

Duration: 3 Hours

Total Marks: 80

- N.B. :** 1) Question Number 1 is compulsory  
 2) Solve any three questions from the remaining questions  
 3) Make suitable assumptions if needed  
 4) Assume appropriate data whenever required. State all assumptions clearly.

**Q.1**

**Solve any four of the following questions.**

- a.** What is a tautology? Check whether the following logical expression is tautology? 5  
 $[(p \rightarrow r) \wedge (\sim q \rightarrow p) \wedge \sim r] \rightarrow q$
- b.** State the Pigeonhole principle and show that if any five numbers from 1 to 8 are chosen, then two of them will add to 9. 5
- c.** Convert the following into CNF form. 5  
 $(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$
- d.** Given  $S = \{1, 2, \dots, 10\}$  and a relation  $R$  on  $S$ , where  $R = \{(x, y) \mid x + y = 10\}$ . Is it reflexive, symmetric, transitive, antisymmetric? 5
- e.** Define the following terms 5  
 1. Planer graph 2. Cut Vertex 3.Chain 4. Monoid 5.Group

**Q.2**

- a.** Let  $A = \{p, q, r, s\}$  and let  $R = \{(p, p), (p, q), (p, r), (q, p), (q, q), (r, p), (q, r), (r, q), (r, r), (s, s)\}$ . Show that  $R$  is an equivalence relation . Determine the equivalence classes and find the rank of  $R$ . 8
- b.** Show that  $A = \{0, 3, 6, 9, 12\}$  is a ring w.r.t. the operation of addition & multiplication modulo 15. 8
- c.** Explain two different types of Quantifiers with example? 4  
 Represent the following sentences using First Order logic  
 i) Some students took GenAI.  
 ii) Every student who takes GenAI passes it.

**Q.3**

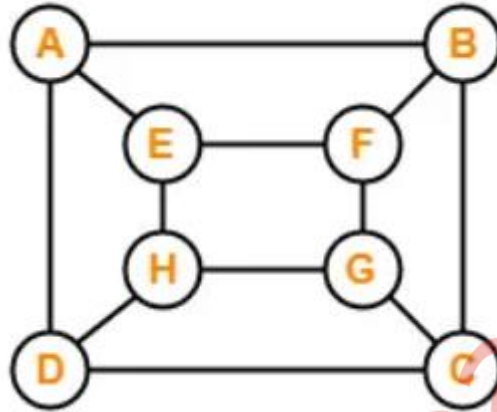
- a.** What is an Abelian Group? Let  $G = \{1, 2, 3, 4, 5, 6, 7\}$  8  
 i) Prepare the composition table w.r.t the operation of multiplication modulo 8.  
 ii) Check whether it is an Abelian group? Justify your answer.
- b.** Define minimum hamming distance. Find the code words generated by the parity check matrix  $H$  given below. 8

$$H = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

c

4

Determine whether the following graph is Eulerian or Hamiltonian or both. Justify your answer.



Q.4 a

Define function. What are three different types of functions.. Consider the function  $f$  and  $g$  from  $N \times N$  to  $N$  given by  $f(x,y) = 2x+y$  and  $g(x,y) = xy$ , identify its type.

8

b

Let  $A = \{a,b,c,d,e\}$  and let  $R$  be a relation on  $A$ .

8

Let  $R = \{(a,a), (a,c), (b,b), (c,d), (c,e), (d,a), (e,b), (e,e)\}$

Compute transitive closure using Warshall's algorithm

c

Prove using Mathematical Induction that sum of cubes of three consecutive integers is divisible by 9.

4

Q.5 a

Let  $X = \{1,2,3,4,6,24,36,72\}$  and  $R = \{(x,y) \in R \mid x \text{ divides } y\}$

8

i) Write the pairs in a relation set  $R$ .

ii) Construct Hasse diagram.

iii) Mention Chains and Anti Chains from above set.

iv) Is it a lattice?

b

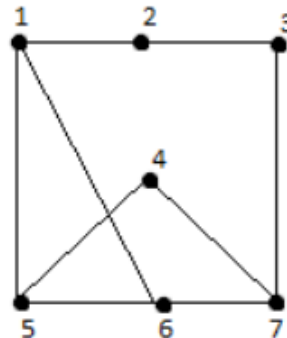
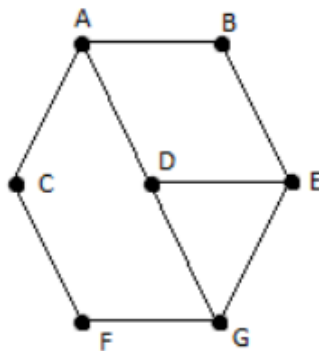
Find the number of integers between 1 to 500 that are not divisible by 5,6, or 8?

8

c

Check whether the following graphs are Isomorphic or not? Justify

4



- Q.6.**
- a** Draw the Hasse Diagram of  $D_{72}$  **8**
    - i) Find the complement of each element
    - ii) Check whether it is a Distributive Lattice
  - b** Let  $f(x) = x + 3$ ,  $g(x) = x - 3$  and  $h(x) = 3x$  for  $x \in \mathbb{R}$ , where  $\mathbb{R}$  is the set of real numbers. **8**
    - Find i)  $g \circ h$  ii)  $f \circ g$  iii)  $g \circ h \circ f$  iv)  $f \circ h \circ g$
  - c** Find the generating functions for the following sequences: **4**
    - a. 0, 0, 0, 1, 2, 3, 4, 5, 6, 7, .....
    - b. 6, -6, 6, -6, 6, -6, .....

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