

(3 hours)

(Total Marks:80)

N.B: (1) Question no.1 is compulsory.**(2) Attempt any three questions from remaining five questions.****(3) Figures to the right indicate full marks.****(4) Assume suitable data if necessary.**1.(a) Find the Laplace Transform of $f(t) = te^{-4t} \sin 3t$. (05)(b) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, find the eigen values of $A^3 + 5A + 8I$. (05)(c) Find half-range sine series for $f(x) = x, x \in (0, \pi)$. (05)(d) Find the constants a, b, c, d, e if $f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$ is analytic. (05)2.(a) Evaluate $\int_0^{\infty} \frac{e^{-t} \sin t}{t} dt$ using Laplace Transform. (06)(b) Show that the function $v = 3x^2y + 6xy - y^3$ is harmonic and find the corresponding analytic function $f(z)$ in terms of z . (06)(c) Find the Fourier Series for $f(x) = x^2, x \in (0, 2\pi)$ and hence, deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ (08)3.(a) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ hence find A^{-1} and A^4 . (06)(b) If $\vec{F} = (x + 2y + az) i + (bx - 3y - z) j + (4x + cy + 2z) k$ is irrotational, then find a, b, c. (06)(c) Find the orthogonal trajectories of the family of curves given by $x^3y - xy^3 = c$. (08)4.(a) Use Gauss's Divergence Theorem to evaluate $\iint \vec{N} \cdot \vec{F} ds$ where $\vec{F} = 4xi - 2y^2j + z^2k$ and s is the surface of the region $x^2 + y^2 = 4, z = 3$ above xy plane. (06)(b) Find the inverse Laplace Transform of $\ln \left[\frac{s^2 + a^2}{s^2 + b^2} \right]$ (06)(c) Show that the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is diagonalisable. Find the transforming and diagonal matrix. (08)

5.(a) Find the Fourier Series for $f(x) = \begin{cases} x + \frac{\pi}{2}, & -\pi < x < 0 \\ x - \frac{\pi}{2}, & 0 < x < \pi \end{cases}$ (06)

(b) Find $L\{\int_0^t u \sin 4u du\}$ (06)

(c) Find $L^{-1}\left\{\frac{s^2}{(s^2+1)(s^2+4)}\right\}$ using Convolution Theorem. (08)

6.(a) Evaluate by Green's theorem $\int (e^{-x} \sin y dx + e^{-x} \cos y dy)$ along the curve C, (06)
 where C is the rectangle with vertices $(0,0), (\pi, 0), \left(\pi, \frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$.

(b) Find the inverse Laplace Transform of $\frac{s+29}{(s+4)(s^2+9)}$. (06)

(c) Find the eigen values and eigen vectors of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (08)
