

SE Comp-IV 'c'

23.11.2023

Time: 3 Hours

Total Marks: 80

N.B.

- 1) Q.1 is compulsory.
- 2) Solve any 3 questions out of remaining 5 questions.
- 3) Assumptions made should be clearly stated.
- 4) Draw the figures wherever required.

Q.1 Solve any four of the following questions.

- a) Prove using Mathematical Induction that $2 + 5 + 8 + \dots + (3n-1) = n(3n+1)/2$ 5
- b) Explain the term poset. Consider a set D_{165} . Find the elements of this set & draw the hasse diagram for this poset. 5
- c) How many strings of length 7 either begin with 2 zeros or end with 3 ones? 5
- d) Explain the term partition set with suitable example. 5
- e) State the Pigeonhole principle and show that If there are 10 marbles in the jar & you have a jar filled with red, green, and blue marbles, you'll always have at least two marbles of the same colour. 5

Q.2

- a) Let $A = \{0, 1, 2, 3, 4, 5\}$ 10
 - i) Explain the term group.
 - ii) Prepare the composition table for the above set w.r.t. the operation of addition modulo 6.
 - iii) Determine whether it is a group.
 - iv) Whether elements of set A are invertible? If yes, then find the inverses of these elements.
 - v) Determine whether it is a cyclic group.

b

Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ and let R be a relation on A whose matrix is :

10

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find out transitive closure of R using Warshall's algorithm.

Q.3

- a) A large software development company employs 100 computer programmers. Of them, 45 are proficient in Java, 30 in C#, 20 in Python, six in C# and Java, one in Java and Python, five in C# and Python, and just one programmer is proficient in all three languages above.

SE Comp III (e)

23.11.2023

82

Determine the number of computer programmers that are not proficient in any of these three languages. 8

b) Explain the terms Conjunctive & Disjunctive Normal Form with suitable examples. 6

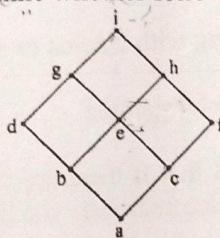
c)

6

Determine the sequence b_n whose recurrence relation is $b_n = 2b_{n-1} + 1$ with initial condition $b_1 = 7$.

Q.4

a) What is a lattice? Determine whether following hasse diagram represents a lattice. 6



b)

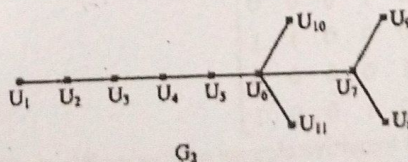
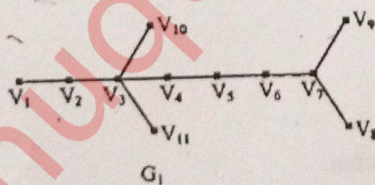
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Consider $(3, 8)$ an encoding function $e : B^3 \rightarrow B^8$ defined as

$e(000) = 00000000$
 $e(001) = 10111000$
 $e(010) = 00101101$
 $e(011) = 10010101$
 $e(100) = 10100100$
 $e(101) = 10001001$
 $e(110) = 00011100$
 $e(111) = 00110001$

How many errors can 'e' detect & correct?

c) What are the necessary conditions for the isomorphism between 2 graphs? Determine whether following 2 graphs are isomorphic. 8



Q.5

a) If the addition & multiplication modulo 10 is defined on a set of integers $A = \{0, 2, 4, 6, 8\}$. Then determine whether this algebraic system is a ring. 8

SE Comp III

C'

23.11.2023

82

b)

A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^2$

Is it i) injective

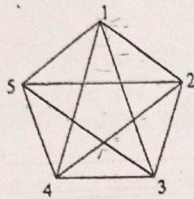
ii) surjective

iii) bijective

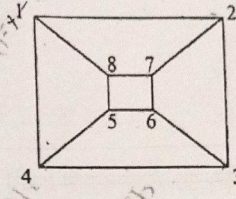
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c) Define the terms Euler path & a circuit.

Determine whether following graphs have Euler path or a circuit.



a)



b)

Q.6

a) Explain the following terms with suitable example (any 4)

- Hamiltonian path & circuit
- Bipartite graph
- Adjacency matrix
- Equivalence relation
- Cartesian product

8

b) Solve the following using the laws of logic

$$p \vee q \vee (\sim p \wedge \sim q \wedge r) \leftrightarrow p \vee q \vee r$$

4

c)

$f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = x^3$

$g: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $g(x) = 4x^2 + 1$

$h: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $h(x) = 7x - 2$

Find the rule defining

i) fog

ii) gof

iii) (goh)of

iv) go(hof)

8