17 DEC. 2025 SE EXTC (SEM-IV) C SCHEME EM-IV QP CODE: 10094281

(Time : 3 hours) **[Total marks: 80]**

Note: - (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q 2 to Q 6.

(3)Figures to the right indicate full marks.

1(a) Evaluate $\int_0^{1+i}(x^2+iy)dz$, along the parabola $y=x^2$. **(5)**

1(b) Find a vector orthogonal to both $u=(1,1,0), v=(0,1,1)$. **(5)**

1(c) Calculate the coefficient of correlation between by X and Y

X	3	5	4	6	2
Y	3	4	5	2	6

. **(5)**

1(d) The Probability function of a random variable X is given by **(5)**

X	-2	-1	0	1	2	3
P(X=x)	0.1	k	0.2	2k	0.3	k

Find (i) k (ii) mean (iii) variance

2(a) Find the extremals of $\int_0^1(xy+y^2-2y^2y')\,dx$. **(6)**

(b) Monthly salary X in a big organization is normally distributed with mean Rs. 3000 and standard deviation of Rs. 250. What should be the minimum salary of a worker in this organization, so that the probability that he belongs to top 5% workers? **(6)**

(c) Find all possible Laurent's expansions of the function $f(z)=\frac{2}{(z-1)(z-2)}$. **(8)**

3(a) A random variable X has the following density function

$$f(x)=\begin{cases}\frac{x}{6}+k, x>0\le x\le 3\\ 0\ ,\ otherwise\end{cases},$$

find k and P(1 <= x <= 2). **(6)**

(b) . Evaluate

$\int_C \frac{sin^6 z}{(z-\frac{\pi}{2})^3}\,dz$, where C is the circle $|z|=2$. **(6)**

(c) Reduce the quadratic form $x^2 + 2y^2 + 2z^2$-$2xy - 2yz + zx$ to normal form through congruent transformations . Also find it's rank, signature and value class. **(8)**

4(a) Construct an orthonormal basis of R^3 using Gram-Schmidt process to **(6)**

$S = \{(3, 0, 4), (-1, 0, 7), (2, 9, 11)\}$.

(b) Find the Probability that at most 5 defective diodes will be found in pack of 600 diodes ; if previous data shows that 3 % of each such diodes are defective . **(6)**

(c)) Solve by Rayleigh-Ritz method the boundary value problem

$I = \int_0^1 (2xy - y^2 - y'^2)\, dx$ given $y(0) = 0$ *and* $y(1) = 0$. **(8)**

5(a) Find the spearman's rank of correlation for the following data

X :	12	17	22	27	32
Y :	113	119	117	115	121

. **(6)**

(b) Using Cauchy's Residue theorem evaluate $\oint_C \frac{e^{2z}}{(z-\pi i)^3}\, dz$ where C is $|z - 2i| = 4$. **(6)**

(c) Find the singular value decomposition of the matrix $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$ **(8)**

6(a) Verify Cauchy-Schwartz Inequality for the vectors

$u = (-4, 2, 1)$ *and* $v = (8, -4, -2)$. **(6)**

(b) Determine whether the set of vectors of the form (a, b, c) *where* $b = a + c$ form a subspace of R^3 under usual addition and scalar multiplication. **(6)**

(c) Obtain the equations of the line of regression for the following data.

X :	5	6	7	8	9	10	11
Y :	11	14	14	15	12	17	16

(8)