SE|comp|sem-III/CBCGS|R-19|C-scheme|Engg.Math-III|SH-24

(Time: 3 Hours) (Total Marks: 80)

QP-1006545

N.B. **(1) Question No. 1 is compulsory.**
(2) Answer any three questions from Q.2 to Q.6.
(3) Use of Statistical Tables permitted.
(4) Figures to the right indicate full marks.

Q1. (a) Find the Laplace transform of $t\,e^{-t}\cosh 2t$ **[05]**

(b) If $u = -r^3 \sin 3\theta$ find the analytic function f (z) whose real part is u. **[05]**

(c) Calculate the Spearman's rank correlation coefficient R

x	85	74	85	50	65	78	74	60	74	90
y	78	91	78	58	60	72	80	55	68	70

[05]

(d) Find inverse Laplace transform of $\frac{1}{s}\log\left(1+\frac{1}{s^2}\right)$. **[05]**

Q2. (a) Evaluate by using Laplace transform of $\int_0^\infty e^{-2t}\frac{\cos 2t \sin 3t}{t}\,dt$. **[06]**

(b) Find the value of k if the function $f(x) = k\,x\,e^{\frac{-x}{3}}$, $x > 0$

$f(x) = 0$ $0 \le x$.

Is a probability density function. find mean and variance. **[06]**

(c) Obtain the Fourier series to represent $f(x) = \frac{3x^2 - 6x\pi + 2\pi^2}{12}$ in $(0, 2\pi)$

Hence show that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \ldots\ldots$ **[08]**

Q3. (a) Find the analytic function whose real part is $u = e^{2x}(x\cos 2y - y\sin 2y)$. **[06]**

(b) Obtain the Fourier series to represent $f(x) = x - x^2$, $-1 \le x \le 1$. **[06]**

(c) Using convolution theorem Find inverse Laplace transform of $\frac{(s+3)^2}{(s^2+6s+18)^2}$. **[08]**

Q4. (a) Obtain the half range cosine series of $f(x) = x(\pi - x)$ in $(0, \pi)$

Hence show that $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} \ldots\ldots$ **[06]**

(b) Find the lines of regression and coefficient of correlation for the data **[06]**

x	65	66	67	67	68	69	70	72
y	67	68	65	66	72	72	69	71

(c) Evaluate by using Laplace transform of $\int_0^\infty e^{-t}\left(\int_0^t u^2 \sin hu \cos hu\,du\right)dt$ **[08]**

Q5. (a) Find the orthogonal trajectories of family of curves $e^{-x}\cos y + x y = \alpha$ where α is the real constant in the x y - plane. **[06]**

(b) A random variable x has the probability distribution [06]

x	0	1	2	3
P (x =x)	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

Find the moment generating function about origin. also find mean and variance.

(c) Fit a second degree parabolic curve to the following: [08]

X year	1965	66	67	68	69	70	71	72
Y profit	125	140	165	195	200	215	220	230

Also estimate the profit in 1973

Q6. (a) Find inverse Laplace transform of $\frac{(2s^2 - 6s + 5)}{(s^3 - 6s^2 + 11s - 6)}$ [06]

(b) Show that the function $v = e^x$ (x sin y + y cos y) satisfies Laplace equation And find its corresponding analytic function and its harmonic conjugate. [06]

(c) A random variable X has the probability function [08]

X	1	2	3	4	5	6	7
P(X = x)	K	2K	3K	K^2	$K^2 + K$	$2K^2$	$4K^2$

Find k , p (X < 5) , p (x > 3), P (0 ≤ X ≤ 5).

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