

Time:3 Hrs

Marks:100

N.B. : (1) Question Number 1 is compulsory

2)Solve any three questions from the remaining questions

3)Make suitable assumptions if needed

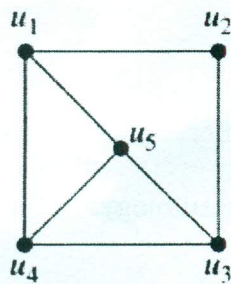
4)Assume appropriate data whenever required. State all assumptions clearly.

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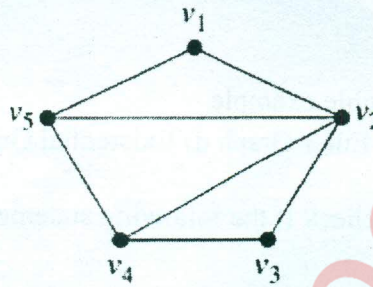
1. a. Define the following with suitable example 5
a) Power Set b) Group c) Euler Graph d) Existential Quantifier
- b. Construct the Truth Table and check if the following statement is tautology. 5
 $(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$
- c. For all sets A, B and C show that 5
 $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- d. Prove by mathematical induction that 5
 $1.1! + 2.2! + 3.3! + \dots + n.n! = (n+1)! - 1$
- 2 a. Define Equivalence Relation. Let A be a set of integers, Let R be a Relation on AXA defined by (a,b) R (c,d) if and only if $a+d = b+c$. Prove that R is an Equivalence Relation 8
- b. Let $A = \{a, b, c, d\}$ Find Transitive Closure of R represented by M_R using Warshall's algorithm. 8
- $$M_R = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
- c. Prove that the set $A = \{0,1,2,3,4,5\}$ is a finite Abelian group under Addition modulo 6 4
- 3 a. Let f,g,h be functions on real numbers R defined as follows: 8
 $f(x) = 2x+5, g(x) = 5x+3, h(x) = 3x$
Find: 1) $g \circ f$ 2) $g \circ h$ 3) $f \circ g \circ h$ 4) $g \circ f \circ h$

- b Give the exponential generating function for the sequences 8
- 1) $\{1, 1, 1, \dots\}$
 - 2) $\{1, 2, 3, 4, \dots\}$
 - 3) $\{1, a, a^2, a^3, \dots\}$

- c Determine whether the following graphs are isomorphic. Justify your answer. 4



G1



G2

- 4 a A Function 8
 $f: \mathbb{R} - \{\frac{2}{5}\} \rightarrow \mathbb{R} - \{\frac{4}{5}\}$ is defined as $f(x) = \frac{4x+3}{5x-2}$

Prove that f is Bijective and find the rule for f^{-1}

- b Show that (2,5) encoding function $e: B^2 \rightarrow B^5$ defined by 8

$$e(00) = 00000$$

$$e(01) = 01110$$

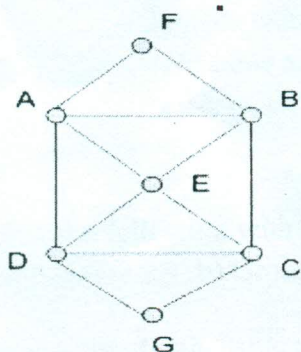
$$e(10) = 10101$$

$$e(11) = 11011$$

is a group code.

- c Find the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3, 4 or 5.

- 5 a Define Euler Path, Euler Circuit, Hamiltonian Path and Hamiltonian Circuit. 8
 Determine if the following diagram has Euler Path, Euler Circuit, Hamiltonian Path and Hamiltonian Circuit and state the path /circuit.



- b State and explain the extended Pigeonhole principle. How many friends must you have to guarantee that at least five of them will have birthdays in the same month. 8
- c Find the complement of each element in D_{42} 4
6. a Draw the Hasse Diagram of D_{72} and check whether it is a Lattice. 8
- b Find the complete solution of $a_n + 2a_{n-1} = n + 3$ for $n \geq 1$ with $a_0 = 3$ 8
- c Define the following with suitable examples. 4
- a) Maximal and Minimal Element b) Partition of a set c) Sub Lattice d) Injective Function
