Paper / Subject Code: 40821 / Engineering Mathematics-IV

13/05/2025 SE EXTC SEM-IV C-SCHEME EM-IV QP CODE: 10082322

(3 Hours) **[Total Marks: 80]**

N.B. : 1) Question No. **1** is **Compulsory**.

2) Answer **any THREE** questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) Evaluate $\int zdz$ where c is unit circle $|z| = 1$ (5)

(b) Find a unit vector orthogonal to both $u = (1,1,1)$ and $v = (1,1,0)$. (5)

(c) Calculate Karl Pearson's coefficient of correlation from the following data. (5)

Price (in $)	5	6	3	4	3
Deman (in units)	10	10	12	11	12

(d) Find k and mean of following distribution. (5)

X	8	12	16	20	24
P(X=x)	1/8	k	3/8	1/4	1/12

Q.2 (a) Find the extremals of $\int_{x_1}^{x_2}(1 + x^2y')y'dx$. (6)

(b) For Normally distributed variate X, with mean 1 and S.D. 3, (6)

Find i) $P(3.43 \le X \le 6.19)$. ii)) $P(-1.43 \le X \le 2.3)$.

(c) Find Laurent's series of $f(z) = \frac{2z-3}{(z-1)(z-3)}$ about $z = 0$. (8)

Q.3 (a) A continuous r.v. X has a P.D.F. $f(x) = ke^{-x}x^2, x \ge 0$ find k and mean. (6)

(b) Evaluate $I = \int_C \frac{z^2+4}{(z-2)(z+3i)} dz$ where c is $|z - 2| = 2$. (6)

(c) Reduce the following quadratic form into canonical form and hence find its rank, index and signature. (8)

$Q = 2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 8x_2x_3 - 4x_3x_1$.

Q.4 (a) Fit a straight line for following data. (6)

X	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

(b) Show that any plane through origin is a subspace of R^3. (6)

(c) Using Rayleigh-Ritz method, solve boundary value problem

$\int_0^1 \left[xy + \frac{(y')^2}{2}\right] dx, \quad 0 \le x \le 1,$ with $y(0) = 0$ and $y(1) = 0$. (8)

Q.5 (a) Given $6y = 5x + 90,\ 15x = 8y + 130$. Find (i) $\bar{x}$ and $\bar{y}$ (ii) r (6)

(b) Find an orthonormal basis for the subspace of R^3 by applying Gram-Schmidt process where $S = \{(1,\ 1,\ 1),\ (2, 1, 0),\ (5,\ 1,\ 3)\}$. (6)

(c) Find the singular value decomposition of $A = \begin{bmatrix} 4 & 4 \\ -3 & 3 \end{bmatrix}$. (8)

Q.6 (a) Verify Cauchy-Schwartz's inequality for $\{u = (2,\ 1,\ -3),\ v = (3,\ 4, -2)\}$ (6)

(b) Find the probability that at most 4 defective bulbs will be found in a box of 200 bulbs if it is known that 2% of the bulbs are defective. (6)

(c) Calculate Spearman's coefficient of rank correlation from the following data. (8)

X	10	12	18	18	15	40
Y	12	18	25	25	50	25

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