(3 Hours) **Total Marks: 80**

N.B. : **(1)** Question No. 1 is compulsory.
(2) Attempt any three questions out of the remaining five questions

Q.1 (a) Given the matrix $A = \begin{bmatrix} 1 & 3 & 3 & 1 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ find null space of A and null space of A^t . **(5)**

(b) A Coin is tossed 4 times. Let X be number of heads obtained Find **(5)**
(i) The probability distribution of X
(ii) Mean of X
(iii) Cumulative Distribution function of X

(c) The number of flu vaccinations reported by a health clinic over a 14-day period is given below: **(5)**
145, 162, 178, 190, 205, 220, 235, 235, 248, 260, 275, 290, 310, 425
Identify the outlier if the outlier lies more than ±2 standard deviations away from the mean.

(d) Obtain the Hessian Matrix for the function **(5)**

$$Z = 12x_1x_2 + 10x_1 - 34x_3 + 3x_1^2 + 15x_2^2 - 16x_3^2$$

Q.2 (a) Find Singular Value of Decomposition of matrix $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ **(10)**

(b) The average commute time for employees at a company is 35 minutes. A sample of 18 employees has a mean commute time of 32 minutes and a standard deviation of 6 minutes. At the 5% significance level, test whether the average commute time has changed. (Given $t_{critical}$=2.110 for 17 degrees of freedom) **(10)**

Q.3 (a) A survey was conducted among 100 students to find out their favorite ice cream flavors. The results are as follows: **(10)**

Flavor	Number of Students
Vanilla	30
Chocolate	25
Strawberry	15
Mango	20
Others	10

Draw a **pie chart** to represent the data.

(b) Conduct a two tailed F Test on the following samples: **(10)**
Sample 1: Variance = 119.76, sample size = 41.
Sample 2: Variance = 69.99, sample size = 21
(Given $F_{((40,20),0.025)=2.287}$, $F_{((40,20),0.975)=0.4836}$))

Q.4 (a) You are a food scientist analyzing fruits. You measure **size** (in cm) and **sweetness** (scale 1–10) of fruits from two types: **Mangoes (C1)** and **Guavas (C2)**. **(10)**

Apply **Linear Discriminant Analysis (LDA)** to project the following data into 1D and derive the **discriminant function**.

Class C1 – Mangoes (size, sweetness):

C1={(5,9), (6,10), (7,8), (8,9), (6,8)}

Class C2 – Guavas (size, sweetness):

C2={(2,4), (3,3), (4,5), (3,4), (5,3)}

(b) You are given the following dataset of two features (variables) for 5 students: **(10)**

Student	A	B	C	D	E
Math Score (X1)	90	80	70	60	50
Physics Score (X2)	85	70	65	60	55

Q.5 (a) Minimize the function $f(x_1, x_2) = 4x_1 + 6x_2 - x_1^2 - x_2^2$ **(10)**

subject to $x_1 + 2x_2 = 6, \quad x_1, x_2 \geq 0$

(b) Find the minimizer of $f(x) = (x-3)^2 + \frac{4}{x}$ using bisection method in (1,5) **(10)**

within a range of 0.2

Q.6 **Attempt any four** **(20)**

(a) Plot the graphs of the following functions over the interval x∈[−2,2] **(5)**

(a) sinh(x) (b) cosh(x)

(b) Explain the **curse of dimensionality** with reference to the following: **(5)**

(a) Volume of a unit hypercube as dimension increases

(b) Why high-dimensional data becomes sparse

(c) How it affects nearest-neighbour algorithms

(c) You are given a distance matrix between five cities (based on travel time): **(5)**

	A	**B**	**C**	**D**	**E**
A	0	3	4	2	5
B	3	0	2	5	4
C	4	2	0	3	3
D	2	5	3	0	2
E	5	4	3	2	0

Use **classical MDS** to find a 2D embedding of the cities such that their pairwise distances are preserved as much as possible.

(d) Write short notes on Non gradient based optimization technique. **(5)**

(e) Define and differentiate between the following supervised learning models: **(5)**

(a) Linear Regression (b) Logistic Regression

(f) Find column Space of $= \begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ **(5)**

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