 5
- Q2**
- A** Using Green's theorem in a plane to evaluate $\oint_C (x^2 - y^2)dx + (x + y)dy$ and C is the triangle with vertices (0, 0), (1, 1) and (2, 1) 6
- B** Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ 6
- C** Show that the function $u = \sin x \cosh y + 2\cos x \sinh y + x^2 - y^2 + 4xy$ satisfies Laplace's equation, also find analytic function. 8
- Q3**
- A** If $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ show that \vec{F} is irrotational and solenoidal. 6
- B** If $v = e^x \sin y$, prove that v is a harmonic function. Also find the corresponding harmonic conjugate. 6
- C** Prove that the matrix A is diagonalisable, also find diagonal form and transforming matrix. 8

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

Q4

A Using Stokes theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ 6
 Where $\vec{F} = x^2\mathbf{i} - xy\mathbf{j}$ and C is the square in the plane $z = 0$ and bounded by $x = 0$, $y = 0$, $x = a$ and $y = a$

B Evaluate $\int_0^\infty \frac{\cos at - \cos bt}{t} dt$, using Laplace transforms 6

C Using Convolution theorem find $L^{-1} \left[\frac{s^2}{(s^2+1)(s^2+4)} \right]$ 8

Q5

A Find $L \left\{ \int_0^t u \sin 4u du \right\}$ 6

B Consider the vector field \vec{F} on \mathbb{R}^3 defined by 6
 $\vec{F}(x, y, z) = y\mathbf{i} + (z \cos(yz) + x)\mathbf{j} + (y \cos(yz))\mathbf{k}$
 Show that \vec{F} is conservative and find its scalar potential.

C Find the Fourier Series for $f(x)$ in $(-\pi, \pi)$ where 8
 $f(x) = x + \frac{\pi}{2} \quad -\pi \leq x \leq 0$
 $= \frac{\pi}{2} - x \quad 0 \leq x \leq \pi$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

Q6

A Obtain Fourier series expansion of $f(x) = 4 - x^2$ in $(-2, 2)$ 6

B Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1} 6
 and A^4 where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

C i) Find $L^{-1} \left\{ \log \left(\sqrt{\frac{s+a}{s+b}} \right) \right\}$ 4

ii) Find $L^{-1} \left\{ \frac{1}{s^2+2s+5} \right\}$ 4
