

Max. Marks: 80

Time: 3 hrs.

N.B. : 1. Q1 is compulsory

2. Attempt any three questions from Q2 to Q6.
3. Figures to the right indicate full marks.

Q1. (a) A r.v.  $X$  assumes the values  $-3, -2, -1, 0, 1, 2, 3$  such that 5  
 $P(X = 1) = P(X > 1) = P(X \leq 0)$  and  $2P(X = -3) = P(X = -2) = 5P(X = -1)$ ,  
 $P(X = 1) = 3P(X = 2) = 2P(X = 3)$ . Find the pmf and the distribution of  $X$ .

(b) The following calculations have been made for closing prices of 12 stocks ( $x$ ) 5  
 on the Mumbai stock exchange on a certain day, along with the volume of  
 sales in thousands of shares ( $y$ ).

$$\sum x = 580, \sum y = 370, \sum x^2 = 41658, \sum xy = 11494, \sum y^2 = 17205$$

From these calculations, find the linear regression equation of volume of sales depending on stock price.

(c) Evaluate the integral  $\int_C \frac{z-1}{z^2+3z+2} dz, \quad C: |z| = \frac{3}{2}$ . 5

(d) Convert the given set of vectors into an orthonormal basis using Gram Schmidt 5  
 process of orthogonalization.  $(2, -1, 1), (1, -1, 3), (1, 1, 2)$ .

Q2. (a) Is the following a subspace of a given vector space with usual addition and 6  
 scalar multiplication? Justify your answer.

(i)  $W = \{(x, y, z) \mid x, y, z \in \mathbb{R}, y = x + z\}$

(ii)  $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{R}, \begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0 \right\}$

(b) A random variable  $X$  has the probability density function 6

$$f(x) = k x (9 - x^2), \quad 0 \leq x \leq 3. \text{ Find } k \text{ and mean of } X.$$

(c) Fit the least square line to the following data i) line of  $y$  on  $x$  ii) line of  $x$  on  $y$ . 8  
 Also find the correlation coefficient using the regression coefficients.

x:	65	63	67	64	68	62	70
y:	68	66	68	65	69	66	68

Q3. (a) At a certain university, 4% of men are over 6 feet tall and 1% of women are 6  
 over 6 feet tall. The total student population is divided in the ratio 3:2 in favour  
 of women. If a student is selected at random from among all those over six feet  
 tall, what is the probability that the student is a (i) woman (ii) man?

(b) Find the extremals of  $\int_{x_1}^{x_2} \frac{(y')^2}{x^2} dx$ . 6

- (c) Find a singular value decomposition of the matrix  $\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$ . 8
- Q4. (a) Find the usual inner product of the two vectors  $(-4, 6, -1, 1)$ ,  $(2, 1, -2, 9)$ . 6  
 Find the norm of each vectors. Are these vectors orthogonal to each other?  
 Verify the triangle inequality and Cauchy Schwarz inequality.
- (b) Evaluate the following integrals using Cauchy Residue theorem, 6  

$$\int_C \frac{1}{z^5} e^{z^2} dz, \quad |z|=1.$$
- (c) The marks of 1000 students in a semester examination of an Engineering 8  
 college are distributed normally with mean 70% and standard deviation 5%.  
 Estimate the number of students whose marks will be i) between 60% and  
 75% ii) more than 75% iii) less than 68%.
- Q5. (a) Find all possible Laurent's series expansion of  $\frac{2z+1}{z^2+5z+6}$  about the origin. 6
- (b) Find the extremals of  $\int_0^1 xy + y^2 - 2y^2 y' dx$ . 6
- (c) Reduce the quadratic form  $x_1^2 - 2x_2^2 + 3x_3^2 + 6x_1x_3 - 4x_2x_3$  to a diagonal 8  
 form using a congruent transformation. Obtain the congruent transformation  
 applied for the reduction. Find the rank, index signature and class value of the  
 quadratic form.
- Q6. (a) An examination of 11 applicant for an accountant post was taken by a finance 6  
 company. The marks obtained by the candidate in reasoning (x) and aptitude  
 (y) test are given below. Calculate the rank correlation coefficient between the  
 performance in the reasoning and aptitude test.  
 $x : 20 \quad 50 \quad 25 \quad 70 \quad 90 \quad 50 \quad 76 \quad 45 \quad 30 \quad 19 \quad 26$   
 $y : 30 \quad 60 \quad 40 \quad 50 \quad 45 \quad 30 \quad 68 \quad 30 \quad 47 \quad 39 \quad 38$
- (b) Evaluate using Cauchy integral formula, 6  

$$\int_C \frac{2z^3 + z^2 + 4}{z^4 + 4z^2} dz, \quad C: |z - 2 - 2i| = 3.$$
- (c) Using Rayleigh-Ritz method, find an approximate solution for the extremal of 8  

$$\int_0^1 2xy - y^2 - (y')^2 dx, \quad y(0) = 0, \quad y(1) = 0.$$