

Duration : 3 hours

Max. Marks: 80

- N.B. (1) Question No. 1 is COMPULSORY.**
(2) Answer ANY THREE question from Q 2 to Q 6.
(3) Use of Statistical Tables permitted.
(4) Figures to the right indicate full marks .

Q 1.

- a. Evaluate the complex line Integral $\int_c (y - x - 3x^2i)dz$ where c is a straight line from $z = 0$ to $z = 1 + i$. (5)
- b. Find a vector orthogonal to $u = (-6, 4, 2)v = (3, 1, 5)$. (5)
- c. The Probability density function of a random variable X is
- | | | | | | | | |
|--------|---|----|----|----|----|-----|-----|
| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| P(X=x) | k | 3k | 5k | 7k | 9k | 11k | 13k |
- Find $P(X < 4), P(3 < X \leq 6)$. (5)
- d. Compute Spearman's rank correlation coefficient for the following data
 $X : 18 \ 20 \ 34 \ 52 \ 12$
 $y : 39 \ 23 \ 35 \ 18 \ 46$. (5)

Q 2.

- a. Find the extremals of $\int_0^1 (xy + y^2 - 2y^2y') dx$. (6)
- b. Fit a straight line of the form $y = a + bx$ to the following data and estimate the value of $x = 3.5$.
 $x : 0 \ 1 \ 2 \ 3 \ 4$
 $Y : 1 \ 1.8 \ 3.3 \ 4.5 \ 6.3$. (6)
- c. Find all possible Laurent's expansions of the function $f(z) = \frac{1}{z(z-2)(z+1)}$. (8)

Q 3.

- a. A continuous random variable with P.d.f. $f[x] = k(x - x^3) \ 0 < x < 1$. (6)
 Find k , mean and variance.
- b. Show that $V = \{(x, y) / x = 3y\}$ is a subspace of R^2 . State all subspaces of R^2 . (6)
- c. Reduce the quadratic form $x^2 + 2y^2 + 2z^2 - 2xy + zx - 2yz$ to normal form through congruent transformations. Also find it's rank, signature and value class. (8)

Q 4.

- a. Find the orthonormal basis for the subspaces of R^3 by applying Gram-Schmidt Process where $S = \{(1, 2, 0)(0, 3, 1)\}$. (6)
- b. If a random variable X follows Poisson distribution such that $P(X = 1) = 2P(X = 2)$. Find the mean and the variance of the distribution. Also find $P(X = 3)$. (6)
- c. Solve by Rayleigh-Ritz method the boundary value problem
 $I = \int_0^1 (2xy - y^2 - y'^2) dx$ given (8)
 $y(0) = 0$ and $y(1) = 0$. (8)

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Q 5.

- a. Calculate the coefficient of correlation between X and Y (6)

X: 8 8 7 5 6 2
Y: 3 4 10 13 22 8

- b. Monthly salary X in a big organization is normally distributed with mean Rs 3000 and standard deviation of Rs.250. What should be the minimum salary of a worker in the organization .So that the probability that he belongs to top 5 % workers ? (6)

- c. Find the singular value decomposition of the matrix $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$. (8)

Q 6.

- a. Verify the Cauchy-Schwartz inequality for the $u = (2, 1, -3), v = (3, 4, -2)$ (6)

- b. Using residue theorem evaluate $\oint_C \frac{e^{zz}}{(z-\pi i)^3} dz$ where C is $|z - 2i| = 4$.

- c. Calculate Spearman's rank correlation coefficient R from the given data (6)

X	12	17	22	27	32
Y	113	119	117	115	121

(8)