

S.E. ECS / SEM-IV / R-19 / S.H. 2024 / 02-12-2024

Q.P. Code - 69946

(3 Hours)

[Total Marks: 80]

N.B.: 1) Question No. 1 is Compulsory.

2) Answer **any THREE** questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) Verify Cauchy Schwarz inequality for the vectors $u=(2,1,1,-1)$ and $v=(1,-2,1,1)$. Find the angle between vectors u and v . (5)

(b) Find p , q and variance of X , if the mean of the following distribution is 16 and (5)

X	8	12	16	20	24
P(X)	1/8	p	q	1/4	1/12

(c) Evaluate $\int_c \frac{1}{z} dz$ where c is unit circle $|z| = 1$. (5)

(d) Find the extremal of $\int_0^{3\pi/2} (y^2 - y'^2) dx$ where $y(0)=0$; $y(3\pi/2)=1$. (5)

Q.2 (a) Evaluate $\int_c \frac{z+3}{(z-1)(z-4)} dz$, where c is the circle $|z-1|=2$. (6)

(b) Fit the second degree polynomial for following data (6)

X	1	2	3	4	5	6	7
Y	-5	-2	5	16	31	50	73

(c) Transform the basis $\{(1,1,1); (-1,1,0); (1,2,1)\}$ into orthogonal basis using Gram-Schmidt process. (8)

Q.3 (a) Check whether the following sets are subspace of \mathbb{R}^3 (6)

(i) $W = \{a, 0, 0\} / a \text{ belongs to } \mathbb{R}$

(ii) $W = \{(x, y, z) / x^2 + y^2 + z^2 \leq 1\}$

(b) Calculate the rank correlation coefficient 'R' for the following data (6)

x	0	2	8	6	5	20
y	2	8	10	5	40	15

(c) Obtain all possible Laurent's series expansion of $f(z) = \frac{1}{z^2 + 3z + 2}$ about $z = 0$. (8)

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- Q.4 (a) If X and Y are independent random variable with $E(X) = 6$ and $E(Y) = -6$, $V(X) = 4$, $V(Y) = 9$ then find (6)
- $E(2X + 3Y - 2)$
 - $V(3X + 2Y + 2)$

(b) Evaluate $\int_0^{1+i} (x - y + ix^2) dz$ along the line from $z = 0$ to $z = 1 + i$. (6)

- (c) Find rank, index, signature and nature of the Quadratic form by reducing it into Canonical form by congruent transformation $x^2 + 3y^2 + 3z^2 - 2yz$. (8)

- Q.5 (a) Three factories A, B, C produce 30%, 50% and 20% of the total production of an item. Out of their production 80%, 50% and 10% are defective respectively. An item is chosen at random and found to be defective. Find the probability that it was produced by the factory A. (6)

- (b) A continuous random variable has pdf $f(x) = k(x - x^2)$, $0 \leq x \leq 1$. Determine k , mean, and variance of the distribution. (6)

- (c) Using Rayleigh-Ritz method solve boundary value problem (8)

$$\int_0^1 (2x^2y - 4y^2 + y'^2) dx, y(0)=0, y(1)=0.$$

Q.6 (a)

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

 (6)

Calculate the Karl Pearson's coefficient of correlation.

- (b) Find the extremals of $\int_a^b (16y^2 - y''^2 + x^2) dx$. (6)

- (c) Given: $6y = 5x + 90$, $15x = 8y + 130$ are regression lines and $\sigma_x^2 = 16$ then find (i) mean of X and Y (ii) correlation coefficient (r) (iii) σ_y^2 . (8)