

Duration: 3 hours

Max. Marks: 80

N.B. (1) Question No. 1 is **COMPULSORY**.

(2) Answer **ANY THREE** questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to right indicate full marks.

Que. 1 a. Evaluate the Complex line integral $\int_C (y - x - 3x^2i)dz$, 5

Where C is the straight line from $z=0$ to $z= 1+i$.

b. Find a vector orthogonal to both $u = (-6, 4, 2), v = (3, 1, 5)$ 5

c. Calculate the coefficient of correlation between by X and Y 5

X	3	5	4	6	2
Y	3	4	5	2	6

d. The Probability function of a random variable X is given by 5

X	-2	-1	0	1	2	3
P(X=x)	0.1	k	0.2	2k	0.3	k

Find (i) k (ii) mean (iii) variance

Que. 2 a. Find the extremals of $\int_0^1 (xy + y^2 - 2y^2y') dx$. 6

b. Monthly salary X in a big organization is normally distributed with mean Rs. 3000 and standard deviation of Rs. 250. What should be the minimum salary of a worker in this organization, so that the probability that he belongs to top 5% workers? 6

c. Find all possible Laurent's expansions of the function $f(z) = \frac{2}{(z-1)(z-2)}$ indicating the region of convergence 8

Que. 3 a. A random variable X has the following density function 6

$$f(x) = \begin{cases} \frac{x}{6} + kx & 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

find k and $P(1 \leq x \leq 2)$.

b. Evaluate 6

$$\int_C \frac{e^{2z}}{(z-1)(z-2)} dz, \text{ where C is the circle } |z| = 3.$$

c. Reduce the quadratic form $2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_3x_1 + 12x_1x_2$ to normal form through congruent transformations. Also find it's rank, signature and value class. 8

Que. 4 a. Construct an orthonormal basis of R^3 using Gram-Schmidt process to **6**

$$S = \{(3, 0, 4), (-1, 0, 7), (2, -9, 11)\} .$$

b. Find the Probability that at most 5 defective diodes will be found in pack of 600 diodes ; if previous data shows that 3 % of each such diodes are defective **6**

c. Solve by Rayleigh-Ritz method the boundary value problem **8**

$$I = \int_0^1 (2xy - y^2 - y'^2) dx \text{ given } y(0) = 0 \text{ and } y(1) = 0 .$$

Que. 5 a. Find the spearman's rank of correlation for the following data **6**

$$X : 12 \quad 17 \quad 22 \quad 27 \quad 32$$

$$Y : 113 \quad 119 \quad 117 \quad 115 \quad 121 .$$

b. Using Cauchy's Residue theorem evaluate $\oint_C \frac{e^{2z}}{(z-\pi i)^3} dz$ where C is **6**
 $|z - 2i| = 4 .$

c. Find the singular value decomposition of the matrix $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$ **8**

Que. 6 a. Verify Cauchy-Schwartz Inequality for the vectors **6**

$$u = (-4, 2, 1) \text{ and } v = (8, -4, -2) .$$

b. Determine whether the set of vectors of the form (a, b, c) where $b = a + c$ form a subspace of R^3 under usual addition and scalar multiplication **6**

c. Obtain the equations of the line of regression for the following data. **8**

$$X : 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11$$

$$Y : 11 \quad 14 \quad 14 \quad 15 \quad 12 \quad 17 \quad 16$$