TIME: 03 HOURS **MAX. MARKS : 80**

Note:

1. Question **No. 1** is compulsory.
2. Attempt **any three** questions out of remaining **five** questions.
3. Assume suitable data wherever necessary.
4. Figures to right indicate full marks.

Q.1 Answer the following Marks

a. Find the Laplace transform of $\frac{\cos 2t \sin t}{e^{-t}}$ 05

b Calculate Karl Pearson's coefficient of correlation r between x and y. 05

X : 28 , 45 , 40, 38 , 35 , 33 , 40 , 32 , 36, 33

Y : 23 , 34 , 33, 34, 30 , 26 , 28, 31, 36, 35.

c. Find the constants a, b, c,d,e if f (z) = $(ax^3 + bxy^2 + 3x^2 + cy^2 + x)$ + i $(dx^2y - 2y^3 + e\,x\,y + y)$ *is analytic function* 05

d. Find inverse Laplace transform of $\log\left(1 + \frac{a^2}{s^2}\right)$ 05

Q.2 a Prove that $\int_0^\infty e^{-t}\left(\frac{\sin 2t + \sin 3t}{t}\right)$ dt = $\frac{3\pi}{4}$. 06

b. The probability density function of a random variable X is 06

X	0	1	2	3	4	5	6
P(X=x)	k	3k	5k	7k	9k	$11k$	$13k$

Find (i) k (ii) p($X < 4$) (iii) p ($3 < X \leq 6$).

c Find the Fourier Series to represent f (x) = x^2 in (0, 2π) . 08

Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2}$ + - - - -

Q.3 a Find an analytic function f (z) = u + iv where 06

u = $\frac{x}{2}$ log ($x^2 + y^2$) – y $tan^{-1}\left(\frac{y}{x}\right)$ + sin x cos hy.

b. Given 6 y = 5 x + 90 , 15 x = 8 y + 130,$\sigma^2 x$ = 16 find 06

(i) mean of x and y (ii) r (iii)

σy.

c. Using convolution theorem Find the inverse Laplace transform of 08

$\frac{s+3}{(s^2 + 6s+13)^2}$.

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Q.4 a Obtain Fourier series of f (x) = 1 - x^2 in $((-1,1)$. 06

b A random variable X has the probability distribution. b

X	0	1	2	3
P(X =x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Find the moment generating function of Xand then find mean and variance.

c. Evaluate by using Laplace transforms of 8

$\int_0^\infty e^{-t}$ ($\int_0^t u^2 \sin hu \cos hu\, du$) dt .

Q.5 a Find the orthogonal trajectories of family of curves 06

$3x^2y + 2x^2 - y^3 - 2y^2 =$ c.

b Find the inverse Laplace transform of $\frac{s+29}{(s+4)(s^2+9)}$. 06

c. Fit a second-degree parabolic curve to the following data and estimate the production in 1982 08

Year x	1974	1975	1976	1977	1978	1979	1980	1981
Production (y)	12	14	26	42	40	50	52	53

Q.6 a Obtain half range Cosine series for f (x) = x(π- x) in $0< x < \pi$. 06

Hence show that $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4}$ ……..

b Show that the function v = e^x(x sin y + y cos y) satisfies Laplace equation 06

and its corresponding analytic function and its harmonic conjugate.

c Find the value of k if the function f (x) = k $x^2(1-x^3)$, $0 \le x \le 1$ 08

f (x) = 0 , otherwise .

is a probability density function. Also find p ($0 \le x \le \frac{1}{2}$) ,find mean and variance.

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