Time: 3 hours **Marks: 80**

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks

Q1 A If $f(t) = \left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^2$, find L[f(t)] and hence find $L\{e^{2t} f(t)\}$ 5

B Find $L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$ 5

C Obtain half-range cosine series for f(x) = x(2-x) in 0<x<2 5

D Find moment generating function of the following distribution. Hence find mean and variance. 5

X	1	3	4	5
P(X)	0.4	0.1	0.2	0.3

Q2 A Find the orthogonal trajectories of the family of curves $e^{-x}[x\sin y - y\cos y] = c$ 6

B Find $L\{t\left(\frac{\cos t}{e^t}\right)^2\}$ 6

C Find the Fourier series expansion for f(x) = 2, -2 < x < 0. 8
=0, 0 < x < 2

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

Q3 A Find $L^{-1}\{\log(1 - \frac{1}{s^2})\}$ 6

B Find the analytic function f(z)= u + iv where $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$, using Milne-Thompson's Method. 6

C Fit a parabola $x = a + by + cy^2$ for the following data: 8

X :	1	2	3	4	5
Y :	10	12	15	14	15

Q4 A The first 4 moments of a distribution about origin of the random variable X are -1.5, 17, -30 and 108. Compute Mean, variance, μ_3 and μ_4. 6

B Consider the equations of regression lines 5x-y=22 and 64x-45y=24. Find $\bar{x}$, $\bar{y}$ and correlation coefficient r. 6

C Find $L^{-1}\{\frac{(s+3)^2}{(s^2+6s+13)^2}\}$ 8

Q5 A Find the Laplace transform of $\cos^3 t \cos 5t$. 6

B Find Spearman's rank correlation coefficient for the data below: 6

X :	32	55	49	60	43	37	43	49	10	20
Y :	40	30	70	20	30	50	72	60	45	25

C Obtain Fourier Series for $f(x) = \frac{1}{2}(\pi - x)$ in $(0, 2\pi)$.

Hence, deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ 8

Q6 A If f(x) is probability density function of a continuous random variable X, find k, mean and variance. 6

$$f(x) = \begin{cases} kx^2, & 0 \le x \le 1 \\ (2-x)^2, & 1 \le x \le 2 \end{cases}$$

B Check if there exists an analytic function whose real part is $u = \sin x + 3x^2 - y^2 + 5y + 4$. Justify your answer. 6

C Evaluate the following integral by using Laplace transforms 8

$$\int_0^\infty e^{-2t}\left[\int_0^t \left(\frac{e^{3u}\sin^2 2u}{u}\right)du\right]dt$$
