

Time: 3 hours

Max. Marks: 80

- N.B. : 1) Question no. 1 is compulsory.
2) Answer any 3 questions from remaining five questions.

1. **Answer any four questions**
- (a) Define Discrete and Continuous random variables by giving examples. **5**
- (b) Explain any two properties of Auto correlation Function. **5**
- (c) Define SSS process. How it is different from WSS? **5**
- (d) Define mathematical, statistical, and axiomatic definitions of probability. **5**
- (e) Explain the central limit theorem. **5**
2. (a) For the following probability density function **10**
- $$f_x(x) = \begin{cases} kx, & 0 \leq x < 2 \\ k(4 - x), & 2 \leq x \leq 4 \\ 0, & \text{otherwise} \end{cases}$$
- (i) Find the value of k for which $f_x(x)$ is a valid pdf.
- (ii) Find the mean and variance of X.
- (iii) Find the CDF.
- (b) Suppose X and Y are two random variables. Define Covariance and correlation of X and Y. When do you say that X and Y are **10**
- i) Orthogonal
- ii) Independent
- iii) Uncorrelated
- Are uncorrelated variables independent?
3. (a) Prove that for a linear time invariant system, if input is a WSS process, the output is also a WSS process. **10**
- (b) The joint pdf of X and Y is given by, **10**
- $$f_{x,y}(x,y) = e^{-(x+y)}; x > 0; y > 0$$
- Find the pdf of $Z = \frac{x+y}{2}$
4. (a) A binary computer communication channel has the following error probabilities: **10**
- $P(R1 | S0) = 0.2$, $P(R0 | S1) = 0.06$ where $S0 = \{ '0' \text{ sent} \}$ $S1 = \{ '1' \text{ sent} \}$ $R0 = \{ '0' \text{ received} \}$ $R1 = \{ '1' \text{ received} \}$ Suppose that 0 is sent with a probability of 0.8, find
- (a) The probability that '1' is received
- (b) The probability that '1' was sent given that 1 is received
- (c) The probability that '0' was sent given that '0' is received.
- (b) Consider a random process $Y(t) = X(t)(\cos\omega_0 t + \theta)$, where X(t) is a wide-sense stationary random process, θ is a random variable independent of X(t) and is distributed uniformly in $(-\pi, \pi)$ and ω_0 is a constant. Prove that Y(t) is wide-sense stationary. **10**

5. (a) Let X be a random variable with PDF. Find Moment Generating Function of X **10**
and Var(X).

$$f(x) = \frac{1}{2} (e^{-x/2}) ; x > 0$$

$$= 0 ; \text{ else}$$

- (b) The density function of two random variables X and Y is; **10**

$$f_{x,y}(x, y) = \begin{cases} \frac{xy}{9}, & 0 < x < 2; 0 < y < 3 \\ 0, & \text{elsewhere} \end{cases}$$

- (a) Show that X and Y are statistically independent.
(b) Show that X and Y are uncorrelated.

6. (a) Find the equation of the regression line from the following data. **10**

X	1	2	3	4	5	6	7
Y	9	8	10	12	11	13	14

- (b) State and prove Chebyshev inequality. **10**
