

(3 Hours)

Total Marks : 80

- Note:** 1) Question 1 is compulsory.
2) Attempt any 3 questions from Question 2 to Question 6
3) Figures to the right indicate full marks.

Q1 Attempt All questions

A If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ then find the Eigen values of $A^3 + 5A + 8I$ **5**

B Find Laplace transform of $f(t) = te^{3t} \sin 4t$ **5**

C Find the half range Fourier sine Series for $f(x) = x^2 + 1$, where $x \in (-\pi, \pi)$ **5**

D Prove that $f(z) = x^2 - y^2 + 2ixy$ is analytic and also find its derivative **5**

Q2

A Using Green's theorem in a plane to evaluate the line integral **6**

$$\oint_C (x^2 - y)dx + (2y^2 + x)dy$$

Around the boundary of the region defined by $y=x^2$ and $y=4$

B Find the Eigen values and Eigen vectors of the matrix **6**

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

C Show that the function $v = 3x^2y + 6xy - y^3$ is harmonic function and find the corresponding analytic function. **8**

Q3

A If $\vec{F} = x^2z\mathbf{i} - 2y^3z^3\mathbf{j} + xy^2z^2\mathbf{k}$ find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ **6**

B Find the orthogonal trajectories of the family of curves $3x^2y - y^3 = c$ **6**

C Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1} and A^4 **8**

where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Q4

A Using Stokes theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ **6**

Where $\vec{F} = 4xzi - y^2j + yzk$ and C is the area in the plane $z=0$ bounded by $x=0, y=0$ and $x^2 + y^2 = 1$

B Evaluate $\int_0^\infty \frac{e^{-t} \sin t}{t} dt$, using Laplace transforms **6**

C Using Convolution theorem find $L^{-1} \left[\frac{s^2}{(s^2+1)(s^2+4)} \right]$ **8**

Q5

A Find $L \{t \cos^3 t\}$ **6**

B Consider the vector field \vec{F} on \mathbb{R}^3 defined by **6**

$$\vec{F}(x, y, z) = (6xy + z^3)\mathbf{i} + (3x^2 - z)\mathbf{j} + (3xz^2 - y)\mathbf{k}$$

Show that \vec{F} is irrotational.

C Expand $f(x) = lx - x^2, 0 \leq x \leq l$ in a half-range (i) cosine series (ii) sine series **8**

Q6

A Obtain Fourier series expansion of $f(x) = 4 - x^2$ in $(-2, 2)$ **6**

B Prove that the matrix A is diagonalisable **6**

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

C i) Find $L^{-1} \left\{ \log \left(\sqrt{\frac{s+2}{s+3}} \right) \right\}$ **4**

ii) Find $L^{-1} \left\{ \frac{s}{s^2+2s+5} \right\}$ **4**