12/11/2024 EXTC SEM-III C SCHEME ENGG.MATHS-III QP CODE: 10065882

(3 hours) **(Total Marks:80)**

N.B: (1) Question no.1 is compulsory.
(2)Attempt any three questions from remaining five questions.
(3)Figures to the right indicate full marks.
(4) Assume suitable data if necessary.

1.(a) Find the Laplace Transform of $f(t) = te^{-4t}sin3t$. **(05)**

(b) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, find the eigen values of $A^3 + 5A + 8I$. **(05)**

(c) Find half-range sine series for $f(x) = x, x \in (0, \pi)$. **(05)**

(d) Find the constants a, b, c, d, e if $f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$ is analytic. **(05)**

2.(a) Evaluate $\int_0^\infty \frac{e^{-t}sint}{t} dt$ using Laplace Transform. **(06)**

(b) Show that the function $v = 3x^2y + 6xy - y^3$ is harmonic and find the corresponding analytic function f(z) in terms of z. **(06)**

(c) Find the Fourier Series for $f(x) = x^2, x \in (0,2\pi)$ and hence, deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$ **(08)**

3.(a) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ hence find A^{-1} *and* A^4. **(06)**

(b) If $\bar{F} = (x + 2y + az)\,i + (bx - 3y - z)\,j + (4x + cy + 2z)\,k$ is irrotational, then find a, b, c. **(06)**

(c) Find the orthogonal trajectories of the family of curves given by $x^3y - xy^3 = c$. **(08)**

4.(a) Use Gauss's Divergence Theorem to evaluate $\iint \vec{N}.\vec{F}\,ds$ where $\vec{F} = 4xi - 2y^2j + z^2k$ and s is the surface of the region $x^2 + y^2 = 4, z = 3$ *above xy plane*. **(06)**

(b) Find the inverse Laplace Transform of $ln\left[\frac{s^2+a^2}{s^2+b^2}\right]$ **(06)**

(c) Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalisable. Find the transforming and diagonal matrix. **(08)**

5.(a) Find the Fourier Series for $f(x) = \begin{cases} x + \frac{\pi}{2}, -\pi < x < 0 \\ x - \frac{\pi}{2},\ 0 < x < \pi \end{cases}$ **(06)**

(b) Find $L\{\int_0^t u\, sin4udu\}$ **(06)**

(c) Find $L^{-}\{\frac{s^2}{(s^2+1)(s^2+4)}\}$ using Convolution Theorem. **(08)**

6.(a) Evaluate by Green's theorem $\int(e^{-x}sinydx + e^{-x}\,cosydy)$ along the curve C, where C is the rectangle with vertices $(0,0), (\pi, 0), \left(\pi, \frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$. **(06)**

(b) Find the inverse Laplace Transform of $\frac{s+29}{(s+4)(s^2+9)}$. **(06)**

(c) Find the eigen values and eigen vectors of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ **(08)**

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