

S.E. / COMP / SEM-IV / CBCGS / R-19 / C' Scheme / Sub: EM-IV / S.H. 2024

(Time : 3 Hours)

Date: - 02/12/2024
(Total Marks : 80)

Gr. Code: - 10067953

Note :

- 1) Q. No. 01 is compulsory.
- 2) Solve any three from Q. No. 02 to 06.
- 3) Numbers to the right indicate full marks.
- 4) Use of statistical tables is allowed.

Q. 1. Solve.

- a) If $A = \begin{bmatrix} -1 & 2 & 38 \\ 0 & 2 & 37 \\ 0 & 0 & -2 \end{bmatrix}$ find the Eigen values of $A^3 + 5A + 8I$. 05
- b) Integrate the function $f(z) = x^2 + ixy$ from A(1, 1) to B(2, 4) along $y = x^2$ 05
- c) Find the Z-Transform of $f(k) = a^{-k}$, $k \geq 0$. 05
- d) If a random variable X follows Poisson distribution such that $P(x=1) = 2P(x=2)$. Find mean and variance of the distribution. 05

Q. 2.

- a) Find the Eigenvalues and Eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. 06
- b) Find the Z-Transform of $\cos\left(\frac{\pi}{4} + k\alpha\right)$, $k \geq 0$. 06
- c) Use the dual simplex method to solve the LPP
Min. $Z = 2X_1 - X_2 + 3X_3$,
 $3X_1 - X_2 + 3X_3 \leq 7$, $2X_1 - 4X_2 \geq 12$, $X_1, X_2, X_3, \geq 0$ 08

Q. 3.

- a) Evaluate $\int_C \frac{z+8}{z^2+5z+6} dz$ Where C is a circle $|z|=5$. 06
- b) Verify Caley-Hamilton theorem and hence find A^{-1} and A^4 where $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. 06
- c) Solve the LPP by Big -M method
Max. $Z = X_1 + 2X_2 + 3X_3 - X_4$
 $X_1 + 2X_2 + 3X_3 = 15$, $2X_1 + X_2 + 5X_3 = 20$, $X_1 + 2X_2 + X_3 + X_4 = 10$, $X_1, X_2, X_3, X_4 \geq 0$ 08

Q. 4.

- a) Find inverse Z transform of $F(z) = \frac{1}{(z-2)(z-3)}$ for i) $|z| < 2$, ii) $|z| > 3$. 06
- b) A certain drug administered to 12 patients resulted in the following change in their blood pressure. 5, 2, 8, -1, 3, 0, 6, -2, 1, 0, 4, 5 Can we conclude that the drug increases the blood pressure? 06

- c) Find all possible Laurent's series expansions of the function $f(z) = \frac{1}{(z+1)(z-2)}$ about $z = 0$ indicating the region of convergence in each case. 08

Q. 5.

- a) Determine all basic solutions to the following problem
 Max = $x_1 - 2x_2 + 4x_3$,
 $x_1 + 2x_2 + 3x_3 = 7$, $3x_1 + 4x_2 + 6x_3 = 15$, $x_1, x_2, x_3 \geq 0$. 06
- b) If X is a Normal variate with mean 10 & s.d. 4, find i) $P(5 \leq X \leq 18)$, ii) $P(X \leq 12)$. 06
- c) Solve the NLPP
 Optimize $Z = 12x_1 + 8x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23$ 08
 Subject to $x_1 + x_2 + x_3 = 10$, $x_1, x_2, x_3 \geq 0$.

Q. 6.

- a) Show that the given matrix is diagonalizable and hence find diagonal form and transforming matrix where $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. 06
- b) Based on the following data if there is a relation between literacy and smoking. 06

	Smoking	Non-smoking
Literacy	83	57
Illiteracy	45	68

- c) Max. $Z = 12x_1x_2 + 2x_1^2 - 7x_2^2$, Subject to $2x_1 + 5x_2 \leq 98$, $x_1, x_2 \geq 0$ by K-T condition. 08
